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Preschoolers' practical thinking and problem solving: The acquisition of an optimal solution strategy

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PRESCHOOLERS' PRACTICAL THINKING AND PROBLEM SOLVING: THE ACQUISITION OF AN OPTIMAL SOLUTION STRATEGY

by

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Abstract

PRESCHOOLERS' PRACTICAL THINKING AND PROBLEM SOLVING:

THE ACQUISITION OF AN OPTIMAL SOLUTION STRATEGY

by

Michael Cohen

Advisor: Professor Katherine Nelson

This study was an investigation of pre-school aged childrens' (age 3 and 4) problem solving strategies that are used to accomplish a practical, mathematically oriented task. Twenty-four pre-school aged children engaged, on three separate occasions, in a mathematically oriented task which consisted of satisfying customer requests for orders of vegetables in a 'play' store setting. Participants also participated in an extensive arithmetic pre and post test. Each order request in the task could be successfully satisfied by a variety of different solution strategies. The number of moves used to satisfy each order, as well as the strategy type employed, was recorded and coded.

Results indicate that participants became significantly more efficient in their solution strategy choice upon repeated exposure to the task. That is, children increasingly chose solution strategies that required fewer physical moves. In order to choose more efficient strategies, children engaged in more abstract arithmetic over the course of the three task trials. In addition, it was determined that children, both individually, and as a group, used a complex repertoire of strategies and these

strategies were identified. Particularly, it was determined that preschoolers acquired and used optimal solution strategies.

The results of this research led to the general conclusion that increased efficiency and the increased use of more complex and sophisticated mental activity is a feature of practical thinking. The increased efficiency results in a decrease in physical moves. The results also confirm prior research findings that pre-school children display a repertoire of strategy solutions and are engaged in sophisticated arithmetic mental activity when such activity functions in the service of successfully completing a larger purposeful goal.

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I would also like to thank Katherine Nelson, my current advisor. Katherine assumed the advisor's role with grace and made a potentially difficult transition not only painless, but pleasurable and rewarding. Katherine's input and guidance throughout the experimental and analytic phases of this research was invaluable. Her personal support certainly made the completion of this research possible.

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TABLE OF CONTENTS

Chapter I

Introduction 1

A. Practical Intelligence within Contemporary Theories of Overall Human Intelligence 3

B. Definitions of Practical Thinking and Identifications of Unique Features 11

C. Practical Problem Solving and Practical Problem Solving Solution Strategies 21

D. Children's Practical Arithmetic 32

Chapter II

Research Goals 40

Chapter III

Method 48

A. Participants 48

B. Pre-test Procedure, Materials and Coding 48

C. Task Procedure and Materials and Coding 51

D. Post-task Interview Procedure, Materials and Coding 57

E. Post-Test Procedure, Materials and Coding 58

Chapter IV

Results 59

Chapter V

Discussion 76

Appendix A: Pretest 94

Appendix B: Task - Overall Design 97

Appendix C: Ordering and Array Placement of Task Problems by
Participant Group 98

Appendix D: Task Problems 103

Appendix E: Possible Strategies and Number of Moves By Problem 109

Appendix F: Post-task Interview Problems 111

Appendix G: Post-task Qualitative/Probe Interview 112

Appendix H: Pre-test to Task Correlations for All Participants 113

References 114

LIST OF TABLES

Table 1 Mean Numbers (and Standard Deviations) of Moves for All Participants (N=24) by Day (Trial) for 12 Problems of Three Arrays 60

Table 2 Mean Numbers (and Standard Deviations) of Moves for all Participants (N=24) by Day (Trial) for Three Problems of Two Arrays 61

Table 3 Mean Numbers (and Standard Deviations) of Moves for all Participants (N=24) by Day (Trial) for 12 Problems of Three Arrays 62

Table 4 Mean Numbers (and Standard Deviations) of Moves and Rate of Improvement for all Participants (N=24) for Individual Problem Types (Difference between Day 3 and Day 1) 65

Table 5 Mean Numbers (and Standard Deviations) of Rate of Improvement for all Participants (N=24) for Order Request (Difference between Day 3 and Day 1) 66

Table 6 Mean Numbers (and Standard Deviations) of Moves Used to Solve 2 Array Problems and 3 Array Counterparts for All Participants (N=24) by Day 67

Table 7 Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Optimal" Strategy Solution by Day 69

Table 8 Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "In-Between" Strategy Solution by Day 70

Table 9 Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Literal" Strategy Solution by Day 71

Table 10 Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Other" Strategy Solution by Day 71

LIST OF FIGURES

Figure 1 Comparison of the Mean Scores and Rate of Improvement by Gender 64

CHAPTER I

INTRODUCTION

A growing body of inquiry has focused attention on practical intelligence, practical thinking, and practical problem solving, as evidenced in everyday life (e.g., Berg & Sternberg, 1985; Rogoff, 1982; Sternberg, 1986). The generally shared view of researchers who adhere to a contextual model of intellectual development is that such mental activity functions in the service of accomplishing a practical goal (e.g., Goodnow, 1986; Scribner, 1986; Wertsch, 1985). It is this goal-directed characteristic that is used most often as a primary criterion for defining practical intelligence and thinking and, by extension, for differentiating it from other types of intellectual activity, particularly academic intelligence. Furthermore, researchers have attempted to delineate the full range of the unique features that comprise practical thinking and to offer a comprehensive contrast of practical thinking with other forms of intellectual activity (e.g., Scribner, 1986; Sternberg, 1986).

Most research has focused primarily on practical and everyday intelligence as evidenced in adult activity (e.g., Cornelius & Caspi, 1987). Recently, scholars have begun to display interest in practical intelligence as evidenced in children. A significant proportion of child-oriented inquiry focuses on the identification of practical skills in cross-cultural settings (e.g., Berry, Dasen & Witkin, 1982), the identification of children's practical problem-solving abilities and the specific strategies that children employ (e.g., Berg, 1989; Rogoff, Gauvain & Gardner, 1987), and the impact of social influences on children's practical thinking (e.g., Perlmutter, Behrend, Kuo & Muller, 1989). With few exceptions, investigators have not explored the psychological processes involved in the development of the young child's practical intelligence, practical thinking, and practical problem-solving abilities.

The first goal of the present research is the identification of specific problem-solving strategies that preschoolers use to accomplish a practical, mathematically-oriented, goal-directed task. By doing so, this research will add to the growing body of literature aimed at the further identification of the range of young children's problem-solving behavior. The second goal is to identify the pattern of solution strategy selections over the course of several trials of a practical task. The third goal of the present research is to specifically explore whether preschoolers engage in the mental reorganization of activity that results in the use of a physical effort saving solution strategy. Such strategies have been identified as an important feature of adult practical activity (Scribner, 1988, 1984), but research has not attempted to identify its appearance in early childhood. The fourth goal of the current research is to identify the correspondence between specific features of the young child's emerging practical arithmetic skills and the child's acquisition of a least-effort solution strategy. Here, the current research is designed to address one aspect of the development of practical thinking in the young child.

PRACTICAL INTELLIGENCE AND CONTEMPORARY THEORIES OF OVERALL HUMAN INTELLIGENCE

Practical thinking has become a legitimate subject for psychological inquiry, in part because several investigators have postulated new theories of human intelligence stressing the necessity of locating and identifying aspects of intelligence that are currently unmeasured by standard psychometric testing (e.g., Berg & Sternberg, 1985; Ceci & Liker, 1986; Sternberg, 1985; Walters & Gardner, 1986). These theorists hold the view that only one type of intelligence, characterized as academic intelligence, is measured by the tasks typically found on standard psychometric tests (e.g., I.Q. tests) and in school settings. These investigators, who have postulated new theories of human intelligence, have most notably identified practical intelligence as one of several types or aspects of intelligence that are currently unmeasured. Although the various theories differ on many counts, practical intelligence is the one 'new' significant type/aspect that is included, in common, as an addition. The tasks that measure academic intelligence are typically characterized as: 1) being formulated by others; 2) having little intrinsic interest; 3) having all necessary information available from the onset; 4) being disembedded from an individual's ordinary experience; 5) being well-defined; 6) having one correct answer; and 7) often having one method of correct solution. Tasks for assessing practical intelligence differ on many of these characteristics.

Sternberg (1985; Berg & Sternberg, 1985) proposed a theory of intelligence that has attracted particular interest and generated subsequent investigation. Sternberg introduced a "triarchic" theory, positing human intelligence to be comprised of three distinct and identifiable (yet interrelated) aspects, parts and/or types. Furthermore, Sternberg grants all three parts equal psychological valance.

The first type of intelligence in Sternberg's theory concerns those mental processes that underlie human intelligent behavior. These processes closely resemble a classic definition of componential intelligence or analytic thinking. This aspect of intelligence is, itself, composed of three mental subprocesses (or components) including: 1) metacomponents, which are the executive processes that coordinate the various procedures used to solve problems and function by planning, monitoring and evaluating problem-solving strategies (i.e., defining the problem; selecting a mental representation; allocating mental resources); 2) performance components, which implement the plans formulated by the metacomponents including inferential processes (e.g., inferring relations between stimuli, applying previously inferred relations to new stimuli, identifying inferential fallacies, etc.) and are commonly manifest and measured in intelligence tests in the forms of verbal and figural analogies, series completion, and classification items; and, 3) knowledge acquisition components, which include all processes that enable people to acquire new information and are used to determine meaning (i.e., selective encoding [locating relevant information]; selective combination [combining information into a meaningful whole]; and selective comparison [interrelating new information with what is already known]).

The second type of intelligence in Sternberg's theory concerns the roles of novelty and atomicity in intelligent behavior and the mental processes involved. Identified "experiential," this type of intelligence is commonly manifest in the ability to acquire knowledge and construct insights when confronted with novel information and/or experiences, the capacity to combine knowledge from seemingly disparate experiences, the application of insight to novel (and/or relatively new) problems, and the capacity to perform operations smoothly without the expenditure of significant conscious effort. Sternberg identifies insight as the most critical developmental skill for confrontation with novelty.

Part three of Sternberg's theory is concerned with the critical importance of the context in which intellectual processes operate. Sternberg identifies an intelligence which is goal-oriented and is inseparable from the context in which it functions. This third type of intelligence includes all mental process operating and functioning in the service of the manipulation of the environment in order to successfully achieve certain ends. This practical intelligence, therefore, includes all mental processes and skills involved and manifest in activities including: adapting to existing environments; shaping existing environments; and selecting new environments.

Overall, Sternberg's theory can be criticized for a variety of failings, most notably for an unsatisfactory identification and exploration of the complex interrelations of the three parts of his theory and for the inclusion of incomplete definitions of several critical phenomena and constructs (e.g., insight and context). However, independent of the theory's shortcomings, Sternberg's contribution offers a new and significant perspective in regard to an overall conception of human intelligence. The identification and inclusion of a 'new' type of intelligence, characterized by mental activity which is functionally goal-directed in 'real-world' contexts, can be historically viewed as a result of the increasing interest in, and a further 'legitimizing' of, psychological investigation of human practical thinking.

Other theorists have also offered general models of intelligence which have included practical intelligence (or its significant features) as distinguishable and significant forms of mental activity. Walters & Gardner (1986) also argue against a unitary notion of intelligence

and offer a theory of multiple human intelligences that differs from Sternberg's. Their theory proposes that overall human intelligence is comprised of seven separate domains. An "intelligence" is defined as such by satisfying a constellation of five criteria, including: the existence of an ability (or set of abilities) that permits an individual to both solve problems and construct products that are of consequence within a particular cultural setting; the evidence of a biological origin; the existence of an identifiable core operation or set of operations; susceptibility to encoding within a symbol system; and being a skill universal to the human system. Using these criteria, Walters & Gardner identify seven distinct human intelligences which together comprise all human intelligence. These are: musical intelligence; body-kinesthetic intelligence; logical-mathematical intelligence; linguistic intelligence; spatial intelligence; interpersonal intelligence; and intrapersonal intelligence.

This theory can also be criticized on a variety of levels (e.g., lack of a comprehensive discussion of, and a model for, both the relationships between intelligences and explanations of the structure and function of each individual intelligence). However, aside from its shortcomings, the theory is partially based on the hypothesis that practical activity and context are defining central features of all human intelligence. Although Walters and Gardner do not posit practical intelligence as a separate form of intelligence per se, it can be argued that their theory postulates all human intellectual activity to be, in essence, practical, since the features of contextual location and goal-directed mental activity are considered defining criteria of all intelligence.

Ceci & Liker (1986) present a contextual model of intelligence stressing the separation of academic from non-academic intelligence. Akin to the investigators cited above, they, too, argue against a unitary theory of human intelligence. I.Q. test performance, in their view, is indicative of intelligence only to the extent that skills required to meet an individual's important environmental challenges overlap with those skills required to meet academic ones (including I.Q. test performance itself). Intelligence develops only in context and appears to be domain-dependent. In an experimental study, expert horse-race handicappers demonstrated a significantly higher degree of cognitive complexity and sophistication, including abstract reasoning, than did counterpart novices with significantly higher I.Q.s. For the purposes of this discussion, the significance of Ceci and Liker's work is the emphasis placed on the functional goal-directed nature and the centrality of context, in regard to intellectual development.

Goodnow (1986) also argues against traditional unitary theories of intelligence and for the inclusion of an everyday or practical intelligence into an overall model. At the core of Goodnow's theory is a theoretical move away from a concept of intelligence as an absolute, towards a notion of intelligence that is relativistic and socially defined. Goodnow identifies the psychological processes of organizing and reorganizing to be the critical features for defining and understanding practical intelligence and its development. Goodnow also stresses that inquiry into the processes of mental organization and reorganization of a task facilitates an understanding of both practical thinking and problem solving as well as offering the vehicle for inquiry without the exclusive reliance on qualitative and/or ethnographic methods.

Doise (1984) argues for a social definition and a social origin for all human intelligence. Doise's argument for a social genesis for human intellectual activity is explicitly located in the tradition of Mead (1934) and Vygotsky (1962). Central to this definition of social is a construct broader than literal interpersonal activity. Within this social construct child development occurs as an ongoing dialectical relationship with the world (social and physical environment) characterized by an ever present and ongoing dynamic of mediation. All of the child's activity is mediated by socially constructed symbol systems and tools. For Mead, the process of the internalization of the external mediating symbol systems, which exist in the child's immediate environment, is critical to the development of thinking. Vygotsky claimed that every function in the child's development appears twice, first between people (interpsychologically) and then inside the child (intrapsychologically). Simply put, development is partially based on two key dynamics: all human activity is mediated by socially constructed symbol systems and tools; and the process of development is characterized by the child's active internalization of socially constructed forms. This process that Vygotsky detailed, and which is incorporated by Doise, applies to all higher intellectual functions. Therefore, all the higher functions of human mental activity originate as actual relations between humans. It is important to note, however, that such relations are, by definition, not limited to the interpersonal. Given that the mediating symbol systems are themselves socially constructed and then internalized by the child, by extension all mental activity originates as social activity. This is a concept of "social" far more expansive than that of literal and distinctly identifiable interpersonal interactions.

Doise's assertion of a socially defined, socially genetic, human intelligence leads to a conclusion that human mental activity is, significantly (if not completely), functionally goal-directed. Doise argues that since human intelligence is socially defined and developed, intelligence can therefore be understood by its functional capacity to successfully adapt to physical and social environments. Doise elevates practical intelligence on the basis of its critical functional performance. Originating from social phenomena and subsequently internalized by the individual, practical intelligence functions to successfully satisfy environmental and social goals.

Ford (1989) locates practical intelligence as an identifiable subset of the larger domain of human intelligence. Ford defines all intelligence as goal-directed and contextual. For Ford, practical intelligence is differentiated, not by its goal-directed characteristic, but rather by its transactional nature and a social determination of the value of goals.

All of the above theories can be criticized on a variety of levels. However, what is shared by all is a significant theoretical shift that includes, within broader theories of human intelligence, an orientation towards an acknowledgment of the significance of practical intelligence. This orientation holds true whether any one particular theory identifies practical intelligence as an identifiable sub-type of intelligence or as a defining criteria for all intelligence. For the purposes of this discussion, the significant feature of these theories are both the critical importance placed on goal-directed mental activity functioning in identifiable contexts and the identification of such an intelligence functioning to successfully accomplish a goal within a specific contextual domain and which differs from, and is contrasted to, academic intelligence. Furthermore, these contemporary theories of human intelligence either imply, or explicitly state, that the development of a practical intelligence may very well proceed differently than former models of intelligence have postulated.

As stated, the theories outlined here break down into two types. Some of the theories hold that all intelligence is essentially social, goal-directed and contexted and that, currently, intelligence has been misspecified as the special sub-type 'academic.' Other theories hold that practical intelligence is a legitimate sub-type of intelligence and has only recently been properly identified and investigated. These two points of view are not mutually exclusive. Intelligence can be conceived as social, contexted and goal-directed and a variety of sub-types can exist. The sub-type of practical intelligence to be explored in the current research refers to mental activity that is embedded in life's daily goal-directed activities and, as a result, is qualitatively different than mental activity undertaken as an end in itself. This definition exists within an overall understanding that intelligence, in general, is social, purposeful and contexted.

DEFINITIONS OF PRACTICAL THINKING AND IDENTIFICATIONS OF UNIQUE FEATURES

Many investigators have explored significant and comprehensive issues in regard to practical intelligence and practical thinking outside the context of complete models of human intelligence. These investigators have attempted a comprehensive definition of practical thinking as well as an identification of its unique features. They stand in contrast to the previously discussed authors, who, for the most part, address practical thinking as a subset of all human intelligence, and who are confronted with intractable problems of terminology and definition central to such an endeavor.

Scribner (1984, 1986, 1989) defines practical thinking as thinking in action. She places inquiry into the nature of the working mind within the general context of Soviet activity theory (Leontiev, 1979) with particular emphasis on the tenet that neither mind nor behavior can be taken as the principal category of analysis in psychology. Rather, the primary object of analysis is the interactive process of humans engaged in, and with, the environment, and this interaction is a synthesis of mental and behavioral processes. Scribner defends a practical/theoretical distinction and suggests that recent interest in practical thinking is the logical result of a body of research whose findings indicate that human intelligence and intellectual accomplishments vary significantly according to three crucial contextual factors: domain, task, and setting. The theoretical construct which defines practical thinking as the mind in action, refers to the kind of thinking that is embedded in life's daily goal-directed activities. The purpose and function of practical thinking is to successfully accomplish the goals of these activities. Practical thinking is, therefore, significantly different from the type of thinking involved in the performance of isolated mental tasks undertaken as ends in themselves.

Scribner (1984, 1989) conducted research on practical thinking with dairy product workers engaged in the task of product assembly and identified significant features of the mental processes involved. This task occurs in the warehouse of a milk-processing plant and:

> ...Assemblers are responsible for locating and taking out to a loading platform various milk products ordered by wholesale drivers for the next day's delivery. The products needed on each route are listed on a computer-generated form which expresses quantities in a metric system specific to the dairy, one composed of cases and units. Dairy products are stored and handled in cases of a standard size which hold a certain number of unit containers of milk. The number of units in a case varies with the size of the container. One case holds four gallons, nine half-gallons, 16 quarts, and so on. If a particular order involves a quantity that is not evenly divisible into cases, the order form represents it as a mixed number: "x cases plus y units" or "x cases minus y units." For example, "1-6" on the order form stands for "1 case minus 6 units"--the exact number of units required for the completed order depending on the type of container specified. After the assembler reads the order (in actuality he or she handles several at a time), he proceeds to the area where the product is stored and pulls out from the array as many cases and units as required. (p. 3)

Based on several criteria, Scribner considered the filling of such an order, in the context of the product assembly job, to be a form of problem-solving. The task has a number of formal features. Primarily, it exists in an environment of identifiable problems "for which there are fixed criteria for solution" (Scribner, p. 4). However, it differs from mathematical problem solving found in schools. Product assembly occurs in an environment that is constantly changing, given that the number of full, empty, and partially full cases available for filling orders continually changes.

Scribner noted that experienced assemblers use these changing conditions to their own advantage. They filled orders using solution strategies that involved the least amount of physical effort. Scribner labels these least physical effort strategies "optimal solution strategies."

Scribner noted that other, less skilled workers;

> ...On some occasions fill(ed) plus orders by adding the specified number of units to an empty case and minus orders by removing the specified number of units from a full case. (p.4)

These solution strategies were "isomorphic to the symbolic expression of the order" (Scribner, 1989, p.4) and such strategies were labeled literal solutions. Simply put, when partially full cases were used to assemble the order, requiring fewer moves, such strategies were labeled optimal solutions, and characteristically required additional mental manipulation of the numerical information in the order (Scribner, 1989, 1984).

From Scribner's studies with product assemblers, several principal outcomes were observed. Skilled assemblers would satisfy orders employing solution strategies that required the fewest possible physical moves. The strategies that these skilled workers displayed are defined as "least effort strategies" or "optimal solution strategies" (Scribner, p.13). The use of an optimizing solution strategy often called for the psychological restructuring of the task, including the reorganization of mental operations to a higher level of complexity, and resulted in a task solution requiring the least amount of physical effort. On the other hand, novices primarily employed solution strategies that used more physical moves for task completion and did not incorporate more complex mental reorganization. These strategies were defined as literal solution strategies. Semi-novice workers, initially, employed both types of strategies, optimal and literal, at a chance percentage level. These semi-novices did increase their use of least physical effort strategy solutions in the second task trial.

Another major finding was that skilled workers displayed flexibility in regard to strategy selection, choosing the appropriate strategy to insure an optimal solution for each practical task. Skill was apparently acquired through practice of the task performance (instruction was not provided). Scribner's inquiry identified dynamics of novice to expert development within the domain of an adult goal-directed practical activity.

Scribner (1989) conducted further research with teenaged students engaged in an experimental simulation of this task. In the simulated study, teenaged students, as compared to workers, were primarily literal problem solvers, although students with higher math achievement scores tended to use optimal solution strategies more frequently, indicating that there may be a correlation between the individual student's formal math ability and their choice of solution strategy.

Like Scribner, other investigators have addressed the domain of non-academic everyday tasks. Although these investigators' definitions of practical intelligence vary considerably, they are all conducting inquiry focused on thinking that is embedded in goal-directed activity. Klemp & McClelland (1986) undertook research in an attempt to delineate the processes that underlie and comprise a particular practical intelligence within a specific domain and identified eight generic interacting competencies displayed by executive business managers.

Wagner & Sternberg (1986) argued that a critical element and a significant underlying process of practical intelligence, applied in work settings, is tacit knowledge. They define tacit knowledge as the ability to acquire and apply information that is never explicitly taught, and is therefore learned without formal instruction. Wagner & Sternberg offer a framework for determining the structure of tacit knowledge, divided by category and orientation. Categories include: management of self; management of tasks; and management of others. Orientations are either local (short-term) or global (long-term). Although the content of tacit knowledge may differ from one task domain to another, Wagner and Sternberg argue that this basic framework does not. Additionally, they clarify that although tacit knowledge structure is functionally generalizable across task domains, this dynamic is not to be confused with either a general unitary factor inherent in academic intelligence, or a construct that includes generalizing tacit knowledge into the realm of academic intelligence. The acquisition of tacit knowledge occurs exclusively in the "real world" of goal-directed activity and cannot be measured by established, standardized academic test performance. Wagner and Sternberg do not, however, offer a model for understanding the developmental processes of the acquisition of tacit knowledge.

Fredrikson (1986) devised psychometric tests to measure, as well as to identify features of, practical intelligence and practical thinking. Research was conducted on subjects engaged in both medical diagnosis and executive business problem-solving tasks. In regard to medical diagnostic problems, Fredrikson discovered that it was primarily the nature of the processes involved in hypothesis formulation the differentiated novices from experts. In regard to executive business problem-solving, he identified eight different factors that contributed to successful solutions, including: preparation for action via becoming informed; managerial skill; etc. Although Fredrikson found some overlap, his research indicates that there are significant differences regarding the competencies needed for the successful completion of practical activity in different task domains. Mercer, et al (1986) also constructed a psychometric instrument with the goal of measuring practical intelligence without sacrificing contextually relevant aspects of the phenomena. Mercer, however, proposed a more limited definition of practical intelligence, defining it solely in terms of social-behavioral, interpersonal skills. He contrasted such skills with academic intelligence. According to Mercer, any exploration of academic intelligence must concentrate on: the individual's internal world; an emphasis on understanding and measuring the cognitive processes and structures that serve to manipulate cultural symbolic systems; mathematical and logical problem-solving; the creation of higher-order concepts; and operation at higher levels of abstraction. Additionally, Mercer claims that this type of intelligence develops only within academically sophisticated environments.

In contrast to academic intelligence, Mercer postulates a social-behavioral intelligence, characterized by the following: an individual's ability to deal with the external world of social structures and interactions; the ability to successfully assume myriad appropriate roles in a variety of significantly different social systems; and the ability to establish and maintain interpersonal relations. Clearly, Mercer situates his notion of practical intelligence in the social domain, with an implicit definition of social as equivalent to interpersonal. The behavioral manifestation of practical intelligence is social competence. Determination of the ability to exhibit successful social competence becomes the basis for the measurement of practical intelligence.

One serious criticism of Mercer's proposal is that the individual's internal cognitive processes are omitted from discussion of sociocultural intelligence, implying both that 'social' is equated with interpersonal and that internal cognitive processes and interpersonal dynamics are mutually exclusive. Mercer's construct locates academic intelligence exclusively as internal mental activity and social/interpersonal activity is treated as exclusively behavioral and takes place somewhere 'outside' the individual's mental activity. Although social-behavioral intelligence may be a legitimate aspect of an overall aspect of a concept of practical intelligence, Mercer's working definitions can at best be viewed as severely limited in scope. Obviously, Mercer's working definitions would be antithetical to Scribner's notion of the "mind-in-action."

Berry and Irvine (1985), working within a cross-cultural research tradition, emphasize the effect and centrality of context. They present a taxonomy of levels of context and paralleling effects. They also provide the construct of a set of criteria which can be used to identify, and differentiate, the factors that comprise practical intelligence and behavior in everyday life. These factors include specific abilities, cognitive styles and general intelligence. The questions raised by Berry and Irvine's research on bricolage, in a variety of cultures, represent three of the most significant themes in the current dialogue regarding the nature of practical thinking and behavior. These themes are: to what extent is practical intelligence content specific; what intercontextual generalities exist and by what processes do such generalities operate; and by what means might the existence of cognitive universals be determined.

Lave (1984), in her work on practical thinking, stresses the centrality and the influence of the external environment on the organization of practical thought. In research on the use of arithmetic in practical daily activities, Lave (1988) studied the cognitive processes involved in grocery shopping. Her research indicated that the supermarket environment had significant impact on the structure and the dynamics of the arithmetic cognitive processes employed by shoppers. Others have also studied practical thinking in the domains of work and everyday living, including tailoring (Reed & Lave, 1979), literacy (Scribner & Cole, 1981), carpentry (Carraher, 1986), and trade and commerce (Petito, 1982; Posner, 1982).

As discussed above, investigation regarding the identification of the specific features of practical intelligence has been included in research that is explicitly oriented to the exploration of thinking embedded in goal-directed activity. Other research, however, has also addressed this aspect of human intellectual activity without formally labelling it "practical thinking."

Greenfield (1969) presented an explicit argument for the inclusion of practical thinking as a legitimate mode of human intelligence without labeling it as such. The presentation of the concept of goal as a significant environmental variable regarding the development of human intelligence represents an alternative to any construct which posits behavior (and underlying mental activity) as preceding intention in development. By extension, Greenfield laid the groundwork for a conception of the development of intelligence as fully practical. If the goal of the infant significantly motivates behavior (including the appropriate mental activity necessary to accomplish behavior), then certain cognitive processes and dynamics, embedded in goal-fulfilling activities, are developing from infancy and the course of such development is theoretically, therefore, traceable.

Other investigators have also addressed thinking embedded in goal-directed activity without explicitly labeling such mental activity practical thought. The more recent theory and research of the Genevan school (Bullinger & Chatillon, 1983) has also seriously addressed and incorporated thinking which is embedded in goal-directed activity. Researchers have attempted to define goal and task, and have addressed the issues of: differentiating between the experimenters and the child's goals; understanding the child's mental representation of a goal; and the relationship of this representation with the formulation and the coordination of the mental activity used to reach it. Genevan research on problem-solving asks serious questions concerning the relationship of the nature of the task goal, its mental representation, and the types of mental activity that the child displays to meet the goal.

As cited above, there has been in recent years a significant theoretical trend in regard to the inclusion of practical intelligence within comprehensive models of human mental activity as well as a growing body of inquiry regarding the identification and understanding of the salient features and mental processes that comprise practical thinking. The development of practical intelligence has received less attention, although it has been addressed by researchers focusing on life-span and adult inquiry.

Dixon and Baltes (1986) apply a contextualist orientation to development and emphasize the functional relationship of active and continuing changing multiple levels as the basis for a reevaluation of the dynamics of growth and decline in life-span intellectual performance. The inclusion of performance and mental skills involved in the pragmatic demands of daily life broadens the concept of models of life-span intellectual development and impacts on the design of future life-span research. Similarly, Willis and Schaie (1986) apply the construct of practical intelligence as an experimental contrast to determinations of mental abilities arrived at via psychometric measurements in regard to issues of intellectual development in later adulthood.

Summary

Although the terminology 'practical thinking' is not always explicitly used in the theoretical work and research discussed above, there emerges, from all, a constellation of dynamics which comprise the defining features of practical thinking and intelligence. These key dynamics include: 1) the centrality of context; 2) the identification of mental activity embedded and functioning in the service of satisfying a goal; 3) the display of complex and sophisticated cognitive activity unmeasurable by standard psychometric tests; 4) the issue of domain-specific competence and cognitive complexity; 5) the issue of knowledge acquisition occurring without instruction; 6) the issue of mental representation of practical problems; and, 7) the relationship of mental organization and reorganization to the cognitive processes involved in life's daily activities.

PRACTICAL PROBLEM-SOLVING AND PRACTICAL PROBLEM-SOLVING SOLUTION STRATEGIES

Many investigators concerned with human practical thinking have focused their attention on problem-solving. As previously stated, it is the generally shared premise of this research that practical thinking is by definition thinking that is embedded in, and functions in the service of, successful goal-directed practical problem-solving activities. Therefore, much research in regard to issues of practical intelligence and practical thinking has legitimately addressed the performance, dynamics, and processes involved in problem-solving and problem-solving strategies.

Adult practical problem-solving

Within the larger body of research on problem-solving in adults (e.g., Newell & Simon 1972), researchers investigated practical thinking as evidenced in adult every day problem-solving (e.g., Cornelius & Caspi, 1987). Child-oriented research has only recently identified and explored thinking embedded in daily, goal-oriented activity. Therefore, it is worth noting some of the adult oriented research that has explicitly examined practical thinking. Research addressed the effects of aging on practical problem-solving (e.g., Williams and Schaie 1986), the determination of the cognitive features of practical social competence (e.g., Goldried & D'Zurilla, 1969), the identification of breadth of knowledge (e.g., Horn 1982) an identification of adults' comprehensive repertoire of problem-solving methods (e.g., Egan & Greeno, 1974), and the development of adult practical intelligent performance (e.g., Dixon & Baltes, 1984). Contextually oriented researchers examined the dialectical relationship of the problem-solving environment, the goal and the solution strategy (e.g., Lave, 1988).

Cross-cultural researchers have identified aspects of adult practical problem-solving activity with an emphasis on the identification of the cognitive processes displayed (e.g., Wagner & Sternberg, 1986), the determination of domain specificity vs. universal aspects of problem-solving abilities (e.g., Mehen, 1984), the nature of transfer regarding practical problem-solving skills (e.g., Ginsbery & Allardice, 1989) and the interaction between schooling and informally acquired problem-solving skills. Others have attempted in-depth explorations of everyday problem-solving skills and activity as evidenced in the workplace (e.g., Scribner, 1984; Laufer, 1984; Beach, 1984) as well as focusing on novice to expert development in problem-solving activities (e.g., Chase & Simon, 1973).

Children's practical problem-solving

The research of Siegler (1983) and Klahr (1978) captures the current status of thinking and debate regarding children's problem-solving and solution strategies. Siegler (1976, 1978) posited a rule-based, rule-governed understanding of children's problem-solving strategies based on research with children using a balance-scale problem. According to Siegler, rules progress in sophistication with the individual's increasing cognitive development. Generally, children progress from the use of simpler to more complex rules as they grow older (become cognitively sophisticated) and rules are identified to be more complex when the rule takes into account more information than is present in the problem situation. Siegler's research was designed so that individual rules lead to individual and identifiable solution strategies and subjects' rules were inferred from task solutions.

Klahr (1978) claimed that a production-based system approach governed children's practical problem solving strategies. He identified a perceptual strategy that involved seven steps: comparing current state to goal state and the determination of items not in their final position; locating the most constrained item that needs to be moved; establishing the goal of moving the item; determining the items that prevent such movement; execute movement if no "culprit" items exist; if "culprit" items exist, replace current goal with the goal of moving the "culprit" to a location that does not inhibit further action; reengage with search for next "culprit" item. Klahr (1981), in a variation of the Tower of Hanoi task, observed that preschool children have acquired the rudiments of problem solving methods and are clearly capable of more than trial and error strategy use. Klahr's research indicated that preschoolers engage in solution strategy selection and are able to apply a range of solution strategies to a novel task.

At their core, Siegler's and Klahr's respective orientations are not in conflict. They agree that young children are engaged in complex problem solving activities that are increasing in sophistication as the children cognitively develop. Rules and production systems function to allow the child to successfully solve problems. Siegler's rule-governed theory implies that children are uni-strategic, that is, that they discover a single solution strategy. However, Siegler (1989) claims that children are indeed multi-strategic, and engage in, simultaneously, a repertoire of solution strategies, each governed by different rules.

However, regarding the overall body of literature focused on young children's problem-solving skills, Klahr and Robinson (1981) noted that, with few exceptions (e.g., Baylor and Gascon, 1974; Greenfield & Schneider, 1977) the wide range of problem-solving methods known to be acquired and available to adults. (such as means-ends analysis, search, evaluation and planning) have not been studied in regard to young children's problem solving processes.

One major finding of both Siegler and Klahr -- that young children are capable of, and display in performance, a large and sophisticated range of problem-solving strategies -- is in concert with the conclusion reached by other researchers investigating children's problem-solving skills. For example, Simon (1975) identified numerous solution strategies that preschoolers use in another version of the Tower of Hanoi task, and Gholson (1989) reported problem-solving solution strategies amongst preschoolers which recall mapping relations in isomorphic transfers. In sum, it has become widely accepted that young children are engaged in complex cognitive problem-solving activities that far exceed previous models of their cognitive capabilities.

ome inquiry into children's problem-solving share the common goal of providing detailed task analyses of the processes studied (Newell, 1977; Brown & DeLoache, 1978). The aim of such analysis is to provide explicit models of cognitive development within a limited task domain including precise descriptions of the initial and final forms of the cognitive processes under investigation. For example, Gelman and Gallistel (1978) provided an analysis of the emergence of counting principles in very young children. A particular strength of this research is its diagnostic power. By considering the errors produced by two- to four-year-olds in a counting task, Gelman could diagnose which counting principle the child lacks -- whether it be one-to-one correspondence, stable ordering, cardinality, etc. Others in this tradition have studied the development of hierarchical complex tree structures (Greenfield & Schneider, 1977) as well as the use of analogies to self-regulate the problem-solving process (Gick & Holyoak, 1980).

Inquiry has also explored other aspects of children's practical thinking and problem-solving activity. Investigators have emphasized the impact of social and peer interaction on children's problem-solving performance (e.g., Wertsch, 1978; Cooper, 1980; Perlmutter, Behrend, Kuo, Muller, 1989). Berg (1989) identified the changing capacities for strategy knowledge regarding solutions for everyday problems among children and adolescents. Other inquiry has focused on the impact of broader social and interpersonal interactions on children's problem-solving (e.g., Carr, Kurtz, Schneider, 1989) including investigation of the impact of formal educational programs on children's practical cognitive-social problem-solving skills and strategies (Battis, 1988) and the impact of family therapy on effecting change in the individual's subsequent, independent choice of practical familial problem-solving solution strategies (Reid, Rotering, Fortune, 1989). However, with the exception of Berg, these researchers have not offered insight into the dynamics of either the changing nature of children's solution strategy selections nor the emergence of new solution strategies into the child's repertoire.

Other investigation has explicitly acknowledged the centrality of motivation and goal-directedness in children's practical problem solving. Inquiry has focused on motivational contributions to object recall (Lange, MacKinnon, & Nida, 1989), the effect of domain specific knowledge on problem solving performance (Schneider, et al, 1989) and the role of inquiry in problem solving (Siegler, 1977).

Other research stressed the importance of the context of goal-directedness, and offers direction and models for examining the child's mind in action. Istomina (1948) provided the framework of a model for a developmentally oriented study of young children's problem-solving in a practical setting. Istomina concluded that the emergence of voluntary memory takes place in the context of a goal-directed activity for the child. In Istomina's study, preschoolers exhibited strategy-like behaviors and improved recall if the memory task is embedded in a game-like context as opposed to the typical laboratory setting.

As stated, goal-directedness as an essential context for practical thinking and problem-solving is stressed by several other practical intelligence theorists (Scribner, 1984, 1986; Sternberg, 1986; Wertsch, 1979). Folds et al. (1990) posit that the quality of young children's strategic behavior varies substantially with the goals set by the experimenter. Baker, Ward et al. (1984) noted that four, five and six-year-old subjects exhibited different behavior in regard to a memory and play task with respect to a change in the articulated goal of the task.

Problem Solving Solution Strategies

Much inquiry into childhood problem-solving activities has focused on the solution strategies employed. The view and evidence that pre-school children use strategies in the service of problem solving has received wide acceptance for over a decade (Siegler, 1990). Bjorkland (1990) claimed that there are significant reasons why children, as early as infancy, function strategically. The role of strategy in problem-solving is to organize knowledge, information, and action. This is clearly evident, in slightly older children, engaged in the Tower of Hanoi experiment (Klahr & Robinson, 1981) where the level of motor skill is trivial and progress is made when a strategy is formulated for organizing a complex sequence of moves. Some investigators suggest that there might even be innate procedures for detailing consistent sequences, eliminating redundancies, and reorganizing a performance to achieve efficiency (Case, 1984, 1985; Karmiloff-Smith, 1979; Klahr, 1984; Klahr & Wallace, 1976). Furthermore, investigations explored effective strategy use via instruction (e.g., Flavell, 1970), the use of verbal mediational strategies via instructional rehearsal (Flavell, Beach & Chuds, 1966), rehearsal of children's memory to enhance strategy selection (e.g., Asarnow & Meichenbaun, 1979), instruction of retrieval cues from memory strategies (e.g., Kobasiqawa, 1974), and a multitude of Piagetian inspired training studies (e.g., Brainerd, 1974; Gelman, 1969). Specific strategies have been identified and their development charted. For example, visual scanning, both as an ability and as a solution strategy has been identified in infants (Salapatek, 1975; Maurer & Salapatek, 1976; Gibson, 1969) and develops through childhood (Zinchenko, Chzhi-Tzin & Tarokanov, 1963).

The emphasis on solution strategies can be traced back two decades. Flavell (1970) identified that children were often unable to use effective strategies spontaneously and labelled this phenomena as production deficiency. Information processing oriented researchers claimed that this "deficiency" was due to the fact that children did not have the necessary mental apparatus or 'hardware.' Piagetian theorists claimed that the children had not yet reached the appropriate cognitive stage of development to make use of certain strategies. However, during the last decade considerable research has identified the sophistication and scope of children's solution strategies and discussions are currently oriented to an appreciation of how much the young child is strategic rather than stressing their deficiencies.

Researchers have long debated some aspects of the criteria that would comprise an exact definition of a problem-solving solution strategy. However, one central set of criteria has been generally agreed upon -- strategies are the child's goal-directed, mental operations that are aimed at solving a problem (Bjorkland, 1990). It is the generally shared view of investigators that inquiry into children's practical problem-solving is, by definition, the investigation of children's goal-directed behavior. Importantly, most contemporary approaches also share the orientation that children's goal-directed behavior is investigated in terms of strategies (Bjorkland, 1990).

The issue of whether or not a strategy must be available to consciousness (i.e., deliberate or intentional) generates serious debate. The shared definition of strategies in the research literature of the past twenty years (e.g., Belmont & Butterfield, 1969; Brown, 1975; Flavell, 1970; Naus & Ornstein, 1983; Pressley, Forest-Pressley, Faust & Miller, 1983) is that they are selective, goal-directed and intentional. Such definition of strategic does not require planning, only the intention of achieving a goal. A more conservative view (e.g., Bismarc & Levevre, 1990; Garner, 1990; Gholson et al, 1990) emphasizes the selection of an effortful operation in the service of satisfying a goal. Therefore, in this view, relatively automatic and efficient processes are not considered strategic. Some researchers further restrict the definition to include child-directedness (e.g., Gholson, 1990). For Gholson, training conditions requiring children to engage in specific activities are not strategic unless such activity is integrated into the child's repertoire. Therefore, he emphasizes the ability of children to execute a procedure on their own, not simply to display the ability of demonstrating that they can do what they are told:

> A strategy is demonstrated when the child deliberately engages in some goal-directed processing, without specific prompting from an external agent (Gholson, p.312).

Definitional problems have also arisen due to the recognition that relatively automatic, and unconscious processes, can result in enhanced levels of performance, often mirroring or exceeding performance attained when deliberate strategies are employed (e.g., Hasher & Zacks, 1979; Shiffrin & Schneider, 1977). Strategic behavior has been observed in children that was apparently mediated by the relatively automatic activation of semantic memory relations (e.g., Bjorkland, 1985; Lange, 1978) although the children display little or no awareness of these techniques (Bjorkland & Zeman, 1982). Furthermore, highly practiced techniques such as retrieval of addition and subtraction facts from memory (e.g., Ashcraft & Fierman, 1982) and skills involved in proficient reading (e.g., Paris et al., 1983) are apparently executed without deliberate conscious selection among many alternative techniques. Ashcraft (1990), in response, has proposed a definition of a strategy to include any cognitive process that serves a goal-related purpose. According to this definition, for example, children's arithmetic fact retrieval constitutes a legitimate strategy.

In an attempt to resolve this definitional conflict, Bjorkland and Harnishfeger (1990) propose the concept of a 'strategy complex' to refer to the multiple cognitive processes, automatic and not, that contribute to the successful satisfaction of a goal-directed task. Furthermore, such a definition defines children's problem solving strategies by a set of criteria that is, on many counts, equivalent to those posited by theorists when defining practical intelligence. The strategies employed by preschoolers to solve practical problems offers, therefore, an excellent vehicle to investigate the child's mind in action.

In light of the current thinking regarding practical intelligence, children's problem-solving strategies, if they are to truly represent cognitive activity that will form the basis of any exploration of the child's mind in action, must: 1) represent thinking that is embedded in the context of, and functioning in the service of, the successful satisfaction of a larger goal-directed activity -- cognitive processes that are not engaged in as ends in themselves; 2) be free of offered or available instruction that may impact on, and enhance, performance; 3) exist in the context of a recognizable goal-directed activity (or approximates one); and, 4) are effective, i.e., successful.

In sum, investigators have conducted extensive research in the domain of adult problem-solving and practical problem-solving. Investigation into adult practical problem-solving and particularly, problem-solving strategies, has been identified as an excellent vehicle for the further understanding of the nature of practical intelligence. Similarly, investigators have conducted extensive inquiry in the domain of children's problem-solving and, more recently, on children's practical problem-solving. Investigators have identified that children exercise practical intelligence, engage in practical problem-solving, and employ practical solution strategies to satisfy the goals of daily life. However, inquiry in this area has primarily emphasized the impact of social and interpersonal interaction on children's practical problem-solving. A comparatively smaller body of research has addressed the issue of children's practical thinking in the form of problem-solving when the child is engaged in an everyday mathematically oriented goal-directed activity. In such activity, the cognitive processes involved and the solution strategies employed, function to satisfy the overall goal and are not undertaken as ends in themselves. Similarly to adult inquiry, the examination of such processes provides an excellent vehicle for the investigation of practical intelligence and practical thinking in young children.

CHILDREN'S ARITHMETIC

Siegler (1990) presents a strong argument for the use of children's arithmetic as an advantageous area in which to study strategy choices. His argument is based on several observable phenomena. First, children do use a variety of strategies to solve arithmetic problems. Secondly, these strategies are distinct, each from the other, and there exist experimental means to assess which strategy is being used. Third, Siegler emphasizes the practical importance of arithmetic and its constant use in the everyday life of the child.

Regarding young children's mathematical skills, a definitional problem exists in the research literature. Researchers typically identify all preschool children's mathematical abilities as practical. This definition apparently arises because the preschool child has not yet entered an academic environment and has not received formal mathematics instruction. However, such a definition is seriously misleading. The proposed research will consider practical arithmetic to refer only to the young child's mathematical abilities when they are embedded, and functioning in, the service of satisfying goal-directed activities.

Practical arithmetic, as discussed above, has been defined by others (e.g., Ginsburg, 1977). In the natural environment children are confronted with a myriad of problems that require arithmetic and numerically based solutions. Preschool children have not learned a formal or written arithmetic to apply to these problems. Given the absence of any formal arithmetic skills, the child develops, via intuition and counting skills, a working practical arithmetic and engages in arithmetic problem-solving. The preschool child possesses a practical arithmetic as a means of informally finding solutions for daily, real life mathematical problems. In a natural environment, preschoolers develop ways of engaging in problem solving, and often select solution strategies that require the use of addition and subtraction. A significant aspect of the child's motivation for developing this practical arithmetic is simply "practical utility" and practical necessity (Ginsburg, 1977). Preschoolers' practical numerosity is goal-directed (Ginsburg, 1977; Wertsch, 1985). In sum, the motivation for the child's acquisition of an informal arithmetic is its practical usefulness, and the development of the practical arithmetic is embedded in the increasing need to use numerosity in everyday activities.

Some research investigated children's use of arithmetic skills in practical activity and included an exploration of the correlation of arithmetic experiences and success in solving realistic verbal reasoning problems (Lyda & Church, 1964), the formation elementary number concepts (Davydov, 1957), differences in computational strategies used by working children engaged in commercial transactions as compared to computational strategies taught in school (Carraher, Carraher & Schliemann, 1985) and the development of mental addition in unschooled Africans (Ginsburg et al, 1981).

In the everyday natural environment, children are spontaneously involved in cognitive learning. Without the benefit of instruction, children learn, and utilize, with increasing sophistication, a functional, simple arithmetic. As Piaget (1953) states:

> It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth. (p.74)

The young child's arithmetic skills are developing free of instruction and are developing in sophistication. Ginsburg (1977) determined that young children are engaged in the spontaneous learning of economical strategies for counting things.

A significant finding of recent research is that preschoolers exhibit greater cognitive capacities than previously thought (Gelman, 1977). Specifically, researchers have determined that preschoolers: can recognize the numerical value of, and manipulate small arrays; can and do count; engage in number operations, specifically addition and subtraction; prefer addition over subtraction; perform addition and subtraction by counting; employ a range of counting strategies in order to perform addition and subtraction; solve mathematically oriented problems; and employ a variety of arithmetic strategies in order to solve problems. These capacities form the basis of the preschooler's mathematical, strategic world.

Ginsburg (1977) determined that children "see" arrays of small numbers (one to three) directly, by immediate recognition, without having to count. Consistent with Ginsburg, Gelman (1978) found that children can identify the number of items in an array when the set size is five or below. Gelman (1977) showed, in her "magic" experiments, that young children can recognize when the number in a small set has been changed while the set is hidden from view. This involves a child's counting the set twice, before and after the change, and then comparing the numbers. Other research concluded that preschool children can correctly identify the number of items in an array when the array set size is relatively small (Beckmann, 1924; Descoeudres, 1921; Gelman, 1972; Gelman & Tucker, 1975; Lawson, Baron, & Siegel, 1974; Smither, Smiley, & Rees, 1974).

Working with small arrays, children can use number operations (addition, subtraction, counting) with apparent ease, as long as the array is physical and the child is not forced to engage in abstract number use or to internalize the array in an abstract form.

Young children's mathematic skills include an integrated elementary concept of quantity. Even though conservation has not been attained, young children display accuracy in respect to judgments of more or less when small numbers of elements are involved. In magnitude comparison tasks, two "target" numbers are named and the subject is asked to decide which is larger or "shows more." Investigators studying children (e.g., Siegler & Robinson, 1982) have established that children can perform this task accurately by the age of 5 or earlier when small numbers are represented.

Specifically in regard to counting, Gelman has identified five basic counting skills that underlie preschoolers' number ability. These include: 1) the one-to-one principle which states that a distinct tag be assigned to each object in an array and only one tag may be assigned to each object; 2) the stable order principle, which states that tags must be arranged in a stable/repeatable order and the number of tags must be as large as the number of objects in the array; 3) the cardinality principle, which states that the tag applied to the final object in an array represents the total number of objects in that array; 4) the abstraction principle, which states that the preceding three principles can be applied to any array or collection, whether the collection is concrete or abstract (very young children do not understand that abstract heterogenous collections can be counted); and, 5) the order-irrelevance principle, which states that the order in which the objects in an array are tagged is irrelevant to the total number, or cardinality of the objects in the array.

Ginsburg (1977) determined that the average four-year-old can count up to nine and by age five can count up to the number twenty. Most importantly, in regard to early number skills, it has been determined that counting forms the core of, and is the basic computational technique of children's arithmetic (Ginsburg 1977, Gelman, 1978). At four or five years, most children do not know and have not remembered the basic addition facts (i.e., the sum of two plus three).

Preschoolers can correctly interpret simple addition and subtraction problems and subsequently implement the correct solution to solve them. For example, Ginsburg (1977) presented preschoolers with two arrays of toys (one array of five, and one array of three). The arrays were removed from the child's sight and two toys were taken from the array of five. The two equal arrays of three toys were then presented to the child. The child was asked to "fix" the situation, i.e., return it to its original state, and was given toys in which to do this. Children were able to solve the problem and the complexity of children's arithmetic was well demonstrated. Preschoolers were able to remember the size of the arrays, notice changes in the array size, could undo subtraction by the use of addition, and used counting to perform the necessary addition.

Children make no use of written notation to solve math problems. Furthermore, such a tool would be a burden, not an aid. Children, however, often use visual cues, tools and aids to facilitate counting. For example, children will count on their fingers, count objects, point, count out loud, etc. Briars and Siegler (1981) indicate pointing to be a crucial part of children's counting strategies and hypothesized that a developmental progression exists in which children first consider counting as requiring a three-way one-to-one correspondence among a word, an entity, and a point. Only later do children understand that it is the word-entity correspondence that is necessary for a correct count.

Internalization of the pointing act occurs with age. Fuson (1983) determined that three-year-olds usually touch the objects when counting, four- and five-year-olds pointed without touching, and some five-year-olds counted to themselves without pointing. This developmental sequence is also reported by Ginsburg and Russell (1981).

Preschool children employ a variety of strategies for performing successful arithmetic calculations (Ginsburg, 1977; Sternberg, 1983; Gelman, 1978; Siegler, 1990; Resnick, 1983). Furthermore, these strategies all appear to be invented by children and all are based on counting.

Regarding the child's performance of simple addition, models of three different strategies have been proposed (Groen & Parkman, 1972; Suppes & Groen, 1967; Resnick, 1980). All strategies share in common the premise of a mental counter that is placed at some value and increased as needed. The first strategy, and arguably the least developmentally sophisticated, involves starting from zero and counting both addends. For example, using the simple arithmetic problem 2 + 3, the child places the mental counter at zero and then counts up to 2, and then counts up three more increments to 5. This strategy is labeled the Sum model.

The second model involves the child placing the mental counter at the value of the smaller addend, in this case 2, and then counting up the value of the larger addend, in this case 3, to arrive at the correct answer. The third model, labeled the Min strategy, involves the child starting from the value of the larger addend and counting up the value of the smaller addend, minimizing (thus the strategy name) the number of increments needed to solve the problem.

Siegler (1990) has demonstrated that preschoolers use a repertoire of strategies and do not rely solely on one. Explanations that would account for the use of a diverse repertoire are primarily based on developmental factors. Siegler has demonstrated that as preschoolers are acquiring arithmetic skills, they are in the process of constructing new strategies, continuing their use of existing strategies, and discarding others. However, Siegler agrees that the general developmental trend is towards the use of more efficient strategies, such as the Min model for preschoolers, and fact retrieval for older grade school children.

Cohen (1989) examined preschoolers' patterning of strategy use in a goal-directed, mathematically-oriented task. In a design derived from Scribner's product assemblers study (1984), three- and four-year-olds performed a play/work task of filling orders for juice. The task environment was predetermined for the child -- they were presented with continually changing arrays from which they were to satisfy a requested order. Orders were successfully accomplished by a variety of strategies. These strategies included: an optimal or least physical move strategy; an in-between strategy requiring more moves than an optimal strategy; a literal or Sum model strategy; and a subliteral strategy -- where random moves were used.

Cohen reported several findings. Importantly, preschoolers chose more efficient strategies over trial within the confines of the task. Error rate was low -- children successfully solved the task. Preschoolers used the full repertoire of strategies, they were not wedded to any single strategy, either as a group or as individual subjects. Preschoolers, to a significant degree, chose more efficient/less physical effort strategies in the second trial than in the first. Furthermore, no subject chose a less efficient strategy in the second trial than in the first. Although some subjects maintained the same strategy choice over trial, change occurred only in the direction of optimality. Instruction was not made available.

Differences were noted among subjects. The youngest three-year-olds tended to be literal and subliteral strategizers at the onset and four-year-olds tended to use in-between strategies and optimal strategies at the onset. Other researchers (e.g., Siegler, 1990) have observed a qualitative shift in mathematical sophistication and abilities between ages three and four. Cohen's research, however, did not indicate a significant age difference in regard to enhanced optimal performance. It is important to note that although three-year-olds did not perform as optimally in Cohen's study, the tendency to choose a more efficient strategy in the second trial was as evident in three-year-olds as four-year-olds. Therefore Cohen concludes that both increased arithmetic sophistication developing ontogenetically, as well as exposure to the task experience itself, impacts positively on efficient and optimal performance.

CHAPTER II

RESEARCH GOALS AND RATIONALE FOR TASK SIMULATION

The goal of the current research is to examine aspects of young children's (three and four year olds) practical intelligence. Specifically, current research is designed to examine both the nature of problem solving strategies employed by preschoolers when involved in a practical, mathematically oriented, goal directed task and the emergence of new strategies into their already existing repertoires. Importantly, this research is also designed to explore the developmental aspect of the acquisition of new strategies, including the nature of the changing pattern of preschoolers' strategy choices.

The research is designed as a study of the patterning of solution strategy selection over time (three task trials). It also includes a comparison of strategy use by gender. The task is also designed to explore the relationship of preschoolers' arithmetic skills and their performance in the mathematically oriented practical task. Finally, the research design allows for an analysis of participants performance within and across task sessions.

A. <u>The nature of children's strategies</u>

The current research shares assumptions with many of the studies reviewed previously. First, it tests the hypothesis that preschoolers do employ a complex repertoire of strategies to solve practical tasks. Therefore, the first goal of the present study is to provide new evidence that preschoolers employ a repertoire of solution strategies. The second goal is to identify the patterning of strategy usage.

The third goal of the current research is to determine whether the development of children's problem-solving skills is characterized by increasingly complex mental reorganizations which are essential to the acquisition of increasingly efficient strategies, culminating in the acquisition of an optimal solution strategy.

Investigators (e.g., Siegler, 1990; Ginsburg, 1977) have determined that three- and four-year-olds employ a variety of arithmetic strategies when solving simple problems. However, comparatively little research has been conducted regarding the use of such strategies in practical activity. Generalizing from Scribner's research on optimal strategy acquisition, several key features for designing research on a mathematical task have been identified. The use of an optimal solution strategy is the primary measure of successful performance in a mathematically oriented practical intelligence task. Such strategies are characterized by increasingly complex mental reorganization, resulting in the use of least physical effort solutions. It is a hypothesis of the research that preschoolers do include efficient and optimal solutions among their repertoire of strategy choices. It is also hypothesized that preschoolers increasingly choose more efficient and optimal solution strategies over time when confronted with the same task.

In prior inquiry, Cohen (1989) presented evidence that three- and four-year-olds employ a repertoire of strategies when solving a practical task, including least physical effort optimal solution strategies. This finding indicates that young children choose solution strategies which require a higher level of mental complexity when other strategies are available to successfully satisfy the goal. In the present research. a variety of strategies will be available from which the subject can choose. They each represent a different level of mental complexity and they each require a different number of physical moves. All strategies lead to the correct solution.

Results from Cohen (1989) indicate that preschoolers expand their strategy repertoire, spontaneously and without instruction, with repeated exposure to a practical task. Cohen also demonstrated that three and four year olds choose more efficient strategies over the course of multiple trials of a practical task. Furthermore, it was found that the same subjects acquired and added new strategies. The current research is designed to determine whether experience, that is, exposure to the task, results over time in more efficient performance. The design differs from Cohen's 1989 study in several ways: participants will be exposed to more task trials, problem design is organized to evaluate the relationship of both numerical operations and strategy selection and the size of the order request and strategy selection. Accordingly, the experiment is also designed to determine if subjects acquire and use new strategies as a result of exposure to the task. Therefore, not only will a variety of solution strategies be present at every trial, but subjects will be tested three times on different days over a week and a half period. This design ensures both that subjects will have sufficient exposure to the task and an accurate observation of emerging strategies and changing patterns of strategy use. Again, in order to verify the change in the selection of strategies, a variety of solution strategies is made available for all task problems.

B. Exposure to Task, and the Emergence of New Strategy

Siegler (1990) conducted inquiry into how young children construct new arithmetic strategies. His experimental task, however, did not explore the construction or emergence of new strategies when they function in the context of satisfying a larger activity. There has been little research that has focused on the construction of mathematically based practical strategies when the child is engaged in a daily life task. The fourth goal of the current research is to identify the emergence of the new problem solving strategies in the context of a practical task. It is the hypothesis of the proposed research that young children will acquire and exhibit the use of new strategies, over trial, spontaneously, in the current experiment.

C. Gender and the Development of Strategy Acquisition

Investigation into preschoolers' arithmetic abilities suggests that there is possibly a gender difference in regard to the development of preschoolers' arithmetic skills (e.g., Siegler, 1990; Gelman & Gallistel, 1978). Therefore, the fifth goal of the current research is to explore performance differences between genders. The research is designed to allow for comparison by gender of performance, strategy usage, and strategy acquisition.

D. The Relationship of Children's Mathematical Abilities and Practical Task Performance

It is a hypothesis of the current research that the development of young children's practical thinking, as evidenced in this task, is highly interrelated to the simultaneous process of the development of the child's arithmetic skills. We believe that young children are not capable of selecting strategies which demand a higher level of mathematical sophistication than the child currently possesses. Therefore, it is necessary to determine the level of complexity and sophistication of each individual subject's arithmetic skills in order to analyze the relationship between these two functional systems -- the child's developing practical intelligence and the child's developing arithmetic skills.

The task is designed to offer a variety of solution strategies for each order/problem that directly correspond to a variety of arithmetic strategies. A mathematical pretest is an integral element of the research design. It is important to note that the proposed task is not a measure of mathematical intelligence, but rather, an examination of how arithmetic is related to, and corresponds to, performance in a goal-directed task. The mathematical pretest is designed to assess the following: the child's counting ability; the child's counting strategies; the child's ability to recognize the numerical value of small arrays; the child's ability to solve simple addition and subtraction problems, including problems in which the question and answer are presented verbally, and addition problems presented with the use of concrete objects where the answer can be given verbally or concretely as a result of the child's manipulation of the objects.

In sum, the sixth goal of the current research is to identify if participants arithmetic skills will, in some aspects, correspond to their performance on a practical task. It is a hypothesis of the current research that such correspondence will be found. However, in light of the findings from adult oriented inquiry, the current research hypothesizes that young children may very well perform at a more sophisticated level in the practical task than on a mathematical test. The pretest design allows for such analysis.

E. Preferences for Addition

Given that prior inquiry has identified specific features of preschoolers' arithmetic skills, current research expects that counting will form the basis of the children's mathematical operations and that children will prefer addition operations over subtraction. The seventh goal of the current research is to determine if addition operations are preferred over subtraction operations.

F. Rationale for Task Simulation

The contextual theoretical perspective of intellectual development guided the present research. Specifically, this research is guided by the model of the mind in action (see Scribner, 1984) which posits that skill, as evidenced in practical problem solving, involves the reorganization of mental activity to a more complex order. The design of the task takes into account several key theoretical constructs. First, practical intelligence, as evidenced in the research cited in the literature review above, typically examines the mind in action in the working environment and/or engaged in daily real world activities. Along with such activities, researchers have also identified a task simulation to be a valid vehicle for such psychological exploration (Bjorkland, 1990; Scribner, 1988; Istomina, 1948). However, in our culture, where young children do not typically engage in work activity, a real world work environment (or a subsequent derived simulation) simply does not exist for preschool children. Preschoolers do, however, continually engage in daily practical activity in which thinking serves to satisfy a practical goal. The research task, therefore, is designed to meet the critical criteria which defines an activity as practical. The play store simulation, which forms the basis of the research design, allows for the creation of a task that meets the following criteria: it is goal-directed -- practical arithmetic serves the larger goal of filling a customer's order; it can be solved successfully by a variety of different and distinct strategies; it can be successfully accomplished, and increasingly efficiently accomplished, without formal instruction; and, can be accomplished successfully by participants -- the emphasis is not on a correct solution, but rather on the solution strategy selected. The task is also derived from a real world working environment (Scribner, 1984). Therefore, it is believed that the play store simulation meets the requirements of being a practical activity for preschoolers.

F. The Relationship of Environmental Constraints and Task Performance

It is hypothesized that the environmental constraints of the task itself may positively impact on performance. To fully test this hypothesis, the task design includes several orders/problems where a literal or Sum model solution strategy is removed. This design feature allows for an exploration of literal strategizers' behavior. The eighth goal of current research is to determine whether environmental constraints of the task (the elimination of a possible literal solution) will enhance performance. Simply put, will literal strategists choose a more efficient strategy or will they choose a sub-literal one?

The research goals can be summarized as follows:

1. To determine if preschoolers use a repertoire of solution strategies and to identify the strategies that comprise the repertoire. It is hypothesized that preschoolers do use a repertoire of strategies.
2. To determine the changing pattern of strategy use and to determine if the pattern of strategy usage includes more efficient strategies over time. It is hypothesized that the patterning of strategy selection includes more efficient strategies over time.
3. To determine if children engage in optimal solution strategies. It is hypothesized that preschoolers acquire optimal solution strategies.
4. To identify the emergence of new solution strategies. It is hypothesized that optimal solution strategies emerge in the children's strategy repertoire.
5. To determine if gender differences exist. It is hypothesized that gender differences, in regard to task performance, do not exist.
6. To determine if arithmetic skills correspond to practical task performance. It is hypothesized that preschoolers' arithmetic skills correspond to their practical task performance.
7. To determine if children prefer addition over subtraction. It is hypothesized that children prefer addition over subtraction.
8. To determine if certain environmental constraints enhance performance. It is hypothesized that performance is enhanced if certain environmental constraints are imposed.

CHAPTER III

METHOD

Participants

Twenty-four pre-school children participated in the research. All children were in attendance in full or half-day programs at child care centers serving middle-class families of diverse ethnic backgrounds in the Greenwich Village and lower Manhattan sections of New York City. All meetings with the participants occurred at the child care centers in rooms or semi-private areas. Meetings were kept separate form the ongoing activity of the facilities. The children ranged from age three years and one month (3;1) to four years and eleven months (4;11). Eight boys and sixteen girls participated and all children were English speakers. Children were selected for participation in the study by availability.

Pretest Procedure and Materials and Coding

Three- and four-year-olds in attendance at the preschools were administered a mathematical pretest.

The goal of the mathematical pretest is to determine the child's overall arithmetic skills and to identify specific features of their arithmetic skills.

The pretest has four parts and consists of the child answering a series of number questions (for the complete pretest, see Appendix A).

Part A -- Counting

Each child was first asked by the experimenter to count out loud from one to six. If the child displayed difficulty counting from one to six, the experimenter intervened, saying "let's see if we can count these toys," putting toy cans of food on the table one by one in an effort to prompt the child. Secondly, the child was asked to count backwards from six, following the same procedure. Whether the child could accurately count forwards and backwards, and whether or not the child answered with or without concrete cues and/or verbal aids (Gelman & Gallistel, 1978; Ginsburg, 1977) was recorded.

Part B -- Determination of "how many"/recognition

There were five recognition questions. The experimenter placed a pre-specified number of toy cans on the table. The number of cans will range from two to six and the arrays will be displayed in random order. The child will be asked "tell me how many toys are on the table." If the child can determine the cardinality of the arrays, the pretest will proceed.

Part C -- Addition

The experimenter asked the child twenty addition questions. The questions were asked in random order. The addition problems will be introduced as follows:

> We are going to do some addition problems today... You can do anything you want to get the right answer, but try to answer the best that you can. You can just say the answer if you know it, or you can count or use your fingers or do whatever you want to do. It doesn't matter how you get the right answer -- you don't even have to figure it out the same way every time -- just as long as you try the best that you can (Siegler, 1990, p. 51).

Ten questions were asked verbally with the simultaneous presentation of toy cans representing the problem presented. The form of these concrete object questions were: "Here are X toys, if I add this many more (Y), how many are there?" The children were probed to determine which strategy they used. Probes to determine employed strategy (i.e., Min, counting from the smaller addend, counting from zero) are taken from Siegler (1990). Ten of the addition questions were asked and answered verbally. The form of verbal question is: "If you had X apples and I gave you Y, how many would you have?"

Part D -- Subtraction

There are eight subtraction problems. Questions are divided similarly to the addition problems (verbal/concrete). There were no probe questions.

Children were observed to see if they can count, recognize an array and solve simple addition and subtraction problems with a minimum of error. Pretest will be videotaped and coded. The pretest was scored as follows.

Pretest Coding

The pretest will be coded for three types of performance.

A. Participants will be scored for correct/incorrect answers, as follows:

- 0 = correct answer
- 1 = incorrect answer on first attempt, correct answer on second attempt
- 2 = incorrect answer on second attempt, correct answer on third attempt
- 3 = incorrect answer on third attempt, correct answer on fourth attempt

- 8 = unable to answer

Task Procedure and Materials and Coding

The problem solving task in this study requires the child, in the context of playing "store," to be the "storekeeper" and fill sixteen separate orders for "vegetables" (see Istomina, 1948). The orders will be given to the child verbally and sequentially. The problem-solving task will be administered three separate times with a two- to three-day interval between trials (see Appendix B for Overall Design for Task Orders).

The task will begin with the experimenter introducing him or herself and an assistant. It will be explained to the child that we are going to play a game of "store," and the child will be the "storekeeper" and the assistant the "customer." The child will be given a storekeeper's apron to wear during the task.

An assistant will be trained to request the orders that will be filled by the children, thereby "freeing" the experimenter to observe, and record by hand, the child's solution strategies. All task trails will be videotaped.

It will then be explained to the child (the storekeeper) that the assistant (the customer) will be asking for orders of vegetables to take home in "cartons." A table will be used as the storekeeper's counter. Tomatoes will be the vegetable requested and used in the study. Three egg carton-like six-space containers will be used as the cartons in which the tomatoes will be placed to fill an order.

It will then be explained that the customer will ask for orders for tomatoes, one order at a time, and it will be the child/storekeeper's job (for each order) to make up any one of the cartons so that it has the same number of tomatoes in it that the customer asks for. It will be explained that there will be three cartons in front of the child most of the time and two cartons in front of the child some of the time. The child will be told that the tomatoes will be in the cartons and arranged in a different way before each new order was asked for. The child will be told that to fill each order they are free to move the tomatoes around, from carton to carton, in any way they want in order to end up with a carton that has the right number of tomatoes in it. The child will be asked to pick up or point to the completed carton when they have filled an order (to avoid confusion about which carton the child intends to be the completed order).

The task will not begin until the experimenter feels assured that the child understands the task. The task will begin with the assistant asking for an order of tomatoes, saying "I need (a number from 3-5) tomatoes to take home with me in a carton." The subject will then fill the order, moving tomatoes to do so, and lifting the carton with the completed order. Errors, though noted, will not be mentioned, and the task will not stop. Arrays will be preset and arranged between orders. Each new order will be requested as "I need to take home (a number from 3-5) tomatoes. Could you please give me that many in a carton?" At the end of each filled order the assistant will say thank you and the tomatoes will be arranged according to preset directions and a new order will be requested. All empty cartons will be preset to a different position for each order. The sequence for the presentation of orders and carton positioning is based on a partial latin square design for both sequence of order presentation and carton positioning (see Appendix C). To insure against either a placement or an ordering effect, participants were separated into four separate groups (six participants per group), and each group was presented with the task trial orders in a different sequence. Furthermore, the sequence differed each of the three days of the task trials. Concurrently, the placement of the arrays was changed for each task problem for each different day. Simply put, over the three task trails participants were never presented with the same problem with the same array configuration.

After sixteen orders, the child was thanked. The task was repeated two more times on subsequent days for all participants. The second and third session procedures differed only in the sequencing of the orders.

The design for the sixteen orders are as follows (see Appendix D for all Task Problems). Twelve of the orders form the core of the task. These twelve orders all involve the use of three arrays placed in front of the child. One of the three arrays is always presented empty to the child at the beginning of each order. These 12 orders are divided into four problem types (three orders per problem type). These four problem types are designed so that each type requires the use of a different numerical operation in order to complete an optimal solution strategy. Problem type A requires the use of an addition operation to complete an optimal solution. Problem type B requires the use of a subtraction operation to complete an optimal solution. Problem type C requires the use of either an addition or subtraction operation to complete an optimal strategy. Problem type D requires the recognition that the correct answer already exists in the array configuration given to the child.

As stated, there are three requested orders that constitute each problem type. For each problem type one order request will be for 3 tomatoes, one order request will be for 4 tomatoes, and one order request will be for 5 tomatoes.

In sum, of the twelve orders that involve three arrays there are four problem types (of three orders each) and three levels of order requests (3, 4 and 5). Each problem type incorporates each order request level. The following chart offers a schematic diagram of the design:

Problem type A Optimal solution by addition; Three order requests for 3, 4, 5	Problem type B Optimal solution by subtraction; three order requests for 3, 4, 5
Problem type C Optimal solution by addition or subtraction; three order requests for 3, 4, 5	Problem type D Optimal solution by recognition; three order requests for 3, 4, 5

These twelve orders were designed for successful solutions via several different strategies (see Appendix E). Each of the twelve orders can be solved by an optimal strategy (least physical moves); by a literal strategy (movement of tomatoes to the empty carton); by an in-between strategy, involving a number of physical moves that falls in between the number of moves required by an optimal and literal solution strategy by adding tomatoes to the carton that has the smaller addend; and by an "other" strategy, involving the use of any number of moves but does not correspond to any of the three defined strategies listed above.

The remaining orders involve the use of two arrays placed in front of the child. No empty array is available for these four orders. These four orders are comprised of two orders from both Problem type A and Problem type B, presented again to the child without the third empty array. The order request levels are for 3 and 4. Given that these four orders do not have an empty carton available for the child, a literal solution is not possible for these orders. This aspect of the design is included to observe whether the environmental constraints of the task impact on performance. Particularly, to see if literal strategists perform more efficiently if a literal solution is not available to them.

Given that research goals focus on an examination of the solution strategies employed, and not the child's capacity to achieve success, the task was designed to insure a low error rate for successful solutions. Order requests will be kept at numerical levels appropriate for the application of preschoolers' practical arithmetic (Gelman & Gallistel, 1978: Ginsburg, 1977).

Children were scored for all trials by the number of moves used to fill the order as well as by type of strategy used. Task performance was videotaped and coded.

Task Coding

Participants were scored for both number of moves used and by strategy type.

A. Number of moves. Scores were tallied for each child in the following manner. Initially, an individual raw score (number of moves used) was recorded for each problem in Day 1, Day 2, and Day 3. A final score was determined. This score was the number of moves used above the optimal strategy (the fewest number of moves necessary to successfully solve the problem). Scores were summed for all problems in each session. There was also a combined sum of scores for all sessions and there was a determined mean score for session 1, 2, and 3.

B. Strategy type. Participants were scored by strategy type selected for each problem in each session. Total scores for the frequency of strategy type used was determined for each task trial and for all sessions combined. Coding was as follows:

1 = optimal strategy

2 = in-between strategy

3 = literal strategy

4 = other

C. Addition/Substraction Preference. Participants were scored for their choice of addition or subtraction on problems where both strategy types are available. Coding was as follows:

0 = NA

1 = Addition

2 = Subtraction

Post Task Interview Procedure, Materials and Coding

Following the third trial, after all orders were completed, each child was asked two interview questions. These interview questions were designed, primarily to determine if the child is aware of more efficient strategies than they employed during the task (Siegler, 1990; Scribner, 1988). The interview consisted of a child filling two orders which are duplicates of orders that have appeared in the task. These were addition orders (with a Min strategy optimal solution) and one substraction order (see Appendix F for Post Task Interview Problems). A series of qualitative questions/probes followed each order request (see Appendix G for Post Task Interview Script). The post task interview was videotapes and coded.

Coding for Post-Task Interview Question

Participants' performance were coded and scored similarly to task problems. In addition, performance was recorded, noting:

A. Description of strategic plan if offered by subject.

B. Notation re: whether an alternative strategy is identified

Yes

No

C. Notation re: which strategy is identified by participant as 'easiest.'

D. Description of participant's explanation of which strategy is easiest (if offered).

Post Test Procedure, Materials and Coding

Subsequent to the culmination of all task trials and the post task interview, each participant was administered a post-test. The post-test is an abbreviated form of the pre-test described above. The post test was scored similarly.

CHAPTER IV

RESULTS

Introduction

The results will be presented in three sections. The first section includes all analyses and results regarding aspects of participants increasing efficiency over time (exposure to the task). The second section includes all analyses and results regarding participants choice of strategy selections, including: the use of a repertoire of strategies in general; the use of an optimal solutions strategy in particular, the changing pattern of strategy use over time (exposure to the task); preference for addition or subtraction; and participants knowledge of alternative strategies. The third section includes all analyses and results regarding both the correlation of pre-test performance to aspects of task performance, as well as pre-test performance in comparison to post-test performance.

Section 1: Efficiency Over Time

The core analysis conducted, in regard to participants' increased performance efficiency over time, was a fully crossed factorial design analysis of variance (ANOVA) with four independent variables. A 4x3x2x3 repeated measures analysis of variance was performed to determine the main effects of problem type (optimal solution by addition, subtraction, addition or subtraction, or recognition), of order request (three, four, or five), of gender, and of day (Day 1, Day 2, Day 3).

A detailed reporting for each independent variable will follow. However, general results for the overall task are: as predicted, the main effect for day was significant, means show that participants used significantly fewer physical moves to satisfy the task on Day 3 than on Day 1; the main effect of gender was not significant; the main effect of order request was significant, means show that participants rate of change regarding efficiency was greater for order requests of four or five than for order requests of three; the main effect of problem type was significant, means show that participants rate of change regarding efficiency was greater for those problems where optimal solutions were solved by subtraction than for other problem types. No interaction effects of any kind were found.

Error Rate. An initial aspect of performance to be addressed was whether children completed the task accurately, regardless of number of moves used or solution strategy employed. The overall error rate was less than 1% for all children (N=24) for all problems (15 problems x three days = 45 problems per participant). Errors consisted entirely of mistakes of plus or minus one tomato in the final order and were corrected by the participant. Corrections were not included in computing task performance scores.

Efficiency Over Time. Each participant was given a score for the number of moves employed to complete the task. Participants were scored for the number of moves employed for each individual problem, for the number of moves employed on each trial day (Day 1, Day 2, Day 3) and for the total number of moves employed for all three days (Day 1 + Day 2 + Day 3). Lower scores, therefore, represent more efficient solutions (i.e., efficiency representing the fewest number of physical moves). Differences between the number of moves employed on Day 3 and Day 1 were calculated. Note that scoring consists of the number of moves employed above the number of moves required to successfully solve the task problem with the use of an optimal solution strategy (the fewest possible moves required to successfully satisfy the task).

For all results to be discussed, analyses were conducted separately for all participants' performance on the twelve task problems that included three arrays for participants' performance on the three task problems that included two arrays (due to experimenter error, the data from one of the four task problems involving two arrays had to be eliminated from all analyses), and for participants' performance on the combined fifteen task problems (twelve problems of three arrays + three problems of two arrays). Results regarding each independent variable (day, gender, order request, problem type) will be discussed in the following order: performance on twelve problems of three arrays; and performance on three problems of two arrays.

Differences Between Trials: For 12 Problems Involving Three Arrays. To determine if any significant performance differences existed between Day 1, Day 2 and Day 3, the performance data from all participants for problems involving three arrays (twelve problems) were entered into a 4x3x2x3 repeated measures analysis of variance.

This analysis reveals a significant difference in performance across trials for all participants considered as one group (N=24) for the twelve problems involving three arrays, $F(2, 44) = 3.74$, $p < .05$ (.032). Table 1 lists the means for all subjects (N=24) performance on the twelve problems involving three arrays on Day 1, Day 2, and Day 3, and the difference in their Day 3 and Day 1 performance. Means show that participants, as a group (N=24), used significantly fewer moves to successfully satisfy the task on Day 3 than on Day 1.

TABLE 1

Mean Numbers (and Standard Deviations) of Moves for All Participants (N=24) by Day (Trial) for 12 Problems of Three Arrays

	Day 1	Day 2	Day 3	Difference between Day 3 and Day 1
All participants	19.66 (13.96)	15.50 (11.63)	13.17 (12.70)	6.50 (13.49)

For day (trial) $F(2,99) = 3.74$, $p < .05$

<u>For Three Problems of Two Arrays</u>. To determine if a significant difference in performance across trials existed for all participants considered as one group (N=24) for the three problems involving two arrays, the data was entered into a two by three repeated measures ANOVA (gender and day). Analysis indicates that the main effect of day was not significant for the three problems that included two arrays, $F(2, 44) = .18$, $p > .05$ (see Table 2 for means).

TABLE 2

Mean Numbers (and Standard Deviations) of Moves for all Participants (N=24) by Day (Trial) for Three Problems of Two Arrays

	Day 1	Day 2	Day 3	Difference between Day 3 and Day 1
All participants	3.83 (3.90)	3.75 (3.70)	3.33 (2.78)	.50 (3.38)

For trial F (2,44) = .18, $p > .05$

Move Toward Efficiency. To further understand the main effect of day, and to determine which trial days significantly differed from one another, a series of t-tests were conducted. T- tests were conducted on the means of all subjects' (N=24) performance comparing Day 1 and Day 2, Day 2 and Day 3, and Day 1 and Day 3. This series of t-tests were conducted separately for all participants' performance for twelve problems involving three arrays. Given that the main effect of day was not significant for the three problems involving two arrays, t- tests were not conducted.

For the twelve problems involving three arrays, Day 2 performance was not significantly different from Day 1, t (23) = 1.73, $p > .05$ (.097), or from Day 3, t (23) = 1.39, $p > .05$ (.179). However, performance is significantly more efficient by Day 3, t (23) = 2.36, $p < .05$ (.027).

Efficiency Over Time -- Differences Between Gender. To determine if any significant performance difference existed between participants by gender, task scores were analyzed using the aforementioned 4x3x2x3 repeated measures analysis of variance. For participants' performance on the twelve problems involving three arrays, the main effect of gender was not significant, at the .05 level $F(1,22) = 3.41$, $p < .078$. (See Table 3 for means.)

TABLE 3

Mean Numbers (and Standard Deviations) of Moves for all Participants (N=24) by Day (Trial) for 12 Problems of Three Arrays

	Day 1	Day 2	Day 3	Difference between Day 3 and Day 1
Girls	22.06 (14.29)	18.56 (12.60)	16.06 (14.22)	6.00 (15.51)
Boys	14.87 (12.77)	9.38 (6.30)	7.38 (6.21)	7.50 (8.99)

For gender $F(1,22) = 3.41$, $p = .078$

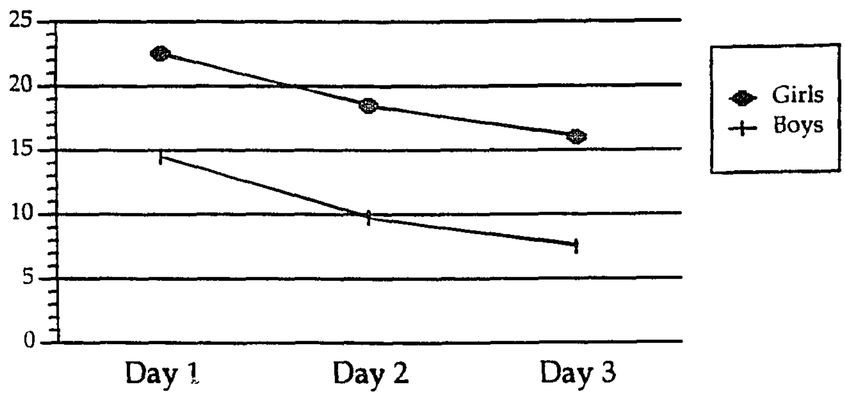
For the three problems involving two arrays, participant task scores were analyzed using a 2x3 repeated measures analysis of variance (gender and day). The main effect of gender for the three problems involving two arrays was not significant, $F(1,22) = 1.38$, $p > .05$.

The means displayed in Table 3 show that both boys' and girls' performance improved over the three trials (Day 1 to Day 3). However neither group's performance independently improved to a statistically significant degree. Secondly, as means show, boys, as a group, started at a more efficient level of performance although it was not statistically significant (Day 1). No interaction effect was discovered between gender and any other variable (day, problem type, order request). Further analytic exploration proceeded by conducting a t-test in order to compare the means of the differences of boys' and girls' efficiency performance (the difference between Day 3 and Day 1). No significant difference was found in the overall rate of change in the efficiency of performance between boys and girls: for the twelve problems of 3 arrays, $t(22), -.30, p > .05$.

Thus, boys performed at a more efficient rate than girls (though not statistically significantly better) and there was no significant difference in the measure of change of efficiency by gender over time. Boys began the task on Day 1 displaying a more efficient performance than girls, but both gender groups became more efficient at a fairly similar rate. The following graph displays the performance comparison for boys and girls (see Figure 1).

FIGURE 1

COMPARISON OF THE MEAN SCORES AND RATE OF IMPROVEMENT BY GENDER



Problem Type

To determine the main effect of problem type (optimal by addition, optimal by subtraction, optimal by addition or subtraction, optimal by recognition), participants' task scores were analyzed by the aforementioned 4x3x2x3 repeated measures ANOVA for the twelve problems.

The main effect of problem type was found to be significant, $F(3,66) = 22.14$, $p < .001$). Participants required fewer moves on Day 3 than on Day 1 for all problem types. Therefore, to further understand the significant main effect of problem type, a series of t-tests were conducted on the mean number of moves of the difference between Day 3 and Day 1 performance for all four problem types for the twelve problems involving three arrays (see Table 4). Although participants used fewer moves over trials for all problem types, they improved at a greater rate over trials where optimal solutions were solved by either subtraction or by the recognition of an existing correct order in an array (see Table 4).

TABLE 4

Mean Numbers (and Standard Deviations) of Moves and measure of Improvement for all Participants (N=24) for Individual Problem Types (Difference between Day 3 and Day 1)

	Day 1	Day 2	Day 3	Difference between Day 3 and Day 1
Problem Type A (Optimal by Addition)	3.04 (2.18)	2.33 (2.51)	3.00 (2.74)	.04 (2.10)
Problem Type B (Optimal by Subtraction)	6.21 (4.80)	4.86 (3.75)	3.92 (4.07)	2.29 (5.09)
Problem Type C (Optimal by addition or subtraction)	4.71 (5.20)	4.25 (5.02)	2.25 (3.21)	2.46 (5.47)
Problem Type D (Optimal by recognition)	5.71 (5.30)	4.04 (3.60)	4.00 (4.36)	1.71 (5.73)

The difference between the measures of improvement for subtraction and addition problems was significant, $t(23) = -4.14$, $p < .001$, as was the difference between the measures of improvement for addition and recognition problems, $t(23) = -3.87$ $p < .001$. The difference between the measures of improvement for subtraction and subtraction/addition problems was also significant, $t(23) = 2.71$ $p < .05$ (.012).

The difference the measures of improvement between addition and addition/subtraction problems approached significance, $t(23) = 1.99$ $p = 0.58$. The difference between the measures of improvement for addition/subtraction problems and recognition problems also approached significance, $t(23) = 2.02$, $p > .05$. The difference between the rates of improvement for subtraction and recognition problems were not significant, $t\ (23) = .29\ p > .05$. Results show that performance increased in efficiency for all problem types. However, increase in efficiency was significantly greater for subtraction problems, and to a lesser extent, for recognition problems.

Order Request

To determine the effect of order request, participants' performance scores were analyzed by conducting the aforementioned 4x3x2x3 repeated measures ANOVA. The main effect of order request for twelve problems involving three arrays was found to be significant, $F\ (2,44) = 9.05\ p < .001$ (see Table 5).

TABLE 5

Mean Numbers (and Standard Deviations) of measure of Improvement
for all Participants (N=24) for Order Request
(Difference between Day 3 and Day 1)

between Day 1	Day 1	Day 2	Day 3	Difference Day 3 and
Order requests for three	4.37 (3.59)	3.50 (3.28)	3.75 (3.23)	.63 (4.04)
Order requests for four	6.79 (6.37)	5.12 (4.45)	4.00 (5.37)	2.79 (7.57)
Order requests for five	8.50 (6.82)	6.86 (5.39)	5.42 (5.46)	3.08 (6.70)

Means show that participants overall used fewer moves on Day 3 than Day 1 for all order requests (3, 4 and 5). To further understand the main effect of order request, a series of t-tests were conducted to compare the measure of improvement for the different order requests. Improvement was greater for order requests of "four" or "five" across all problem types than requests of "three." Improvement on order requests of "five" compared to "four" was not significant $t(23) = 1.18$ $p > .05$; "five" compared to "three" was significant, $t(23) = 2.94$ $p < .05$; and "four" compared to "three" was significant, $t(23) = 2.81$ $p < .05$.

Comparison of Two Array and Three Array Problems

One prediction of the current research was that the removal of the possibility of a literal solution strategy (by removing the empty array) would enhance performance. Therefore, a comparison was made of the two array problems and their three array counterparts. Table 6 presents the means.

TABLE 6

Mean Numbers (and Standard Deviations) of Moves Used to Solve
2 Array Problems and 3 Array Counterparts for All Participants (N=24) by Day

	Day 1	Day 2	Day 3	Difference between Day 3 and Day 1
2 Array	3.83 (3.99)	3.75 (3.76)	3.33 (2.78)	.53 (1.72)
3 Array	5.03 (3.82)	3.07 (3.40)	3.42 (3.53)	1.59 (1.08)

To determine if participants significantly differed in their performance on the two types of problems, a t-test was conducted. Results indicate that there was no significant difference found in performance, $t(23) = -1.30$, $p > .05$. Means show, however, that although task performance was not statistically significant, group performance was more efficient on two array problems, although the measure of improvement was greater for three array problems.

Section II Solution Strategy Selections

This section includes all analyses and results regarding participants' choice of strategy selections, including: their use of a repertoire of strategies; the use of an optimal solution strategy in particular; the changing pattern of strategy use over trials; their preference for addition or subtraction; and their knowledge of alternative strategies.

In order to understand participants' solution strategies, test performance was coded for strategy type as well as the number of moves used. For each problem, participants' performance was coded as "optimal," "in-between," "literal" or "other." "Other" strategies include all solutions that are not exact manifestations of "optimal," "in-between" or "literal" solutions.

In order to determine if any significant performance differences existed regarding participants use of particular strategy type across trials, the data was analyzed by separate ANOVAs.

Participants' performance was entered into a 2x3 repeated measures ANOVA (gender and day). A separate analysis was conducted for each strategy type. The main effect of gender for all strategy types was found to be not significant. Furthermore, no interaction effects were found.

The Use of a Variety of Strategy Types. As predicted, participants engaged in the use of the full repertoire of the strategy solution types. 21 out of 24 participants engaged in the use of all strategy solution types and all 24 participants engaged in the use of at least 3 out of 4 strategy solution types.

The Changing Patterning of Strategy Selection: Optimal Strategies. As predicted, participants engaged in the use of optimal strategies. Twenty-three out of 24 participants used optimal strategies at least once in the course of the three days. Furthermore, means show that participants' use of optimal strategy increased in frequency over the course of the three trials but was not statistically significant, $F(2,46) = .84$, $p > .05$. Table 7 presents the means of participants' use of an optimal strategy.

TABLE 7

Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Optimal" Strategy Solution by Day

	Day 1	Day 2	Day 3
All participants	6.92 (3.98)	7.67 (3.57)	7.67 (4.27)

For trial $F(2.46) = .84$, $p > .05$

In-between Strategies: Twenty-three out of 24 participants engaged in the use of an in-between strategy as part of their solution strategy repertoire. Use of an "in-between" strategy did increase over time, but the increase is not significant, $F(2,46) = .24$, $p > .05$ (see Table 8).

TABLE 8

Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "In-Between" Strategy Solution by Day

	Day 1	Day 2	Day 3
All participants	2.26 (1.46)	3.25 (1.96)	3.00 (1.82)

For trial $F(2.46) = .24$, $p > .05$

Literal Strategies: Twenty-one out of 24 participants engaged in the use of "literal" strategies as part of their repertoire of solution strategies. Although participants' use of a "literal" strategy did increase over time, the increase is not significant, $F(2,46) = .05$, $p > .05$ (see Table 9)

TABLE 9

Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Literal" Strategy Solution by Day

	Day 1	Day 2	Day 3
All participants	1.75 (2.83)	1.75 (2.63)	1.88 (3.03)

For trial F (2.46) = .05, p > .05

<u>The Other Strategies</u>: All participants engaged in the use of "other" solution strategies. A decrease in the use of "other" strategy solutions was significant by Day 3, $F(2,46) = 6.76$, $p < .05$ (see Table 10).

TABLE 10

Mean Numbers (and Standard Deviations) for All Participants (N=24) of Problems Solved by an "Other" Strategy Solution by Day

	Day 1	Day 2	Day 3
All participants	3.33 (2.88)	2.33 (2.30)	1.50 (2.09)

For trial F (2.46) = 6.76, p > .05

Addition/Subtraction Preference: Overall, participants did not display any significant preference for addition or subtraction when either numerical operation would result in a successful optimal solution strategy. Participants who chose the optimal solution strategy on "C" type problems (problems on which an optimal solution can be reached by either subtraction or addition) chose, on an average, an addition operation 48% of the time and a subtraction operation 52% of the time.

Awareness of Alternative Strategy: In a post-test interview, the vast majority of participants (22 out of 24) displayed no awareness of any alternative solution strategy other than the one the child had chosen during the actual task. The two participants who reported an awareness of an alternative strategy both offered less efficient solution strategies.

Section III: Pre-Test and Post-Test Analyses

This section includes all analyses and results regarding 1) the correlations of pre-test scores to aspects of task performance, and 2) the correlation of pre-test scores to post-test scores. Participants' pre-test performance was scored by whether or not they correctly solved the problems.

Pre-test Scores. Two features of the participants' performance are particularly noteworthy. First is that all participants successfully completed all aspects of the pre-test that involved concrete cues. The second feature is that none of the participants successfully completed all of the abstract questions and approximately 75% of the participants were unable to answer any abstract questions.

Pre-Test and Gender. To determine if any significant performance differences existed by gender on the pre-test, a t-test was conducted on the means of participants pre-test scores. There was no significant gender difference regarding pre-test performance, t (22) = .56, $p > .05$.

Pre-test to Task Correlation. Pearsonns' correlations were performed to determine if a correlation existed between participants' pre-test scores and participants' task performance. Contrary to predictions, no significant correlation was found to exist between overall pre-test scores and overall task performance with all participants considered as one group, (N=21, r = -.1880, $p >.05$. Further analysis was conducted to determine if correlations existed between aspects of participants' pre-test performance and aspects of task performance. No significant correlation was discovered. Appendix H lists all pre-test to task correlations.

Pre-test to Post-test Comparisons

To determine if any significant performance difference existed between the pre-test and the post-test, analyses were conducted comparing participants' pre-test and post-test performance. No significant difference was found, t (23) = .00, $p > .05$. Pre-test to post-test comparisons were considered between two aspects of pre-test and aspects of post-test performance and were found to be not significant: pre-test abstract to post-test abstract, t (23) - .30, $p > .05$; and pre-test concrete to post-test concrete, t (23) .20, $p > .05$.

CHAPTER V

DISCUSSION

A primary goal of this study was to determine if pre-schoolers engage in the use of a repertoire of available successful solution strategies when engaged in a practical task. It was hypothesized that pre-schoolers would engage in the use of a variety of successful strategies. This hypothesis, based on the premise that children do not discover one successful strategy and engage in its use exclusively, is in accordance with recent thinking of researchers investigating childhood problem solving (i.e.Siegler,1989).

Repertoire

Results clearly show that, as predicted, children select from, and engage in the use of, a full repertoire of solution strategies when solving a mathematically oriented practical task. Participants as a group, on all trials, engaged in the use of the full range of defined strategies in order to solve the task. Importantly, all individual participants engaged in the use of the full repertoire of strategies. Interestingly, both individually, and as a group, participants used the full repertoire of strategies right from the beginning of the task on Day 1. Although the patterning of strategies changed over the three days of trials (as will be discussed shortly), children displayed the use of the full repertoire at the onset of the task.

Some theorists have previously posited that preschoolers' development proceeds differently than from what these results suggest. Siegler, prior to 1989, emphasized the rule as a basic unit of analysis and depicted development as a sharp qualitative change from use of one rule to use of another, resulting in solution strategies replacing, rather than co-existing with, previous ones. Siegler (1989) began to depict development differently, emphasizing a gradual change in the distribution of strategy use, with new strategies co-existing with previous ones.

Error Rate

Furthermore, as predicted, the repertoire of selected solution strategies were all successful. All participants selected strategies that led to a successful completion of the task. This high level of accurate task completion suggests both that the goal directedness inherent in practical problem solving requires the selection of successful strategies and that the cognitive demands of the strategies fall within the range of the children's cognitive ability. The selection process, obviously, requires the simultaneous rejection of unsuccessful strategies.

The Identification of Specific Strategies

Another goal of this study was to identify the specific solution strategies that comprise the strategy repertoire. Results show that children engaged in the use of the following defined strategies: an optimal strategy where the task was solved using the fewest number of physical moves (this strategy, as stated, depending on the individual problem, either required a min addition or subtraction operation, or the recognition of an existing correct array); an in-between strategy (which required an addition operation based on adding the larger of two addends to the smaller one); and a literal solution (which required the counting of the correct amount into an empty array). Importantly, children also chose less defined "other" strategies. Simply put, these "other" strategies consisted of movements of tomatoes that successfully solved the task but were not an exact optimal, in-between or literal strategy.

The frequency of the use of the "other" strategy was unanticipated at the onset of the study. It was initially hypothesized that the participants would select a strategy that did not conform to a strictly defined optimal, in-between, or literal strategy only when the child was attempting a literal strategy but used extraneous moves.

However, results show that many of the "other" strategy choices appear to be attempts at optimal and in-between strategies as well. Many of the "other" strategies involved the use of fewer moves than a literal solution required, and yet the strategy did not meet the exact criteria that would label it an optimal or in-between strategy. Obvious speculation would conclude that these "other" strategies were attempts at an optimal, an in-between or a literal strategy, and, moreover, at a significant frequency, participants chose an "other" strategy for a particular problem during one trial and subsequently chose a defined strategy for the same problem in the next trial. However, it would be presumptuous to state definitively that these "other" strategies were "attempts," and therefore it was decided to identify such strategies, en masse, as "others" rather than to sub-group them as attempted optimals, attempted in-betweens, or attempted literals.

Efficiency

It was the critical organizing feature of the study design to determine if children used more efficient strategies over time, that is, did they, with increasing frequency, choose strategies that required less physical moves to solve the practical task.

As predicted, results show that participants over time choose more efficient strategies to solve the practical task. Efficiency was measured in two ways: 1) the raw number of physical moves used to successfully complete the task; and 2) the solution strategy employed.

Efficiency: Number of Moves

Results indicate that over the course of the three task trials, participants as a group used significantly fewer moves to successfully solve the practical tasks. Although the move towards efficiency was evident on each successive task trial the movement toward efficiency was not significant between any two successive days. A two-day task trial design would not have resulted in any significant findings. By extension, it can be predicted that efficiency would have increased even further if more trial days had been included in the design. It can be assumed that at some point there would be a leveling off.

Again, it is important to note that the task is designed in order that participants could successfully complete it using a variety of strategies. The task is carefully constructed so that no motivational conditions were imposed on the participants to engender efficiency. No time pressure existed, no instruction was offered, and the saving of physical energy was not relevant to this task. The finding, therefore, is as follows: without any observable or identifiable external motivation to do so, preschoolers naturally and spontaneously move toward more efficient solution strategy selections when exposed to a mathematically oriented practical task.

There are significant features to this movement towards efficiency. It is important to understand the effect of gender, problem type and order request to more fully understand the tendency towards efficiency.

Gender

No significant difference with regard to performance by gender was found. There were, however, noticeable differences. Primarily, both boys and girls performed more efficiently at a similar rate over the course of the three task trials. Furthermore, neither boys nor girls, as a group, used significantly fewer moves by Day 3. The one aspect of performance, with regard to gender difference, that was considerable, was how much more efficiently boys performed at the onset of the task. Simply put, boys and girls progressed in efficiency at a similar rate. Yet, boys begin the task using fewer moves. Why?

It was not within the research scope of this study to explore the complex constellation of factors that could be considered in answer to this question. Obviously, both the social and cultural environmental context and the possibility of innate predisposition might be entertained in any current debate. However, the finding of importance here is that gender was not a significant variable with regard to on-going performance upon exposure to a mathematically oriented practical task.

Problem Type

Results show that children as a group used fewer moves by Day 3 of the task for all problem types. Only on problems where the optimal solution was reached with the use of subtraction or the recognition of the current order in an existing array did children use significantly fewer moves on Day 3 than on Day 1. The mean scores reveal a simple explanation for this result. Participants, as a group, performed at a highly efficient level on addition problems right from the onset. They started off well and became somewhat more efficient. Performance on subtraction and recognition problems was considerably less efficient at the onset and then progressed in efficiency to levels approaching, but not reaching, performance levels on addition problems. This more dramatic increase in efficiency accounts for the significant results.

As prior research has shown (Ginsburg, 1977; Gelman 1978), three- and four-year-olds have more "comfort" with addition operations which may account for their initial efficient performance on problems that required addition.

A different interpretation is necessary in order to understand the children's performance on the array recognition problems. Although it was not explicitly stated to the participants, it is possible that on Day 1 of the task participants had a pre-conceived orientation that some tomatoes had to be moved in order to satisfy the order and therefore were biased against searching for a correct existing array. Although the context was a play store, the children knew that they were being observed and may have formulated the game as requiring some manipulation of tomatoes in order to satisfy the customer's request. By the completion of Trial 1, children would have come to understand that some correct orders existed in the presented arrays. Some of the extra moves on Day 1 for recognition problems may have been simply that the child did not attend to the possibility that correct order was present and subsequently began the process of moving tomatoes.

Order Requests

Results show that children as a group used fewer moves by Day 3 of the task for all three levels of order request. However, children used significantly fewer moves on Day 3 than on Day 1 for requests of 4 and 5. Similarly to the dynamic cited for the addition problems above, children began the task on Day 1 at a highly efficient rate for order requests of 3. As studies regarding early childhood math skills show, children manipulate arrays of three and less with considerably more ease than arrays of 4 or 5. Children's rate of improvement with regard to efficiency was considerable for order requests of 4 and 5. However, children performed consistently at a more efficient rate for orders of 3.

Participants' use of a significantly fewer number of moves over the course of the practical task on both subtraction problems and for order requests of 4 and 5 is confirmation of Scribner's "head for hand" theory (1988). Scribner's study showed that dairy workers engaged in a more sophisticated mental organization of activity that resulted in the use of a physical effort-saving solution strategy requiring fewer moves. Young children, in the current study, increasingly engaged in the efficient use of subtraction operations which are identified as more cognitively complex than addition operations. Children, therefore, over trials, engaged in more sophisticated mental activity and saved physical effort. Children's increased efficiency in regard to order requests of 4 or 5 is evidence of a similar dynamic. The successful manipulation of these larger arrays with the use of fewer moves requires a more sophisticated mental reorganization of the task that results in a physical effort-saving strategy.

By focusing on the task variables, it becomes evident that the children's significantly increased efficient performance required more sophisticated mental activity.

Efficiency: Solution Strategies

Importantly, another goal of this study was to identify the changing nature of the child's choices regarding their selection of solution strategies as the children were exposed to the task over time. Specifically, the study was designed to explore whether children used the most efficient, or optimal, strategy more frequently over time. This movement towards the increasingly frequent selection of an optimal strategy over time, when other successful strategies are simultaneously available, has been determined to be a significant feature of adults' practical thinking (Scribner, 1988). A hypothesis of the present study stated that pre-schoolers will specifically select optimal strategies more frequently upon repeated exposures to a practical task. An important aspect of this hypothesis is that there exists no apparent external motivation for the child to make such selections. A variety of strategies, from the most to the least efficient, all result in successful solutions. As stated, there were no time constraints placed on the child. Furthermore, differences in time between strategy choices would be minimal. There was no instruction, no modeling, no demonstration, nor any other indication to suggest that an optimal solution is more desirable.

Unlike the exploration of optimal solutions with adults (Scribner, 1988), the present study did not offer the motivation of the savings of any considerable physical energy. For Scribner's dairy workers the selection of an optimal strategy saved the workers considerable expenditure of physical labor. This is a highly significant factor for a manual worker with a full day's worth of physical work ahead of them. Neither the expenditure, nor the savings, of hard physical work can be considered a factor for the three- and four-year-olds engaged in the present study. The savings of the physical energy involved in the movement or two of tomatoes cannot be considered a motivating factor that would drive efficient strategy choice.

Results show that children used optimizing solution strategies when confronted with a task that could be successfully solved by a variety of solution strategies. It was hypothesized that the study would reveal the emergence of an optimal solution strategy during the course of the task. Contrary to expectations, children chose optimal solution strategies as one of their repertoire right from the onset of the trial. Children did choose optimal solutions more frequently over the course of the study but not to a significant degree. They also chose other defined strategies (e.g., in-between and literal) more frequently over the course of the study, but, again, not to a significant degree. Simply put, children began the task using the full repertoire of defined solution strategies and increasingly used more efficient strategies over the course of the three trials. However, the significant and unanticipated finding in regard to the pattern of strategy selections is the sharp decrease in the use of the "other" strategy over the course of the task. Clearly, the children moved, at a significant rate, from using the undefined "other" strategy to using one of the three defined strategies over the course of the study. Whereas it had been hypothesized that children would move from literal to in-between to optimal strategizing as evidence of their increased proficiency, rather, the children moved from "other" to a defined strategy (whether it be literal, in-between, or optimal). This finding differs from Scribner's study with dairy workers and Cohen's (1989) prior study with preschoolers, where novice-to-expert problem solvers shifted from choosing literal solutions to optimal solutions upon exposure to the practical task. This dynamic may well have become evident if more trials were included in the current task design, but results show the novice-to-expert development in the current study was defined by movement from less defined to defined strategy selections.

It was also expected that new strategy selections would emerge over the course of the present study. Emergence did take place, but not as predicted. Rather than more efficient defined strategies emerging from less efficient defined strategies, emergence took the form of defined strategies of all levels of efficiency emerging from unformed "other" strategies. The significant decrease in the use of "other" strategies (characterized by extraneous moves), shows that the course of development in regard to efficiency is partially characterized by the ongoing discovery of efficient, defined strategies and the simultaneous rejection of strategies that require unnecessary physical movement.

Another goal of the present study was to determine if children prefer addition over subtraction as a mathematical operation. Contrary to prediction, children did not prefer addition. In the Type C problems, which were designed to specifically test this hypothesis, children who chose optimal solutions showed an almost equal preference for addition or subtraction operations. One possible explanation for this behavior is that children who chose optimal solutions had already, prior to the onset of the current study, established some comfort with subtraction operations and the "stage" of addition preference had already passed.

Awareness of Alternate Strategies

Children displayed almost no awareness of alternate strategies. This aspect of the study's design was incorporated to ensure that children were not performing at a less efficient level than they were capable of. The results from the qualitative interview of the model questions indicates that they were not.

Removal of Literal Solutions

It was expected that the removal of the empty array from several of the task orders might enhance performance in regard to efficiency. Specifically, the expectation was that literal problem solvers would choose a more efficient solution strategy if the possibility of a literal solution did not exist. However, results show that performance was not significantly enhanced. Rather, the "other" strategy type was often chosen to replace a literal strategy. The "other" strategy was not more efficient in regard to the number of moves required to successfully solve the task. Means showed that on Day 1, children as a group performed better on two array problems (where the empty array was removed). However, these two array problems come at the end of the Day 1 trial, which may account for more comfort, due to task exposure, and resulted in a slightly more efficient performance.

Children's Arithmetic and Practical Tasks

It was predicted that there would be a correlation between children's arithmetic skills and their performance on the practical task. Contrary to prediction, no correlation was found. The explanation for this lack of correlation may be explained by the fact that participants as a group performed similarly on the mathematics pretest. Therefore, there was little variance in the pretest scores that precluded finding correlations with task performance. Participants generally performed well on all aspects of the pretest that involved the child's use of concrete cues and, conversely, participants were generally unable to successfully answer any of the mathematics problems that were presented in an abstract form. Therefore, a correlation between young children's arithmetic abilities and their performance on a mathematically oriented practical task may exist, but the similar performance by all participants on this arithmetic pretest did not allow for a correlation of differentiation in individuals' arithmetic abilities and their practical task performance.

Cohen (1989), in a prior study, reported findings that suggested that the child's capacity to engage in the abstract manipulation of numbers, particularly the internalized use of an array, is a prerequisite for the development of an ability to acquire an optimal solution strategy. The psychological reorganization of a task, intrinsic to a least-effort solution strategy which involved the use of extra, internal, mental steps, can only occur if internalized and abstract number use is available. Further, strategy selection in Cohen's study (1989) appeared to be strongly influenced, if not determined, by the child's capacity for abstract number use. Two of the basic operations that comprise preschoolers' practical arithmetic, subtraction and the ability to recognize and manipulate an array, require the capacity for a more sophisticated abstract number use, in contrast to counting and addition. Findings suggested that the child's capacity to mentally manipulate an array is a significant feature of practical abstract numerosity.

The current study does not disprove these findings. However, it was unsuccessful in differentiating individuals' capacity for abstract number use.

Summary

Three- and four-year-old children, both individually and as a group, demonstrated that they select from a range of solution strategies in order to successfully accomplish a practical task. One of their selections is an optimal solution strategy. Furthermore, the pattern of their strategy selection suggests an orderliness. Characteristics and patterns of their strategy selection include the following:

1. Children's selected solution strategies are successful. All subjects selected strategies that led to a correct completion of the task.

2. Performance, over several exposures to the task, becomes significantly more efficient -- participants choose solution strategies that require fewer physical moves to successfully complete the task.

3. Children engage in a more complex mental reorganization of the task. The use of fewer physical moves is partially the result of an increasingly efficient use of both more sophisticated arithmetic operations and larger numerical arrays.

4. The pattern of strategy selection was orderly. As a group, children use more efficient solution strategies upon each new exposure to the task. Secondly, children use

less defined strategies upon each exposure to the task and simultaneously consolidate more defined strategies.

These characteristics and patterns are consistent with other research focused on adult practical thinking. Firstly, the high level of accurate task completion suggests that the goal directedness of practical problem solving requires the selection of successful strategies. This dynamic would also require the rejection of less efficient strategies. Similarly, this dynamic would demand the rejection of any strategy that requires mental capabilities not present in the problem solver. Secondly, development from novice to expert has been identified, partially, by the acquisition of an optimal solution strategy and its spontaneous appearance appears to be the result of task practice over trial (Scribner, 1984).

The mental activity studied here meets the evaluative criteria previously discussed that defines practical thinking. It is contexted mental activity embedded in a larger, purposeful daily activity. It is offered free of instruction and it is successful.

The findings of this study lead to certain other theoretical conclusions, conjectures and questions. Findings indicate, consistent with current research (Gelman & Gallistel, 1978; Ginsburg, 1977), that preschoolers are users of a practical, sophisticated, informal arithmetic, which functions in the service of solving everyday mathematical problems.

The outcome of the current research contributes to the debate and dialogue concerning both the definitions and the relationship of the intelligence subtypes 'practical' and 'academic.' Children's practical task performance, in all the dimensions already discussed, show that there is intellectual activity (manifested in both physical action and mental activity), which differs qualitatively from academic performance (as indicated by performance on the pre and post arithmetic tests).

Simply put, children manifested more sophisticated use of number operations and increasingly used more abstract arithmetic when engaged in a practical task. This finding suggests that children may, in other contexts as well, be engaged in more sophisticated arithmetic activity when such activity serves a larger, purposeful task, as compared to arithmetic operations performed as an end in themselves. This conclusion suggests that the development of young children's practical thinking proceeds differently than the young child's academic thinking. This is not to imply that the development of the child's practical and academic intellectual activities are completely separate and independent from each other. The relationship, however, remains unclear. Present research findings strongly suggest that the child's capacity to engage in the abstract manipulation of numbers, particularly, the internalized use of an array, is a prerequisite for the development of an ability to acquire an optimal solution strategy. The psychological reorganization of a task, intrinsic to a least-effort solution strategy, which involves the use of extra, internal, mental steps, can only occur if internalized and abstract number use is available. This concept of an internalized, abstract number use is in the theoretical tradition of Vygotsky's (1962) concept of inner speech.

The present findings suggest that one characteristic of developing practical thinking in the young child is its ongoing tendency towards efficiency. Essential to the manifestation of the increased efficiency is a more complex mental reorganization of the task. Simply put, mental work replaces physical effort.

This finding confirms Scribner's 'head for head' theory regarding the development of practical thinking in adults. We can assume, however, that the development of preschoolers' practical problem solving differs from adults in a significant way. For the child, there appears to be simultaneous development of skill within the domain-specific contingencies of a particular task, as well as a developing capacity for abstract number use and a developing ease for working with numbers in general.

The evidence of the movement towards more abstract thinking is as follows: 1) children increasingly chose subtraction operations; 2) children increasingly worked efficiently with larger arrays; 3) children increasingly chose solution strategies that demanded abstract manipulations of arrays.

These findings indicate unique aspects of the child's mind in action. The most significant is this correlation between the child's capacity for internalized, abstract number use and the acquisition of an optimal strategy. As stated, strategy selection appears to be strongly influenced, if not determined, by the child's capacity for abstract number use. Two of the basic operations that comprise preschoolers' practical arithmetic, subtraction and the ability to recognize and manipulate an array, require the capacity for a more sophisticated abstract number use, in contrast to counting and addition. Findings suggest that the child's capacity to mentally manipulate an array is a significant feature of practical abstract numerosity.

These characteristics and patterns are consistent with other research focused on adult practical thinking. Firstly, the high level of accurate task completion suggestions that the goal directedness of practical problem solving requires the selection of successful strategies. This dynamic would also require the rejection of less efficient strategies. Similarly, this dynamic would demand the rejection of any strategy that requires mental capabilities not present in the problem solver. Secondly, development from novice to expert has been identified, partially, by the acquisition of an optimal solution strategy and its spontaneous appearance appears to be the result of task practice over trial (Scribner, 1984).

The findings of the current research lend considerable support to an overall understanding of practical thinking. It appears that movement towards efficiency (i.e., the "trading" of physical action for mental activity) is an essential and inherent characteristic of practical activity. Scribner concluded that this dynamic, as evidenced in adults, can be attributed to a desired diminishing of physical work. However, such motivation does not account for the appearance of the movement towards efficiency in young children. If we assume that movement towards efficiency is an inherent feature of practical activity, the question arises: why is this dynamic not manifest in all observed practical behavior? People are not always efficient nor do they necessarily become more efficient in regard to their daily activities. It is hypothesized that the manifestation of efficiency may not always be apparent due to competing and opposing developmental processes. The existence of cognitive and affective dynamics, functioning simultaneously within the individual, may be creating the opposing forces that are inhibiting or masking the manifestation of efficiency. Certainly, certain levels of cognitive sophistication are requirements for the individual to be able to choose more efficient solution strategies. If development in some cognitive areas has not been attained, more efficient strategy solutions cannot be acquired. Psychodynamic and affective processes may also be providing goals that are opposing and inhibiting to a movement towards the solution of more efficient solutions.

Given these theoretical concerns, certain key questions and issues present themselves as topics for further research. If the movement towards efficiency is considered a key characteristic of practical thinking, when, how and why do new, more efficient strategies emerge in the child's repertoire? Specifically, do children always acquire optimal strategies and at what point, if any, does the child become, exclusively, an optimizer? What is the relationship of the child's academic and practical thinking? Lastly, future research is needed to study the acquisition of optimal solution strategies in other contextual domains.

APPENDIX A

PRETEST

A. Counting 1 to 6

Counting backwards 6 to 1

B. Determination of "how many"/recognition of an array

2

3

4

5

6

These are presented in random order and concrete cues are available.

C. Addition -- There are 20 addition questions. 10 asked and answered verbally, and 10 asked verbally, but concrete objects will be presented simultaneously. The form of the verbal questions will be: "If you had x cans of food and I gave you y, how many would you have?" The form of the concrete object questions will be: "Here are x cans of food, if I add this many more (y), how many are there?" Probe to determine employed strategy (i.e., min, counting from the smaller addend, counting from zero) are taken from Siegler (1990).

1. 1+1
2. 1+2
3. 1+3
4. 1+4
5. 1+5
6. 1+6
7. 2+2
8. 2+3
9. 2+4
10. 3+3

Subtraction

There will be 8 subtraction problems. Questions will be divided similarly to the addition.

Subtraction problems

1. 6-1
2. 5-2
3. 4-2
4. 3-1

APPENDIX B

TASK -- OVERALL DESIGN

Task consists of 3 sessions (Day 1, Day 2, Day 3).

Each session consists of:

- 16 orders
- 2 novel questions (Day 3 only)

The make-up of the 16 orders includes:

- 3 orders where the optimal solution is achieved by an addition/min strategy
- 3 orders where the optimal solution is achieved a subtraction strategy
- 3 orders where the optimal solution is achieved by either an addition or subtraction strategy
- 3 orders where the optimal solution is achieved by array recognition
- 2 orders where the optimal solution is achieved by an addition strategy, with no literal solution available
- 2 orders where the optimal solution is achieved by a subtraction strategy, with no literal solution available.

APPENDIX C

Ordering and Array Placement of
Task Problems by Participant Groups

EXAMPLE

PARTICIPANT GROUP 1

Problem # 1 3A3 FME

- 1 — The sequence in which the problems were presented
- 3A3 — Order request; Problem type; Number of arrays present
- FME — The placement of arrays

F = the "fullest" carton

M = the "medium" filled or less filled carton

E = the empty carton

GROUP 1

Problem #:		DAY 1		DAY 2		DAY 3	
	1	3A3	FME	3B2	FM	4D3	MFE
	2	4B3	EMF	4A2	MF	5A3	EMF
	3	5C3	FEM	5A2	FM	3D3	EMF
	4	3B3	MFE	3A2	MF	4A3	MEF
	5	4C3	EFM	5B3	EFM	5B3	MEF
	6	5D3	MEF	3C3	MEF	4B3	EFM
	7	3C3	MFE	4B3	FME	5C3	FME
	8	4D3	EMF	5D3	FEM	3B3	MEF
	9	5A3	FME	5D3	FEM	3B3	MEF
	10	3D3	MEF	3A3	MFE	4C3	FEM
	11	4A3	EFM	4D3	FEM	5D3	MFE
	12	5B3	FEM	5A3	MEF	3C3	FME
	13	3A2	FM	3B3	EFM	4A2	FM
	14	4A2	MF	4A3	MFE	5A2	MF
	15	3B2	FM	4C3	FME	3A2	FM
	16	5A2	MF	3D3	EFM	3B2	MF

GROUP 2

		DAY 1		DAY 2		DAY 3	
Problem #:	1	4D3	MFE	3A2	FM	5B3	EFM
	2	5A3	EFM	4A2	MF	3C3	MEF
	3	3D3	EMF	3B2	FM	4B3	FME
	4	4A3	MEF	5A2	MF	5C3	EFM
	5	5B3	MEF	3A3	FME	5D3	FEM
	6	4B3	EFM	4B3	EMF	3A3	MFE
	7	3A3	EMF	5C3	FEM	4D3	FEM
	8	5C3	FME	3B3	MFE	5A3	MEF
	9	3B3	MEF	4C3	EFM	3B3	EFM
	10	4C3	FEM	5D3	MEF	4A3	MFE
	11	5D3	MFE	3C3	MFE	4C3	FEM
	12	3C3	FME	4D3	EMF	3D3	EFM
	13	4A2	FM	5A3	FME	3B2	FM
	14	5A2	MF	3D3	MEF	4A2	MF
	15	3A2	FM	4A3	EFM	5A2	FM
	16	3B2	MF	5B3	FEM	3A2	MF

GROUP 3

		DAY 1		DAY 2		DAY 3	
Problem #:	1	5B3	EFM	4A2	FM	3A3	FME
	2	3C3	MEF	5A2	MF	4B3	EMF
	3	4B3	FME	3A2	FM	5C3	FEM
	4	5C3	EFM	3B2	MF	3B3	MFE
	5	5D3	FEM	4D3	MFE	4C3	EFM
	6	3A3	MFE	5A3	EFM	5D3	MEF
	7	4D3	FEM	3D3	EMF	3C3	MFE
	8	5A3	MEF	4A3	MEF	4D3	EMF
	9	3B3	EFM	5B3	MEF	5A3	FME
	10	4A3	MFE	4B3	EFM	3D3	MEF
	11	4C3	FME	3A3	EMF	4A3	EFM
	12	3D3	EFM	5C3	FME	5B3	FEM
	13	3B2	FM	3B3	MEF	3A2	FM
	14	4A2	MF	4C3	FEM	4A2	MF
	15	5A2	FM	5D3	MFE	3B2	FM
	16	3A2	MF	3C3	FME	5A2	MF

GROUP 4

		DAY 1		DAY 2		DAY 3	
Problem #:	1	3A3	FME	4A2	FM	5B3	EFM
	2	4B3	EMF	5A2	MF	3C3	MEF
	3	5C3	FEM	3A2	FM	4B3	FME
	4	3B3	MFE	3B2	MF	5C3	EFM
	5	4C3	EFM	5D3	MFE	5D3	FEM
	6	5D3	MEF	5A3	EFM	3A3	MFE
	7	3C3	MFE	3D3	EMF	4D3	FEM
	8	4D3	EMF	4A3	MEF	5A3	MEF
	9	5A3	FME	5B3	MEF	3B3	EFM
	10	3D3	MEF	4B3	EFM	4A3	MFE
	11	4A3	EFM	3A3	EMF	4C3	FME
	12	5B3	FEM	5C3	FME	3D3	EFM
	13	3A2	FM	3B3	MEF	3B2	FM
	14	4A2	MF	4C3	FEM	4A2	MF
	15	3B2	FM	5D3	MFE	5A2	FM
	16	5A2	MF	3C3	FME	3A2	MF

APPENDIX D

TASK PROBLEMS

Problem Type A -- Optimal Solution by Addition

1. Order Request for 3

X	X

X	

2. Order Request for 4

X	
X	X

X	

3. Order request for 5

X	
X	X

X	X

Problem Type B -- Optimal Solution by Subtraction

4. Order Request for 3

X	X
X	X

X	

5. Order Request for 4

X	
X	X
X	X

X	X

6. Order request for 5

X	X
X	X
X	X

X	X

Problem Type C -- Optimal Solution by Addition or Subtraction

7. Order Request for 3

X	X
X	X

X	X

8. Order Request for 4

X	
X	X
X	X

X	
X	X

9. Order request for 5

X	X
X	X
X	X

X	X
X	X

Problem Type D -- Optimal Solution by Recognition

10. Order Request for 3

X	
X	X

X	X

11. Order Request for 4

X	X
X	X

X	
X	X

12. Order request for 5

X	
X	X
X	X

X	X

Optimal Solution by Addition -- No literal available

13. Order Request for 3

X	X

X	

14. Order Request for 4

X	
X	X

X	

Optimal Solution by Subtraction -- No literal available

15. Order Request for 3

X	X
X	X

X	

16. Order Request for 4

X	
X	X
X	X

X	

APPENDIX E

POSSIBLE STRATEGIES AND NUMBER OF MOVES BY ORDER

	1	2	3
Optimal	1(+)	1(+)	2(+)
In-between	2(+)	3(+)	3(+)
Literal	3(+)	4(+)	5(+)
Subliteral	>3	>4	>5

	4	5	6
Optimal	1(-)	1(-)	1(-)
In between	2(+)	2(+)	3(+)
Literal	3(+)	4(+)	5(+)
Subliteral	>3	>4	>5

	7	8	
Optimal	1(-) or 1(+)	1(-) or 1(+)	1(-)
or 1(=)			
In between	-	-	-
Literal	3(+)	4(+)	5(+)
Subliteral	>3	>4	>5

	10	11	12
Optimal	0	0	0
In between	1(+)	1(+)	3(+)
Literal	3(+)	4(+)	5(+)
Subliteral	>3	>4	>5

	13	14
Optimal	1(+)	1(-)
In between	2(+)	3(+)
Subliteral	>3	>4

	15	16
Optimal	1(-)	1(-)
In between	2(+)	3(+)
Subliteral	>3	>4

(+) = Addition

(-) = Subtraction

APPENDIX F

Post Task Interview Problems

1. Order Request for 4

X	
X	X

X	

2. Order Request for 3

X	X
X	X

X	

APPENDIX G

POST TASK QUESTIONS -- QUALITATIVE/PROBE QUESTIONS

For each of the 2 orders.

1. I would like you to give me an order of X. Please go ahead and make up the order.
2. Let's pretend that your friend wants to do the same thing. How would you explain to him what you did?
3. Let's pretend that your friend is also a storekeeper, and they have to fill the same order, but they don't want to be a copycat and do it the same way as you. How else could they do it? What other way would you show them?
4. If the child does not describe or perform another solution strategy, the interviewer performs an alternative.
5. What of these ways is the easiest way to fill this order?
6. Why is that the easiest way?

APPENDIX H

Pre-test to Task Correlations for All Participants

Overall Pre-test to Overall Task Performance	r = .1880, p > .05
Overall Pre-test to Day 1 Task Performance	r = .2455, p > .05
Overall Pre-test to Day 2 Task Performance	r = .2338, p > .05
Overall Pre-test to Day 3 Task Performance	r = .0111, p > .05
Pre-test Addition Problems to Task Problem Type A	r = .2273, p > .05
Pre-test Subtraction Problems to Task Problem Type B	r = .1238, p > .05
Overall Pre-test to Problem Type C	r = .1302, p > .05
Overall Pre-test to Problem Type D	r = .1910, p > .05
Overall Pre-test to Overall Task Rate of Improvement	r = .1927, p > .05
Pre-test Concrete Addition to Task Problem Type A	r = .1230, p > .05
Pre-test Abstract Addition to Task Problem Type A	r = .0210, p > .05
Pre-test Concrete Subtraction to Task Problem Type B	r = .1810, p > .05
Pre-test Abstract Subtraction to Task Problem Type B	r = .1415, p > .05

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