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**The efficiency of forward rates as forecasts of future interest rates**

**Biolsi, Robert A., Ph.D.**

**City University of New York, 1989**

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THE EFFICIENCY OF FORWARD RATES AS FORECASTS  
OF FUTURE INTEREST RATES

BY  
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A

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## Abstract

THE EFFICIENCY OF FORWARD RATES AS  
FORECASTS OF FUTURE INTEREST RATES

by

Robert A. Biolsi

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The purpose of this paper is undertake an empirical investigation of forward interest rates as forecasts of realized ex post interest rates. In particular, the attempt will be made to reconcile the Expectations Hypothesis of the Term Structure of Interest Rates with the past literature that has been highly critical of the Hypothesis based upon its seeming incompatibility with the data. By using various testing methodologies, it is shown that Forward Rates can be viewed as sufficient statistics for future interest rates.

Forecast accuracy is first explored as a means of testing this efficiency. Risk premia are implicitly accounted for

in this analysis. It is shown that forecast accuracy increases with the forecast horizon. This is followed by evaluating the general slope of the yield curve as a predictor of future changes in interest rates. The results confirm the notion that the predictive power of the Term Structure is dependent upon the forecast period in question.

The paper then proceeds to evaluate the forecast error of the forward rates. It is shown that to a large extent, these errors can be attributed to factors that could not have been plausibly anticipated under efficient markets. Two such factors are explored in depth: unanticipated inflation and policy changes of the Federal Reserve. The purpose here is to illustrate that efficiency is not solely a function of forecast accuracy, but rather how well the bond markets assimilate new information.

The paper concludes by postulating and examining empirically a mean-reversion model of interest rate behavior. By developing such a model, not only is a theoretical framework constructed to address the question of expectations formation, but in addition helps explain the pattern of forecast accuracy given above.

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## CHAPTER I.

Section I: Introduction.

The Expectations Hypothesis of the Term Structure of Interest Rates has been broadly established in Economics/Finance literature since Fisher's Theory of Interest.<sup>1</sup> Based on the notion that markets are rational and efficient, it postulates that the relationship among interest rates on bonds of different maturities contain forecasts of future short-term interest rates. The object of this paper is to examine empirically the extent to which the forecasts of future interest rates embedded in the term structure have met the criteria of efficient markets. That is, how well in the past have bond prices reflected information available to investors as opposed to reflecting market rigidities, speculation, or other sources of market inefficiency. The focus in the paper will be to counter arguments of some researchers who have concluded that the bond market has failed various tests of its efficiency.

In the strong form of the Expectations Hypothesis, the current term structure at a point in time contain unbiased predictions of future interest rates. These predictions, which according to the hypothesis are

implied by the slope of the term structure, will be referred to in this paper as forward rates. The hypothesis itself revolves around the process of bond prices reaching equilibrium by the act of bond dealers (speculators), buying and selling bonds of different maturities based upon their future expectations. Implicit in the argument in its pure form is a competitive market with homogenous expectations among borrowers and lenders. For example, if two-year interest rates were higher than one-year rates, the Hypothesis asserts that the market is predicting a higher one-year rate one year in the future. If this were not the case, economic profits could be made. Presumably, bond dealers would realize the opportunity, borrow at the one-year rate and lend at the two-year rate. Excess profits would be made by such transactions if rates did not rise. The rebalancing of portfolios on the part of participants to reap the excess expected profits would thus raise two-year bond prices, lower one-year prices. This would continue until prices adjusted such that the expected future interest rate would become equal to the implicit forward rate. For the actual calculation of these implied rates from the maturity structure, refer to Appendix A.

Since the 1950's, the Expectations Theory has been subjected to numerous assaults. Most of the empirical

studies (e.g. Culbertson (1957) <sup>2</sup>, Hamburger and Platt (1975) <sup>3</sup>, and Shiller, Campbell and Schoenholtz (1983) <sup>4</sup>) have focused on the forecasting accuracy of the implied forward rates with regard to future interest rate movements. In general, these studies have concluded that the Hypothesis fails empirical testing because of the poor forecasting accuracy of these forward rates. Numerous other models have been suggested in these papers, which have been proposed as better alternatives because of greater forecasting accuracy. These have ranged from simple extrapolative models (Hamburger & Platt (1975)) to polynomial distributed lags (Modigliani and Sutch (1966) <sup>5</sup> )

Market efficiency, however, does not necessarily imply forecasting accuracy. Rather, it revolves about the notion that the market incorporates available information into the structure of asset prices. This notion was summarized by Hamburger and Platt (1975), "Rather, the analysis would proceed along a second route to determine whether the forecasts assumed to be represented by the forward rates are based on all information available at the time the forecast was made. <sup>6</sup>" A poor forecast can be justified within the context of efficient markets if the forecast error is due to the subsequent addition of information that was

not available at the time prices were determined. The focus of this paper, therefore, is not only to examine the forecast accuracy of forward rates as a test of the bond market's efficiency, but also to analyze the source of the forecast error. If forecast errors can be traced to the addition of information that became available only after the forecast was made, then it is possible to validate the bond markets as rational. The definition of rationality being the efficient incorporation of available information in bond prices.

In examining the Expectation's Hypothesis in terms of market efficiency, our plan is as follows: In Chapter 2, the forecast accuracy of forward rates will be examined. Several tests will be performed to determine the predictive accuracy of forward rates, including nonparametric tests, regression analysis, and moving average models. In this chapter, our analysis allows for uncertainty by recognizing the existence of risk premia. In addition, a broader spectrum of forward rates will be utilized than has generally been used in similar works. The purpose here will be then to distinguish the forecast performance of forward rates from other empirical studies. Chapter 3 will then explore the relationship between interest rates from opposite ends of the interest rate spectrum, and its

import for future interest rate movements. Here, individual forward rates will not be analyzed, but rather the general slope of the yield curve as a predictor of future interest rates.

Once the forecast accuracy of forward interest rates is analyzed, Chapters IV will examine the forecast error. In this chapter, the focus will be on explaining the error of the forecast in terms of new information available to investors. The factor that will be used to explain the forecast error will be unexpected inflation. In Chapter V, the Expectations Hypothesis will be tested against an explicit alternative hypothesis in terms of Federal Reserve policy. The focus here again will be to trace forecast error to information received subsequent to the pricing of the bonds from which the forward rates were determined. This will be followed in Chapter VI, by a model which will be developed to explain how the market determines forward rates. The expectations-generating model will be used as a means of giving theoretical content to the pattern of prediction errors of forward rates. It is hoped that in doing so, the criticisms of the Expectations Hypothesis in past empirical studies can be reconciled to the present results. A summary would then follow in Chapter VII reiterating the major conclusions of the paper.

## Section II. Literature Review

The Expectations Theory was first formally expounded by Irving Fisher (1930) <sup>7</sup>. Assuming a world of certainty, long-term interest rates would be an average of current and future short-term interest rates. This relation between short and long-term rates would be brought about by arbitrage activities of investors. The Hypothesis received its first rigorous formulation by Lutz (1940), who asserted "We can conceive of the long-term rate as a sort of average of the future short-term rates."<sup>8</sup> Its simple intuitive appeal, combined with a relatively stable interest rate environment made the theory difficult to challenge.

Hick's (1946) <sup>9</sup> offered the first major criticism of the theory with his claim that liquidity risks have a significant impact. According to this line of thinking, short-term investments entail less risk, and hence more liquidity relative to longer rates. Consequently, long-term rates will rise above short-term rates because of premiums attached to compensate for their lack of liquidity. "Hicks argued that even if short-rates are ...expected to remain unchanged, the forward rate can be expected to exceed the current short-rate by a risk premium which must be offered the holder of the bond to compensate him for assuming the

risks of price fluctuations.<sup>10</sup>" Cox, Ingersoll, Ross (1985) <sup>11</sup> refined the analysis to make a rigorous case for time varying term premia. In addition, unlike Hicks, they pointed out that the premia need not be monotonically increasing the time to maturity, nor uniformly positive. This model was tested empirically by Ramaswamy and Gibbons (1987) <sup>12</sup> which lent statistical support for the time-varying risk premia model of Cox, Ingersoll, and Ross.

However, the first pointed dissent was offered by Culbertson (1957) in an empirical study. It concluded: "We find the expectational theory unsatisfactory on theoretical grounds and inconsistent with the behavior of the rate structure in the postwar period.<sup>13</sup>"

Another important observation made in the paper concluded: "Holding period yields for long-term debt fluctuate over a much wider range than those for short-term debt...<sup>14</sup>.", foreshadowing later criticism of the Hypothesis. The basic premise here was that investors divide the bond market into different markets, or segments. These different institutional preferences for borrowing and lending maturities play the predominant role in determining the term structure. Culbertson's of the decomposition of the bond market into separate markets led this line of reasoning to be

dubbed the Market Segmentation Theory. It was summarized by Modigliani and Sutch (1966) as follows: "The proponents of this approach suggest that both lenders and borrowers have definite preferences for instruments of a specific maturity, and for various reasons, partly due to institutional factors and regulations constraining financial intermediaries, will tend to stick to securities of the corresponding maturity without paying attention to rates of return on other maturities. 15"

They modified this theory with the third major alternative to the Expectations Hypothesis, which they referred to as "The Preferred Habitat" theory. This differs primarily from the Market Segmentation approach in that the rigidities do not effect entirely separate markets but rather entail an additional interest premium to entice agents to leave their "Preferred Habitat".

This was followed by a series of empirical studies using different time series and countries whose consensus conclusion was that forward interest rates (extrapolated from the yield curve) were poor predictors of actual future short-term interest rates. In fact, according to some of these studies, simple

extrapolative models provide greater forecasting accuracy than forward rates do! Some of the more prominent studies that fall into this group include Modigliani and Schiller (1973) <sup>16</sup>, Hamburger and Platt (1975) <sup>17</sup>, and Schiller (1985) <sup>18</sup>, Mankiw and Summers (1983) <sup>19</sup>. The evidence of poor predictive power suggested that the expectations theory failed an important test of rationality. Alternative expectation models were summarized by Dobson, Sutch, and Vanderford (1976). <sup>20</sup> The overall thrust of these papers was to place the onus of proof on proponents of the Expectations Hypothesis to explain how investors could be persistently wrong within an efficient markets context.

In Schiller (1979)<sup>21</sup>, dissent from the theory took on a different slant. In that paper, the evidence seemed to show that short-term rates historically displayed less variability relative to long-term rates than could be consistent with the Expectations Hypothesis. This would then conflict the theory which necessarily implies that long-term rates being averages must be characterized by less volatility, as first suggested by Culbertson.

Attempts to rehabilitate the theory have come primarily

from Fama. In studies from 1976 <sup>22</sup>, and 1981 <sup>23</sup>, it was shown that the predictive power of forward rates when adjusted for time-varying term premiums to be much more accurate than previously thought. In 1987 Fama and Bliss <sup>24</sup>, introduced the notion that the predictive power of forward rates increases as the time horizon involved lengthens. Fama 1988 <sup>25</sup> decomposed forward rates into future rate forecasts, as well as forecasts of real rates and risk premia.

## Chapter II.

### The Forecast Accuracy Of Forward Rates

#### Section I. Introduction

The Expectations Hypothesis basically revolves about the notion that expectations of future interest rates are embedded in the current slope of the yield curve. These expectations can, according to the theory, be calculated from the relationship of yields of different maturities. In Appendix A, an explanation of the calculation of these implied forward rates is given. In this chapter, the predictive power of forward rates is analyzed.

While such analyses permeate the literature, the presentation here is done for several reasons. First, account will be taken for the Hicksian risk premium. It will be shown that such premia obscure the predictive accuracy of implied forward rates. Secondly, a broader spectrum of forward rates will be presented here than in most of the literature, and which have certain desirable robustness properties. Following on the results of Fama and Bliss (1987) <sup>1</sup>, it will be asserted that the predictive accuracy of forward rates is not uniform. Rather, a clear pattern emerges when a range of forward rates is analyzed. Third, some tests that are not found in the literature will be performed in order to obtain additional insights into the

information contained in forward rates.

In the present chapter, we use the following notation:

$S(t, t+1)$  will be used to denote the one year spot yield to maturity on a bond, priced at time  $t$ .

$F(t, t+\tau, t+\tau+1)$  will denote the one year forward rate of return at time period  $t + \tau$ , implied by the yield curve at time  $t$ .

In this paper, The Expectations Hypothesis, as formulated by Hicks (1947), will be used as a model. "The forward rate of interest for any particular future week ... will be have to be higher than the short rate expected by these speculators to rule in that week, since otherwise they would get no compensation for the risk they are incurring; it will indeed have to exceed it by a sufficient amount to induce the marginal spectator to undertake the risk. The forward short-rate will thus exceed the expected short-rate by a risk premium which corresponds exactly to the 'normal backwardation' of the commodity markets. If short rates are not expected to change in the future, the forward rate will exceed the current short rate by the extent of this premium; if short rates are expected to rise, the excess will be greater than this normal level; it is only if short rates are expected to fall that the forward rate can lie below the current rate. " That is, the one-year interest rate  $r$  periods in the future is given by the  $\tau$  period forward rate at time  $t$  less a risk premium. Thus we have:

$$E\{S(t+\tau, t+\tau+1)\} = F\{(t, t+\tau, t+\tau+1)\} - \pi + u(t) \quad (1)$$

where  $\pi$  represents a Hicksian risk premium to compensate for the inherent uncertainty and should be positive according to the Hypothesis.  $u(t)$  is an error term with an expected value of zero.

The the no expected excess profit argument that yields equation (1) would go as follows: suppose the expected future interest rate at time,  $\tau$ , were less than the forward rate by an amount greater than necessary to induce agents to hold the risks of price change. Then bond dealers and other investors would purchase  $\tau + 1$  period bonds at time  $t$ , while selling short-term bonds. Such trading activity would then serve to lower the  $\tau$  period yield and consequently the  $\tau$  period forward rate until it reached equality with the expected rate plus an adjustment for risk. Conversely, if the forward rate were less than the expected one-year rate at time  $\tau$ , then speculators would seek to profit by selling  $\tau + 1$  period bonds at time  $t$ , buying short-period bonds with the proceeds. This would serve to depress the prices of  $\tau+1$  period bonds and consequently raise their yields so as to increase the  $\tau$  period forward rate until it reached equality with the expected rate adjusted for risk.

Subtracting  $S(t, t+1)$  from both sides of (1) then gives:

$$E\{S(t+1, t+2) - S(t, t+1)\} = F(t, t+\tau, t+\tau+1) - S(t, t+1) - \pi_t + u_t \quad (2)$$

It is common to refer to the difference between current forward and spot rates as the term premium. However, to avoid confusion in terminology with the above risk premium, it will be referred to as the Forward-Spot spread. Thus, the expected change in interest rates (the left argument) is seen to be a function of the difference between the current forward and spot rates and a risk factor. According to the Expectations Hypothesis then, if forward rates are higher than spot rates, expectations are for interest rates to rise. Conversely, a negative spread between forward and spot rates imply expectations of a fall in interest rates.

As was mentioned in the previous chapter, the forecast accuracy of forward rates has been subjected to much analysis. It has been the poor forecasting ability of forward rates, especially in terms of other expectations generating models, that has lead to the widespread disenchantment with the theory.

However, there are two serious flaws with simply estimating the degree of correlation between future spot rates and current forward rates. First, there is a very high degree of serial correlation with interest rate data, as has been

pointed out by Hamburger and Platt (1975) <sup>3</sup> among others. This problem is exacerbated by the fact that most of the empirical studies have relied almost exclusively on short-term forecast horizons. Consequently, simple extrapolative models, which have been used as basis of comparison for the Expectations Hypothesis, (e.g. Hamburger and Platt (1975) ) tend to generate impressive results using short-term data because of the first order autocorrelation. However, if the forecast period is extended beyond a year, the forecast accuracy of extrapolative models deteriorate rapidly.

The point to be emphasized here is that past researchers apparently operated on the assumption that if forward rates were shown to be relatively poor short-term predictors, then it should follow that longer forecast horizons would also lead to inaccurate forecasts. However, this paper is especially concerned with the forecast performance of forward rates over various time horizons. It will be shown that as the forecast horizon is extended, a much different portrayal of the predictive power of forward rates emerges.

The second major difficulty in empirical testing is that  $\pi_t$ , the risk premium is unobservable. Many of the empirical studies have tended to ignore this factor implicitly setting it to zero. As Fama and Bliss (1987) <sup>4</sup>

have pointed out, it is difficult to determine to what extent the risk premium obscures the correlation between current forward rates and future spot rates. "Thus the slope...estimates the split of variation in the forward-spot spread between the two terms the forecasted change in the one-year spot rate from  $t$  to  $t+x-1$  and the premium... expected return on an  $x$  year bond over the time  $t$  yield on an  $(x-1)$ -year bond. " "

To overcome these estimation difficulties, the approach in this section will be to use the first differences of equation (2). By subtracting the previous period's values, monthly values in this paper, the resulting equation becomes:

$$\delta\{S(t+1) - S(t)\} = \delta\{\text{Forward-Spot Spread}\} + v(t) \quad (3)$$

where  $v(t) = u(t+1) - u(t)$

The assumption being made here is that the risk premium does not change significantly from month to month. Note that this is not assuming a constant risk premium through time. In past studies, such as Fama and Bliss, risk premia were modeled explicitly. This technique, however, presented other problems. "This means....the forward-spot spread has power to forecast the change in the spot rate; but the regression fitted values only track all variation in the forecasts when the expected change in the spot rate and the expected premium...always vary in fixed proportions. This is a limitation ...."

In the subsequent tests, the ex post realized interest rate is substituted for  $E\{S(t+1)\}$ . Also, using first differences allows for various testing methods which would not be valid otherwise. That is taking the first differences removes the overlapping error terms for year-to-year changes in interest rates. The observations are now independent. For example, a  $\tau$  period observation of a one-year change in interest rates would include error terms  $\epsilon_t, \epsilon_{t-1}, \dots, \epsilon_{t-1,1}$ . The prior period's observations would include  $\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-1,2}$ . By taking first differences, all but  $\epsilon_t$  and  $\epsilon_{t-1,2}$  remain. The subsequent observation would then include only error terms  $\epsilon_{t+1}$  and  $\epsilon_{t-1,1}$ , which are independent of the prior period's.

Thus according to (3), if the forward-spot spread is positive, a negative first difference would imply a negative first difference in the one-year change in interest rates. That is, interest rate changes are becoming less positive or more negative. A result consistent with the Expectations Hypothesis. Conversely, a positive first difference in the forward-spot spread would imply future interest rate changes to be more positive or less negative. If this were not the case, then either market expectations, as embodied in the forward rate were erroneous, or the risk premium changed from period to period. As stated above, it will be assumed

that the premium does not change significantly from month-to-month. A test to determine whether this assumption approximately hold will be presented.

### Section II. A Regression Analysis

By using the first differences in equation 3, overlapping disturbances are removed. Consequently, the equation can be estimated using Ordinary Least Squares since disturbances are now independent. Also in the equation, the risk premium has been assumed to be canceled out. This will be a good assumption even if it is not strictly constant through time as long as the rate of change in the risk premium is not great.

Using a monthly time series from 1970 to 1987 then, the OLS regressions were run on Equation 3 using six different forward rates. The use of these forward rates in the analysis differs from that of most of the empirical tests of the Expectation's Hypothesis. As in Fama and Bliss (1987) <sup>7</sup>, the analysis here seeks to gauge the explanatory power of forward rates over longer time horizons. By being able to discern a pattern to forward rates of various time periods, it may be possible to reconcile the relatively poor forecasting ability of short-term forward rates with a broader interpretation of the Expectation's Hypothesis. Moreover, a pattern of increased forecast accuracy with time would also enhance the efficiency aspects of the by showing

that excess profit opportunities diminish over the long-term. The results from the various regressions are reported in Table I.

Table I  
Forecast Accuracy of the Forward-Spot Spread

<u>First Difference in:</u>	<u>R<sup>2</sup></u>	<u>S<sub>em</sub></u>	<u>a</u>	<u>b</u>	<u>DW</u>	<u>T-STAT</u>	
						<u>b=1</u>	<u>b=0</u>
F(t,t+1,t+2)-S(t,t+1)	.22	0.10	.02	.75	1.61	2.57	7.72
F(t,t+2,t+3)-S(t,t+1)	.27	0.09	.01	.78	1.69	2.36	8.42
F(t,t+3,t+4)-S(t,t+1)	.36	0.09	-.01	.92	1.66	0.86	9.90
F(t,t+4,t+5)-S(t,t+1)	.38	0.09	-.03	.92	1.60	0.88	10.19
F(t,t+5,t+6)-S(t,t+1)	.38	0.08	-.03	.81	1.46	2.28	9.77
F(t,t+6,t+7)-S(t,t+1)	.44	0.08	-.04	.85	1.60	1.88	10.59

There are several points worth noting from these results. First, longer forward rates generally display stronger results than near-term forward rates. A qualification here though is that the four-year forward rate is more significant than the five-year. Secondly, the forward-spot coefficients are uniformly positive, consistent with the Expectation's Hypothesis. Also, the Durbin-Watson statistics indicate that autocorrelation is not a serious problem here. In addition, the R<sup>2</sup> are relatively low. So while the forward rates display some explanatory power, to assert that bond markets are rational would require an analysis of the unexplained variation. Such will be the subject of subsequent chapters.

For the strong form of the Expectations Hypothesis to be true, the coefficient of the spread variable should be one

while the constant term should be zero. This is tested with restricted least squares estimation. As defined in Johnston (1984) <sup>10</sup>, the test of a hypothesis that the coefficients of a regression obey a set of linear restrictions is an F-test equal to

$$\{ (RSS - USS) / q \} / \{ USS * (n - k) \} \quad (4)$$

where RSS represents the Restricted Sum of Squares and USS represents the Unrestricted Sum of Squares.  
 q represents the number of restrictions  
 n represents the number of observations  
 k represents the number of parameters to be estimated

The results of this test are reported in Table II:

Table II

Restricted OLS Regression of Forecast Accuracy

<u>First Difference in:</u>	<u>RSS</u>	<u>USS</u>	<u>q</u>	<u>n-k</u>	<u>F-STAT</u>	<u>95% F</u>
F(t,t+1,t+2)-S(t,t+1)	192.8	186.7	2	201	3.28	3.04
F(t,t+2,t+3)-S(t,t+1)	167.4	162.7	2	189	2.72	3.05
F(t,t+3,t+4)-S(t,t+1)	147.8	147.2	2	177	0.36	3.05
F(t,t+4,t+5)-S(t,t+1)	144.3	143.4	2	165	0.51	3.05
F(t,t+5,t+6)-S(t,t+1)	126.5	122.1	2	.155	2.79	3.05
F(t,t+6,t+7)-S(t,t+1)	114.8	111.9	2	.143	1.85	3.06

In Fama and Bliss (1987), it is pointed out "The results suggest that when forward-spot spreads ... is mostly variation in the term premiums in current 1-year expected returns, and forward-spot spreads do not predict yield changes one year ahead." That is, that while Fama and Bliss find evidence for shifting risk premiums in their regressions, the above results are more supportive of the Expectation's Hypothesis. The negligible constant coefficients along with the positive slope coefficients are evidence for the notion that variations in the forward-spot

spread are rational predictions of future changes in interest rates.

It should be noted that the conclusions drawn by Fama and Bliss are a result of a different regression. In their analysis, regressions are run on ex post realized excess returns, which are in turn estimates of the risk premia. The attempt then was to take the risk premia into account explicitly rather than removing them from the regression as is done here.

In Table II, the joint hypothesis that the constant is zero and that the slope is one cannot be rejected in five of the six tests. This is evidenced by the F-Stats in the table. This provides some evidence that the Expectations Hypothesis, even in its strong form, can be supported by the empirical evidence. The fact that the constant coefficient is insignificantly different from zero while the slope is positive and significant contradicts the Fama and Bliss finding that changes in the forward-spot spread are mostly changes in risk. Here the evidence is the opposite. The change in the forward-spot spread is an unbiased predictor of the change in the change of future one-year interest rates.

In addition, the failure to reject the constant being equal to zero helps justify the assumption that the risk premia do not change significantly from month-to-month. As has been noted previously, other researchers have found evidence of a time-varying risk premium. This does contest those studies and in fact time-varying risk will be incorporated explicitly in Chapter IV.

### Section III. Nonparametric Tests

To test whether forward rates help to call changes in interest rates; nonparametric tests were performed to complement the analysis in Chapter II. The tests are designed to capture both the direction and the relative magnitude of changes in a dependent variable (changes one-year interest rates) versus the spread between between current implied forward rates and one-year spot rates.

These tests differ from the regressions previously run in that unlike OLS regressions, they tend to maintain their power even if the underlying probability distribution is not normal. The purpose here then is to counter arguments made by some researchers such as Roll<sup>10</sup>, who claimed that interest rate changes have fat-tailed distributions and consequently the significance of the associated T-Statistics are suspect.

#### 1. Sign Test

As mentioned previously, the Expectations Hypothesis implies that the first differences of the forward-spot spreads and the changes in interest rates be of the same sign. To perform a sign test then, values of the first differences in the change in one-year rates were divided by the first differences in the forward-spot spread. Counts were taken as to whether the resulting quotients were either positive or negative. The sign test is a simple measure of the number of positive values that were obtained

relative to negative values. Zero values were excluded from these tests.<sup>21</sup> The results are recorded in Table III for the various forward-spot spreads.

$$H_0: \text{Change/Spread} < 0$$

$$H_1: \text{Change/Spread} > 0$$

TABLE III  
Sign Test of Changes in One-Year Rates  
Divided by Forward-Spot Spread

<u>First Difference</u>	<u>Total #</u>	<u>No. Pos. Quotients</u>	<u>Z Value</u>
F(t,t+1,t+2)-S(t)	203	115	1.82
F(t,t+2,t+3)-S(t)	191	125	4.20
F(t,t+3,t+4)-S(t)	179	127	5.53
F(t,t+4,t+5)-S(t)	167	118	5.26
F(t,t+5,t+6)-S(t)	155	110	5.14
F(t,t+6,t+7)-S(t)	144	100	4.58

$$* Z = (\# \text{ pos} - (n/2)) / (\sqrt{n/2})$$

Under the null hypothesis of no relation, Z is distributed as a standard normal deviate. The above results serve to reinforce those of the previous analysis. For each of the forward-spot spreads, the null hypothesis that the two variables are of opposite signs can be rejected at 95% confidence. One qualification here, however, is that the Z values do not increase uniformly. While this somewhat detracts from the previous results, it is still true that the longer forward rates are more significant than the one and two year forward rates. The overall thrust of the test then is to confirm the fact that previous researchers may have been led to conclusions that were too general in

dismissing the predictive power of forward rates based solely on short-term predictions. The above results suggest that by failing to take into account longer time horizons, the predictive power of forward rates was understated.

## 2. Test of Wilcoxon <sup>12</sup>

In this test, a value of one is assigned to positive quotients, -1 for negative. These values are denoted Z. The values of the quotients are then ranked in terms of their absolute values from lowest to highest and are denoted R(i). The test is thus designed to not only look at the signs of the changes, but also at their relative magnitudes. The statistic W is then calculated, where

$$W = \sum Z(i) \cdot R(i) ; \text{ under the null hypothesis, } W \text{ is distributed normally } \{0, ((n \cdot (n+1) \cdot (2n+1)) / 6) \cdot .5\}$$

$$H_0 := W < 0$$

$$H_1 := W > 0$$

The following are the results of the test:

Table IV

Wilcoxon Test of Changes in One-Year Rates  
Divided by Forward-Spot Spread

<u>Forward-Spot Spread</u>	<u>W</u>
$\delta F(t, t+1, t+2) - S(t, t+1)$	3.59
$\delta F(t, t+2, t+3) - S(t, t+1)$	3.93
$\delta F(t, t+3, t+4) - S(t, t+1)$	4.33
$\delta F(t, t+4, t+5) - S(t, t+1)$	4.80
$\delta F(t, t+5, t+6) - S(t, t+1)$	5.37
$\delta F(t, t+6, t+7) - S(t, t+1)$	6.06

Again, the above results take into account not only the sign of the change but also their relative size. Table IV more fully confirms the regression analysis of Section II. For each forward-spot spread, the null hypothesis can be

rejected at 99% confidence levels. Moreover, as with the previous tests, the longer the forward rate that is used, the more confidently one can reject the null hypothesis that forward-spot spreads give little or erroneous forecasts of short-term interest rate changes. This is regardless of the underlying distribution of the error term.

#### Section IV: The Farsighted Bond Markets

The notion that the predictive power of forward rates improves as the time horizon increases is not novel. As has been noted, Fama and Bliss (1987) have made this claim. In addition, Mankiw and Summers (1984) in a different context alluded to this pattern. "Taken at face value, the results imply that the market is hyperopic: the market gives too little weight to the current rate and too much weight to the expected future rate or its determinants."<sup>13</sup> While it has been suggested previously, it tends to run against the grain of accepted economic theory. For example, Stein in a paper on futures pricing asserted "The longer the period between the production decisions and the subsequent consumption decisions, the greater the bias between the forward price and the subsequent spot price."<sup>14</sup> The object of this section then is to offer some explanation as to why such a counter intuitive idea would seemingly hold.

Using the 1970 to 1987 data set, differences were calculated

between forward rates stretching from one to six years and the one-year spot rate. Changes in spot rates equivalent to the time horizon of the implicit forward rate was also taken. Unlike the regressions of Section II, however, the error terms in Equation (3) were taken explicitly into account. As noted previously, the data is plagued by serial persistence. The objective here then is to use the error process to yield insights on the predictive pattern of forward rates.

Equation (2) is tested using a Moving Average Model. Moving Average models of various lengths, were tested. but there was a distinct tendency to obtain the best results from a third order moving average. Regressions were thus run on the following model:

$$S(t+\tau, t+\tau+1) - S(t) = \alpha + \beta \{F(t, t+\tau, t+\tau+1) - S(t)\} + MA(1) + MA(2) + MA(3) \quad (5)$$

The coefficient  $\beta$  is expected to be positive. That is, if forward rates are less than short-term spot rates (or short rates are higher than long-rates) the market is forecasting interest rates to fall. Conversely, if the spread between forward and spot rates is positive than a rise in rates is expected. The following are the results:

Table VMoving Average Model of Changes in Interest Rates

<u>Term</u> <u>Spread</u>	<u>Coefficient</u>	<u>R<sup>2</sup></u>	<u>T-Stat</u>	<u>DW</u>
F(t,t+1,t+2)-S(t)	0.13	.76	1.68	1.83
F(t,t+2,t+3)-S(t)	0.30	.86	4.52	1.55
F(t,t+3,t+4)-S(t)	1.04	.90	14.73	1.42
F(t,t+4,t+5)-S(t)	1.32	.88	17.06	1.45
F(t,t+5,t+6)-S(t)	1.54	.88	17.47	1.51
F(t,t+6,t+7)-S(t)	1.54	.89	19.73	1.66

As can be seen from Table V, weak results as evidenced by the T-statistic, is found for the one-year forward-spot spread. This is consistent with the results of previous research that suggests forward rates show no improvement over simple extrapolative models in forecasting future short-term spot rates over short time horizons. That is, while the forward rate's coefficient displays little explanatory power, the R<sup>2</sup> is relatively high. This suggests that the error terms provide much of the explanatory power over the short-term forecast period. This is consistent with studies that show that simple extrapolative models have substantial explanatory power over short time intervals. However, as the time horizon of the forward rate lengthens, the significance of the forward spot spread increases considerably, while the overall fit of the model levels off. This would suggest that the the moving average component of the model loses explanatory power as the forecast period lengthens.

These results confirm the long-term view that the market takes. The longer the implied forward rate, the more significant it seems to be in explaining the future course of interest rates. More to the point, the focus here is on the significance of the individual coefficients. The combined pattern of the  $R^2$  and the forward-spot t-statistics yields further insight. That is that for short term forecasts, the error components dominate forward rates in explaining future movements in interest rates. This is apparent from the low t-statistics for the one and two-year forward-spot spreads. However, the situation is reversed for longer-term forward rates. The t-statistics improve much more than the  $R^2$ .

A plausible explanation for such an observation would be that shocks to the financial markets overwhelm the markets predictions over short time intervals. This paper will take the stance, however, that the explanatory power of forward rates improve with time because of the ability of the policy makers to offset these shocks over time. This can be seen by the steady improvement in the significance of the coefficients of forward rates as the forecast period lengthens.

### Section V: Summary

In this chapter, the Hicksian model of the Expectation's Hypothesis was tested. In these tests, the focus was on decomposing the Forward interest rate into two parts: a risk premium and an expectation of future one-year interest rates. By using first differences in the analysis, the assumption was made that the expected change in the future spot interest rate could be isolated. The validity of this assumption withstood testing.

Having put the expectation component in testable form, it was tested for its explanatory power of actual changes in future interest rates. The results of varied tests are summarized as follows:

- (1) Forward interest rates offer significant explanatory power in terms of future interest rate changes.
- (2) This explanatory power directly with the length of the forecast horizon.
- (3) Past studies which relied on the forecast of power of forward rates which relied on forecast horizons of less than one-year were overly broad in their conclusions.

## Chapter III

Long and Short RatesSection I. Introduction

The Expectations Hypothesis has been frequently criticized on the grounds that the slope of the yield curve, especially the relationship between short-term and long-term interest rates, often gives the wrong signal concerning the future course of interest rates. For example, Shiller (1985) observes "... when long-term rates are above short-term rates, the long term interest rate shows a tendency to decline subsequently rather than rise."<sup>1</sup> If correct, Shiller's observation could cast serious doubt on the Hypothesis that the slope of the yield curve varies directly with the expected future change in interest rates.

Since Shiller's observation runs directly contrary to the intuition underlying the Expectations Hypothesis, it is important to reconcile this it with the previously reported results. The data set used previously in the paper was used to test Schiller's observation. A regression was consequently run on the following equation:

$$(1) \quad \text{Change}_t = a + b * \text{Spread}_t + u_t$$

where Change represents the one-year change in the rate on thirty year bonds, while the Spread represents the

difference between thirty-year and ninety-day bills. Under the Expectations Hypothesis the coefficient of the spread variable is expected to be positive. The results as reported below in Table I are seen to weakly confirm Schiller's observation.

Table I: Change In Long-Rates vs. Term Spread

b: -.3  
t: -4.52  
R<sup>2</sup>: .09  
D.W.: .128

It is obvious from the above results that nothing conclusive can be extrapolated from the regression. Moreover, this finding is not necessarily contrary to the Expectations Hypothesis. As previously pointed out, most models of the term structure include a risk premium for longer-term bonds, such as that proposed by Hicks (1947)<sup>2</sup>. Furthermore, this premium can obscure the true implicit forecast of future rates.

In the above results, as well as in studies by Shiller (1985)<sup>3</sup> and Mankiw & Summers (1984)<sup>4</sup>, the one-year change in long-rates is taken as the dependent variable. However, the Expectations Hypothesis of Lutz (1940)<sup>5</sup>, Malkiel (1962)<sup>6</sup>, and Meiselman (1962)<sup>7</sup>, implies that the long-rate on a coupon bond contains forecasts of multi-period changes in short-rates. (See Appendix B.)

Formally,

$$(2) S(t, t+30) = \bar{\Phi} + w_1 * S(t, t+.25) * \sum_{i=1}^{119} w(i) E(r(t))$$

where  $w(i)$  is a weighting factor.

$\bar{\Phi}$  is a risk premium

$r(t)$  is a future short-rate at time  $t$

That is, the long rate is a weighted average of expected future short-rates, where the weighting factor declines geometrically with time. Regressions such as that performed by Shiller and Mankiw & Summers on equation (1) effectively assume static expectations through time. In Chapter VI, the notion of revised expectations upon the receipt of new information will be explored. In this chapter, risk will be incorporated into the regression on (1) to examine the forecast power of the slope of the yield curve.

### Section II. Accounting For Risk

Given the discrepancy in the results of Chapter II with that of Shiller and Mankiw & Summers, further investigation would seem to be warranted. In this section, risk will be incorporated into the regression on (1). Support for this comes from varied sources such as Kane (1984) who observed, "Evidence developed in this paper indicate that term premia vary over time with the level of interest rates."<sup>10</sup>. Thus the claim to be made in this section is that the spread between long and short-rates can be decomposed into an expected change in long-rates as well as a time varying risk premia. Moreover, the relative magnitude of the two components changes through time depending upon the level of rates.

In particular, the higher the absolute value of interest rates, the lower the risk premia for holding long-term bonds relative to short-term bonds. Intuitively, when interest rates are low, it is plausible to conceive of investors forming expectations of a rise in interest rates. Conversely, when interest-rates are high, investors can be seen to be expecting a fall in rates. Such was the expectations generating model of Malkiel (1962) <sup>7</sup>. Consequently, the risk premium should vary inversely with the level of rates.

More formally, however, it can be shown that a bond price is more sensitive to interest rate changes when rates are low. Hence, the inherent risk is higher when interest rates are low. Modeling premia in this way is consistent with the model developed by Cox, Ingersoll, and Ross, who claimed "The instantaneous return premium on a bond is proportional to its interest elasticity."<sup>10</sup>

A measure of this shifting bond price sensitivity (risk) to changes in interest rates is known in finance circles as the Adjusted Duration of a bond. This weighted average of the time to maturity of a bond's cash payments, varies inversely with the level of interest rates. Formally,  $\mu = \text{dur}/(1+r)$

where  $\mu$  represents the interest elasticity of the price of a bond;  $dur$  is the Macauley duration, and  $r$  denotes the yield-to-maturity of the bond.

The Macauley Duration is itself equal to the weighted average of the discounted cash flows of the bond. (See Appendix C.) From the above relation then, the Adjusted Duration, and hence interest rate risk of a long-term bond, falls as interest rates rise. As stated in a standard textbook, "The percentage of change in bond price is equal to the change in yield times adjusted duration."<sup>11</sup>

In terms of the analysis here, the risk premium declines as rates increase. Not coincidentally, the spread between long and short rates tends to become negative when rates have risen to record levels (e.g. 1974, 1981). Not only do expectations tend to be revised in the opposite direction when rates reach extreme levels, but the risk premium of long bonds falls relative to that of short bonds due to the fall in their Adjusted Duration.

To observe whether in fact the unaccounted for risk premium in the above cited works can explain those results, the adjusted duration of thirty-year bonds is added as an explanatory variable to (1). The purpose here is to test whether by controlling for risk, the negative

coefficient of the term spread found by Shiller and Mankiw & Summers can be accounted for.

In the following regressions, first differences are taken on the relevant variables to correct for the serial correlation found in the data. In addition, various time horizons for the change in long-rates are taken.

Regressions were run with the adjusted duration, but after further testing it was found that the log of the adjusted duration produced better results. The results are reported in Table III.

TABLE III

Change In Long Rates vs Term Spread and Adjusted Duration

<u>Change in 30 Year Rate</u>	<u>30 Year-3 Month Spread (T-Stat)</u>	<u>Log of Adjusted Duration (T-Stat)</u>	<u>R<sup>2</sup></u>	<u>Durbin- Watson</u>
One-Year	-0.037 (-0.426)	.43 (2.48)	.02	1.34
Two-Year	0.04 (1.18)	13.52 (13.76)	.50	1.33
Three-Year	0.004 (.13)	12.32 (11.93)	.44	1.32
Four-Year	0.067 (1.81)	13.24 (12.24)	.48	1.35
Five-Year	0.01 (0.29)	12.66 (10.90)	.43	1.29

From the above results, two observations can be made.

First, changes in risk, as measured by the log of the adjusted duration, accounts for the largest proportion of the changes in long-rates. As measured by the T-statistics, this variable overwhelms the explanatory power of the difference in long and short-rates. Moreover, the coefficients of the adjusted duration variable are

highly positive, tending to confirm the textbook notion that as rates rise (adjusted duration falls), long-rates subsequently fall at least partly due to the reduced risk. Secondly, while not significant, the spread coefficients are now positive in four of the five regressions. Moreover, the one negative coefficient is actually statistically insignificant from zero. Controlling for the risk as measured by adjusted duration, the term spread generally signals the correct subsequent change in long-rates. It is thus somewhat possible to maintain the validity of the Expectations Hypothesis even with the results of Shiller and Mankiw and Summers by controlling for risk. In keeping with the previous chapter and encouraged by the previous sections results, regressions are run with just the first differences of the spread variable against the change in long rates. The assumption here again is that first differences will cancel out the risk premium. This will be true if the premia does not change significantly on a monthly basis. In addition, it alleviates the autocorrelation in the unadjusted data. The results are reported in Table IV.

Table IV

First Differences in Change in Long-Rates vs Term-Spread

<u>Change in long-rates</u>	<u>b</u>	<u>t-Stat</u>
1 year	-.3	-4.52
2 year	-.29	-3.03
3 year	-.11	-0.95
4 year	.4	2.99
5 year	.88	5.99

The point of the above table is illustrate that for short time intervals, the coefficient is of the wrong sign and counter to the intuition of the Expectation's Hypothesis. However, as the time interval lengthens, the results become consistent with the Expectations Hypothesis. This bolsters the central conclusion of the previous chapter that the bond markets tend to be farsighted.

### Section III. Summary

The focus of this chapter has been to address a criticism of the Expectations Hypothesis that has recurred in the literature. Specifically, the slope of the yield curve, as measured by the difference between very short and long-term interest rates, has persistently sent the wrong signal regarding the future course of interest rates. While confirming that the data appears to weakly support this assertion, the argument here has been to separate the risk inherent in such a relation from future expectations. This has been done here through utilizing the notion of adjusted duration. When controlling for risk using the log of adjusted duration, the analysis was designed to show that the evaluation of the results of works such as that of Shiller and Mankiw & Summers must be tempered by the

fact that risk was not accounted for in their papers.

The results here suggest that the spread between short and long rates is not a particularly useful measure of the validity of the Expectations Hypothesis, since that difference is mostly a measure of risk. In addition, the coefficient of the slope variable turns more significantly positive as the forecast horizon is extended. This complements the results of Chapter II.

However, it is recognized that these results only counter particular arguments against the Expectations Hypothesis. Since they do not provide strong empirical support, it is important, therefore, to analyze the relationship between long and short-rates more thoroughly. This will be the subject of Chapter V. There the addition of a model of expectations revision will be employed to explain the changes in long-rates and their relationship with short-rates.

## CHAPTER IV

### Unexpected Inflation as a Source of Forecast Error

#### Section I

##### Efficient Market Theory and the Expectations Hypothesis

In this paper, the Expectations Hypothesis has been based on the notion that the bond markets are rational and efficient. If so, then it is important that the large forecast errors that historically have been observed must be explained by factors that could not have plausibly been anticipated in a rational world. One such factor could be inflation shocks. Economics/Finance theorists have almost universally accepted interest rates as being a function of inflation since Irving Fisher's Theory of Interest (1930): "...that when the rate of price change falls from one period to the next, the money rate of interest usually falls, and when the rate of price change rises, the interest rate usually rises also.<sup>1</sup>" Using this as the underlying dynamic for interest rate determination then, unanticipated inflation could be used as a possible explanation of why the forward rates do not accurately anticipate future interest rates.

In this chapter the concept of unanticipated inflation as a source of forecast error will be explored. The inability to foresee price shocks does not imply irrational markets,

but rather represents the inherent "noise" in the environment. Consequently, relating the forecast error of forward rates to unanticipatable price changes will help confirm forward rates as rational predictors of future interest rates. The problem inherent in any such analysis, of course, is to quantify the notion of expected inflation. The approach taken in this paper will be to use interest rates themselves as unbiased forecasts of future inflation. Again using Fisher as a reference, "Our investigations thus corroborate convincingly the theory that a direct relation exists between prices and interest rates...".

Inherent in the Expectations Hypothesis in conjunction with the notion of an efficient markets, is that expectations about future interest rates are based upon the same factors that determine current interest rates. That is, assuming that spot interest rates are determined by current inflation expectations, efficient markets imply that expectations of future interest rates are themselves functions of inflation expectations. This concept was summarized by Hamburger and Platt (1975), "The rational expectations hypothesis may be expressed as follows: expectations (or, more generally, the subjective probability distribution of outcomes) tend to be distributed for the same information set, about the

prediction of the theory (or the "objective" probability distribution of outcomes). An implication of the hypothesis is that aside from random variation, expectations are generated by the same process through which the actual variable is determined. □"

For purposes of this chapter, if spot interest rates incorporate expectations of future inflation, then under efficient markets, interest rate expectations will also be determined by expectations of future price changes. Since the Expectations Hypothesis maintains that forward rates are themselves these expectations, forward rates by inference should incorporate inflation expectations. Using this line of reasoning then, a test of market efficiency would be if the prediction error of forward rates can be "explained" by the error in forecasting inflation, then forward rates can be validated as rational predictors. An alternative way of viewing this would be that interest rates, being based on expectations of inflation, are revised through time as the information set about inflation is incremented. Consequently, future spot rates, having assimilated this additional information, will vary from the initial expectation due to the receipt of this new information. The new information set is hypothesized here to include errors in the initial inflation forecast.

In order to quantify this incremental information, a measure of expected inflation must be estimated. Such a measure will be obtained from spot interest rates themselves. As a first approximation constant real interest rates will be assumed. Subsequently, a model will be developed allowing for time-varying real interest rates.

### Section II. Constant Real Interest Rates

The analysis begins with the assumption of a constant real interest rate. To obtain a simple estimate of the inflation expected over a particular time horizon, the real interest rate is estimated by forming a series of compounded interest yields over that time horizon and then subtracting the ex post inflation that takes place over that period. So for example, to get the two year expected inflation, the holding period return of the two-year spot rate is first calculated. From this figure is subtracted the inflation realized over that two year period, as measured by the CPI. Averaging over time yields the average real rate of interest which in this section will be assumed to be constant. This constant factor is then subtracted from the appropriate spot Treasury yield to get an estimate of ex ante expected inflation.

The actual inflation rate over the particular time period is then subtracted from this expected inflation rate to

get a measure of unanticipated inflation. In the model presented here, the forecast error of forward rates is a linear function of unexpected inflation over the forecast period. More formally:

$$(1) \quad F(t, t+\tau, t+\tau+1) - S(t+\tau, t+\tau+1) = \pi + b_1 \text{Unexp}(t+\tau)$$

where Unexp represents actual inflation as a percent over time period  $t + \tau$ , less expected inflation for period  $\tau - t$  formed at period  $t$ . As before  $\pi$  represents a risk premium. Equation (1) models the prediction error of forward rates as a function of unexpected inflation and risk.

In Fama (1988) <sup>4</sup>, it was suggested that the forward rate itself contains a forecast of not only expected inflation, but also a forecast of the real interest rate. It should be noted that the approach here is different for several reasons. First, Fama himself recognized one limitation to his approach, "...the split of the forecasted change in the spot rate between its components ... is constant through time. It is unlikely that the actual period-by-period split is constant." <sup>5</sup> Second, the methodology leads to estimates of real interest rates that are negative for substantial lengths of time. This is difficult to reconcile with commonly accepted notions of real rates. Finally, real interest rates are unobservable. If market participants have been shown to have been

lacking in their ability to forecast observable nominal interest rates, it is hard to attribute to it the ability of predicting such unobservable quantities as real interest rates. This is evidenced by the relatively poor significance levels obtained in the results of that paper.

As a test of market efficiency then, regressions for various forward rates and their associated forecast horizons are run on equation (1). A graph of the two-year forecast error vs. the two-year cumulative unexpected inflation included in Appendix D. The results for this regression are reported in Table I.

Table I

Forecast Error Regressed Against Unexpected Inflation

<u>Forecast Error</u>	<u>Unexpected Inflation</u>			<u>Durbin-Watson</u>
	<u>b<sub>1</sub></u>	<u>t-Stat</u>	<u>R<sup>2</sup></u>	
F(t,t+1,t+2)-S(t+1,t+2)	.44	14.61	.54	0.26
F(t,t+2,t+3)-S(t+2,t+3)	.27	17.73	.62	0.17
F(t,t+3,t+4)-S(t+3,t+4)	.17	19.33	.67	0.15
F(t,t+4,t+5)-S(t+4,t+5)	.12	15.40	.58	0.13
F(t,t+5,t+6)-S(t+5,t+6)	.09	14.36	.57	0.11
F(t,t+6,t+7)-S(t+6,t+7)	.07	13.01	.54	0.12

While the results show strong explanatory power, it is apparent that serial correlation is a major problem in interpreting the results. Consequently, the regressions were rerun using a Cochrane-Orcutt procedure. Those results are reported in Table II.

Table IICorrected Forecast Error Regressed Against Unexpected Inflation

<u>Forecast Error</u>	<u>Unexpected Inflation</u> <u><math>b_1</math></u>	<u>t-Stat</u>	<u>Durbin-Watson</u>
$F(t, t+1, t+2) - S(t+1, t+2)$	.42	6.35	1.72
$F(t, t+2, t+3) - S(t+2, t+3)$	.22	6.30	1.85
$F(t, t+3, t+4) - S(t+3, t+4)$	.17	7.42	1.78
$F(t, t+4, t+5) - S(t+4, t+5)$	.08	4.25	1.80
$F(t, t+5, t+6) - S(t+5, t+6)$	.06	3.61	1.58
$F(t, t+6, t+7) - S(t+6, t+7)$	.03	2.35	1.63

From the above results, two points are worth noting. First, the coefficients of the unexpected inflation variable are all positive and significant. This is especially true for the first three forecast horizons. Secondly, although significant, it is obvious that a substantial part of the forecast error still needs to be explained. This will be attempted subsequently by accounting for the risk variable more explicitly, and analyzing a second information variable. At this point, the same test will be performed allowing for time varying real interest rates.

Section III. Time Varying Real Interest Rates

The analysis is now refined to allow for a time-varying real interest rate. It has generally been accepted that real interest rates have risen over the last two decades. For example, The Federal Reserve of St. Louis notes "Another way to state this is: lenders, if they are risk averse, require that a greater "risk premium" be added to

interest rates in order to offset the greater uncertainty associated with real returns." <sup>6</sup> The stochastic process that will be used to estimate such parameters is referred to as ARCH-M, for Autoregressive Conditional Heteroskedasticity-Mean. The methodology which was used in Engle, Lilien, and Robins (1987) <sup>7</sup> to estimate time-varying forward-spot interest rate risk premia, will be employed here to estimate a time varying real interest rate. From the estimates of real interest rates, estimates of expected inflation will be derived.

The ARCH-M methodology basically assumes that premia are functions of the most recent errors in the forecast. As a consequence, the premia become functions of the variance of these errors. As summarized by Engle, Lilien, and Robins, "We assume that the most useful information to agents are the previous innovations or surprises  $\epsilon_t$ . If these have been large in absolute value then, extending Mandelbrot's observation, they are likely to be large in the future."<sup>8</sup> The methodology used here is a variation of theirs due to the fact that a different particular parameter is being estimated. The methodology starts with the calculation of the current real interest rate by subtracting the year-to-year percent change in the CPI from the current spot rate of interest,  $S(t)$ . Then, the

realized real rate of interest is calculated as in the previous section. Thus the compounded yield of period  $t+\tau$  is calculated, and from this value is subtracted the realized compounded inflation rate over the same period. The difference between the ex ante and ex post real rates is taken, and is denoted the innovation or  $\epsilon(t)$ . The  $\epsilon(t)$  are then squared. These innovations are estimated with a maximum likelihood technique. The fitted real rate becomes a weighted average of the estimated innovations. Formally:

$$(2) \text{ REALI} = \sqrt{\sum w(i) y(t-i)}$$

where REALI is the estimated real rate at time period  $t$ .  $w(i)$  are declining weights distributed over time periods  $i=1$  to  $5$   
 $y(t)$  are the maximum likelihood estimated innovations.

The estimated real rates are subtracted from the  $\tau$  period bond yield and then compounded to get the expected ex ante inflation rate over the life of the bond. From this quantity is subtracted the ex-post realized inflation rate over  $\tau$  periods, and becomes the estimate of unexpected inflation. The error of the forecast of  $\tau$  period forward rates, defined as  $F(t, t+\tau, t+\tau+1) - S(t+\tau, t+\tau+1)$ , is then regressed against this measure of the unexpected inflation.

As has been noted previously, a recurring problem associated with such regressions is that they are plagued by serial correlation. Consequently, the results reported in Table III, include both the untransformed and Cochrane-Orcutt estimated results.

Table III

Forecast Error Regressed Against Unexpected Inflation

Forecast Error	Unexpected Inflation			Cochrane-Orcutt		
	$b_1$	t-Stat	$R^2$	$b_1$	T-Stat	
$F(t, t+1, t+2) - S(t+1, t+2)$	0.49	16.13	.58	0.42	7.09	
$F(t, t+2, t+3) - S(t+2, t+3)$	0.27	16.28	.62	0.20	5.04	
$F(t, t+3, t+4) - S(t+3, t+4)$	0.28	18.17	.70	0.10	4.18	
$F(t, t+4, t+5) - S(t+4, t+5)$	1.59	18.04	.74	0.47	4.93	
$F(t, t+5, t+6) - S(t+5, t+6)$	0.16	21.79	.85	0.04	1.86	
$F(t, t+6, t+t) - S(t+6, t+7)$	0.09	16.88	.82	0.02	1.82	

The unaltered t-statistics and  $R^2$  point toward unexpected inflation explaining a substantial portion of the forecast error. A source of concern, however, is the instability of the coefficient of unexpected inflation. This is probably due to the large loss of data points used in each successive year in the ARCH-M estimation. Adjusted for the serial correlation, unexpected inflation still remains highly significant for the first four forward rates. The successive loss in significance of these transformed coefficients again could be explained by the shorter data sample where the serial correlation becomes more of a factor. In addition, inflation shocks tend to be smoothed out over the long-run, which by inference would lead to less explanatory power for unexpected inflation in the longer forecast periods. For these reasons, the focus of this analysis remains on the shorter forecast time horizons.

Section IV. The Bias of Forward Rates with Unexpected Inflation

In this section, Forward rates are used as regressors along with unanticipated inflation in a multiregression to explain ex post spot rates. The model is as follows:

$$(3) S(t, t+r, t+r+1) = \alpha + \beta F(t, t+r, t+r+1) + \Gamma \text{Unexp} + \epsilon(t)$$

This differs from the regressions in the previous two sections in that the magnitude of the individual coefficients are analyzed for their compatibility with the Expectations Hypothesis. That is, it would assist the overall analysis if with the addition of unanticipated inflation as a regressor, forward rates could be shown to be unbiased estimators of future spot rates. The null hypothesis thus entails setting  $\alpha = 0$  and  $\beta = 1$ . It is expected also that  $\Gamma$  be between zero and unity.

Regressions using various forward rates were run on equation (3), where unanticipated inflation is estimated using constant real interest rates as in Section II. The results are reported in Table IV.

TABLE IV

Ex-Post Spot Rates vs Forward Rates and Unexpected Inflation

Forward Rate	$\alpha$ (Std Err)	$\beta$ (Std Err)	$\Gamma$ (Std Err)	Cochrane-Orcutt $\beta$	t-stat
F(t,t+1,t+2)	-3.33 (.60)	1.32 (.07)	.61 (.04)	.20	2.38
F(t,t+2,t+3)	-6.45 (1.09)	1.68 (.08)	.39 (.03)	.19	0.24
F(t,t+3,t+4)	-8.60 (1.16)	1.87 (.13)	.30 (.02)	.34	2.18
F(t,t+4,t+5)	-10.59 (1.55)	2.19 (.17)	.25 (.02)	.08	0.52
F(t,t+5,t+6)	-10.76 (1.59)	2.25 (.18)	.20 (.01)	.16	0.88
F(t,t+6,t+7)	-7.32 (1.96)	1.95 (.23)	.14 (.01)	.21	1.15

It is obvious from the above results that forward rates are seen to be biased estimates of ex post spot rates. The  $\beta$  parameters fluctuate about 2 and thus the null hypothesis that they are equal to unity can be rejected. Moreover, the  $\alpha$  coefficients are seen to be highly negative while the  $\Gamma$ 's are significantly less than one. Taken together, this would seem to suggest the presence of a time varying risk premium that has the effect of distorting the parameters. Consequently the analysis is refined to take into account time varying risk premia. The regressions on (3) are rerun using the ARCH-M methodology of Engle, Lillien, and Robins (1987) to estimate time varying risk premia. In addition, the analysis incorporates the methodology to estimate unanticipated inflation from time varying real interest rates as in Section III. Only three forward rates are included because the large amount of degrees of freedom that are used up in employing ARCH-M limited the use

of the longer term forecast horizons. The results are reported in Table V.

TABLE V  
ARCH-M Forecast of Ex-Post Spot Rates

<u>Forward Rate</u>	$\alpha$	$\beta$	$\Gamma$	F-Statistic	
	(Std Err)	(Std Err)	(Std Err)	$\alpha=0$	$\beta=1$
F(t,t+1,t+2)	-2.03 (.62)	1.25 (.09)	.51 (.05)	11.01	
F(t,t+2,t+3)	-3.13 (1.54)	1.61 (.19)	.20 (.04)	31.29	
F(t,t+3,t+4)	-6.44 (1.81)	1.87 (.20)	.29 (.03)	217.00	

The results here are somewhat more supportive of the Expectations Hypothesis. While it would appear that the strong form of the Expectations Hypothesis is lacking, as evidenced by the high F-Statistics, it remains true that the forward rates do explain a significant proportion of the variability of spot rates when combined with unexpected inflation. The downward bias to both the constant term and the unanticipated inflation term, combined with the upward bias for the forward rate would suggest that the ARCH-M methodology does not completely account for the time-varying risk premia.

#### Section V: Summary

The purpose of this chapter has been to show how the addition of new information can account for a significant portion of the forecast error of forward rates. This additional information is modeled to be price shocks. The unexpected inflation in this chapter is derived from spot

interest rates themselves. Thus, the accuracy of the analysis here hinges upon the Fisherian notion of spot rates incorporating unbiased estimates of expected inflation. Building on this notion then, the empirical results in the chapter support the Rational Expectation's Hypothesis in the sense that to the extent that forward rates inaccurately predict future spot rates, it is largely due to the addition of new information. While the evidence advanced does not confirm the strong-form of the Expectation's Hypothesis, that is of unbiased expectations, it does suggest that the theory is valid in a broad context. That is in the sense of bond prices incorporating available information on such variables as price expectations.

Since the residual variation in forecast error, even when accounting for price shocks is still significant, the analysis is extended in the following chapter for an additional information variable. The Rational Expectations Hypothesis can be validated if the prediction error of forward rates can be "explained" by the addition of information made available after the determination of the forward rates. This will be again taken up in Chapter V with the inclusion of a simple measure of policy changes.

## Chapter V

### Policy Changes as Additions To Information

#### Section I. Introduction

In Culbertson (1957)<sup>1</sup>, an alternate hypothesis was proposed as an explanation for the shape of the term structure of interest rates. This has generally come to be known as the Market Segmentation Theory. Recall the summary by Modigliani and Sutch in Chapter I. One of the implications of the theory would then be that induced changes in interest rates at one segment of the term structure will not lead to changes at another point on the spectrum of interest rates. This point has been emphasized by Harris, "The model's implication for policy are that the central bank can alter the term structure of interest rates by altering the relative supplies of long and short bonds. Moreover, the central bank cannot influence the long rate by altering the supply of short bonds alone. That is in direct contrast to the policy implications of the pure expectational theory."<sup>2</sup>

In this chapter, the impact of policy changes on the part of the Federal Reserve will be examined in terms of their consistency with alternative views of the term structure. By offering an alternative hypothesis in which to interpret test results, it is hoped to increase the statistical power of those results. Moreover, it is hoped

that the inclusion of these policy changes, when included in the information set of bond market participants, can be used to validate the efficiency of forward rates as predictors of interest rates. In addition, it is hoped by doing so, a tractable means of deriving a model for the formation of expectations. This being to counter a recurring criticism of the Expectations Hypothesis, that it does not provide for a model of expectations formation. This will be the subject of Chapter VI.

### Section II: Formulating the Null and Alternative Hypotheses

In this section, as well as in subsequent sections of this chapter, changes in short-term interest rates will be hypothesized as being the result of Federal Reserve policy. The corresponding changes in forward, as well as long-term spot rates, will then be shown to offer a testable means of comparing competing theories of the shape of the yield curve. For example, if short rates fall, policy will be considered to have taken an expansionary stance. Conversely, higher short rates will be considered to point toward a monetary tightening.

The resultant changes in interest rates due to policy changes will be the basis of the tests. The hypothesis to be presented here is that policy changes (changes in short rates) have an impact on expectations of future rates. The

measure of this impact can be discerned from the corresponding impact on long rates if the Expectations Hypothesis is correct. If no pattern of changes on long-rates can be discerned, then it can be seen as evidence in support of the Market Segmentation Theory. In addition, the analysis can be related to that of Chapter III. Recall that in that chapter, the analysis hinged around the predictive accuracy of the relationship between long and short rates. In that chapter, the assertion was made that by accounting for variable risk, much of the forecast error of the term spread can be accounted for. The analysis in this chapter will complement that chapter by focusing on the importance of the short-rate. Typically in the term structure literature, the short-rate is modeled as a rate of return on a particular bond holding. In this chapter, it will be presented as a policy variable which has the ability to change expectations of future interest rates. Looked at in this manner, it is hoped that the changes in that rate can be included in the information set of investors and consequently can explain the critical results of other researchers. It will be modeled here as being under the direct influence of the Federal Reserve.

In Fisher's Theory of Interest, the observation was made that "A rate on a five-year contract may be considered as

a sort of average of five theoretically existing rates, one for each of the five years covered."<sup>23</sup> A more precise, as well as more general, formulation of the above statement can be found in Shiller, Campbell, and Schoenholtz (1983)<sup>24</sup> (see Appendix B) and is reproduced in Equation (1):

$$(1) S(t, t+\tau) = \sum w(\tau) E_t[f(t, t+\tau, t+\tau+1)]$$

where the summation takes place from 0 to  $t+\tau-1$   
 $f(t, t+\tau, t+\tau+1)$  denotes a future expected short rate at time  $\tau$  as of time  $t$ .  
 $w(\tau)$  represents a weighting factor and is equal to  
 $(g^\tau * (1-g)) / (1-g^\tau)$   
 $g = (1+S(t, t+\tau))^{-1}$   
 $0 < w(\tau) < 1$

It should be noted that the above formula takes into account the fact that the bond in question is coupon bearing. Recognizing that the first term in the above summation is the spot short-rate (3 month bill for purposes of this paper), while the left hand variable will be the thirty year rate, (1) can be rearranged as

$$(2) S(t, t+30) = w(.25) * S(t, t+.25) + \sum_{\tau=1}^{119} w(\tau) F(t, t+\tau, t+.25)$$

Subtracting the previous periods relation from (2) we get

$$(3) \delta S(t, t+30) = \delta\{w(.25)*S(t, t+.25)\} + \delta\{\sum w(\tau)*F(t, t+\tau, t+\tau+.25)\}$$

Subtracting the weighted change in short-term spot rates from both sides then yields:

$$(4) \delta S(t, t+30) - \delta\{w(.25)*S(t, t+.25)\} = \delta\{\sum w(\tau)*F(t, t+\tau, t+\tau+.25)\}$$

From (4), we see that the change in long-rates should change by a fraction of the change in short-rates (policy variable) in the absence of changes in future expectations. This allows for the establishment of an alternative hypothesis. Under the Market Segmentation

Theory, changes in short-rates should have no perceived effect on long rates outside of the above weighted change. Thus if the left side of (4) is regressed against the change in short rates, the coefficient of the change in short-rates should be less than zero under the Market Segmentation Theory. A coefficient significantly positive would conversely suggest that changes in short rates lead to changes in expectations. If it can be established that changes in short rates do in fact change expectations, it would consequently be a vehicle for further testing of the Expectations Hypothesis.

The difference in the change of long-rates less the weighted change in short-rates is regressed against changes in short-rates over various time horizons. The point of the test being to measure the impact of policy changes on the changes in expectations as defined in (4). The results are then reported in Table I.

Table I  
Adjusted Change in Thirty-Year Bonds

Time Period of Change in	Coefficient of Change	T-Stat	Cochrane-Orcutt		
			$R^2$	B	T-Stat
<u>Short-rates</u>	<u>in Short-Rate</u>				
3 Months	.27	10.49	.34	.18	6.62
6 Months	.32	12.58	.43	.16	6.39
1 Years	.41	13.99	.49	.12	4.39
2 Years	.46	17.00	.60	.17	5.85
3 Years	.45	16.97	.62	.14	5.10
4 Years	.48	19.24	.69	.18	6.47
5 Years	.54	24.02	.79	.15	4.77

A few observations on Table I. First, the data conclusively rejects the notion that changes in short-rates have no impact on long-rates. Not only are the coefficients highly significant even when adjusted for serial correlation, but there is a clear, consistent pattern to them. Secondly, the significance of the change is directly related to the time horizon involved as evidenced by the  $R^2$ . In the context of the theory presented above, this can be interpreted as meaning that the more sustained the change in policy, the more effective its ability to influence expectations.

Third, the coefficient of the change in short-rates is significantly less than one. Adjusted long-rates change in the same direction but not in the same proportion as changes in short-rates. This suggests that not all the changes in future expectations are of the same magnitude and some are probably in the opposite direction. Since a high degree of serial persistence characterizes interest rates, it is reasonable to conclude that the near-term changes in expectations are of similar magnitude and direction as the actual changes in short-rates. Note by (1), these near-term expectations account for the largest weights in the determination of long-rates. Therefore, in order for the coefficients to be significantly less than one (on order of .5), there must be some offsetting

changes in longer-term expectations. Since these have smaller weights in (1), at least some would presumably be in the opposite direction. This would be consistent with the observation of Modigliani and Sutch, "Furthermore, the prevailing expectations of long-term rates involve a blending of extrapolation of very recent changes and regression toward a long-term level."<sup>4</sup>. This point will be pursued more fully in the following chapter.

### Section III. Changes in Short-Rates as Expectational Variables

In Shiller 1979 <sup>5</sup>, equivalent results to the ones reported in the previous section were interpreted as an overreaction on the part of long-term rates to changes in short rates. This work was summarized by Mankiw and Summers "Robert Shiller examines this smoothing property of the expectations hypothesis and finds that long rates are too volatile to be consistent with the theory." <sup>6</sup>

The stance to be taken here is that rather than being an overreaction, changes in short-rates, due to their status as a policy variable, have the ability to change expectations of future rates. Consequently, once the policy change is known, it would seem plausible to conclude that the information set of investors is incremented now they know what recent policy has been. It is common knowledge that the Federal Reserve takes great pains to keep its intentions confidential. "Fed Watchers"

may be able to make educated guesses as to what these intentions in actuality are, but there is hardly any doubt that these expectations are usually revised as the policy actions become public knowledge. Of course the actual knowledge of these policy changes occur only with a lag. It is the intention of this section to determine whether the changes in long-rates relative to that of short-rates are due to "overreaction" (and hence evidence in favor of irrational markets) or instead be attributed to rational changes in expectations.

In Schiller, Campbell, and Schoenholtz (1983) it was claimed: "The martingale property of forward rates implies that the change,  $f_{t+s}(n,m) - f_t(n,m)$  over any time interval,  $s$ , of linearized forward rates applying to a particular time,  $t+n$ , should depend only on information available between  $t$  and  $t+s$  and not on information available before  $t$ ." In short, the change in forward rates can be interpreted as the change in available information.

In order to test whether changes in long-rates as a function of changes in short-rates are a sign of irrational markets or due to "rational" changes in expectations, the effect of changes in short-rates on changes in forward rates will be examined. The rationale

behind this test is that there is no a priori connection between current changes in short-rates and forward rates outside of the serial correlation inherent in interest rates. However, due to the time lags involved with the forward rates being used in this paper, this characteristic of rates should not be significant. For example, there is no statistical correlation between changes in current short-rates and those of one-year rates three years into the future. Consequently, for a change in three-month rates to impact on forward rates at different time horizons can reasonably be held to be due to changes in expectations.

This is similar to a test employed by Meiselman<sup>7</sup> who used past forecast errors as the variable which incorporates the addition to investor information. He concluded, "Changes in forward one-year rates classified by maturity are, in fact, highly correlated with the forecasting error for the fifty-four annual observations covering the 1901-1954 period."<sup>8</sup> The test to be employed here is to regress the change in forward rates (the change in information) against changes in short-rates. Formally,

$$(5) \quad \delta F(t, t+\tau, t+\tau+1) = \alpha + \beta_1 \text{Pol}(t+\tau) + \epsilon(t)$$

where Pol represents the change in short-rates.

The results are reported in Table II:

Table II

The Change in Forward vs The Change in 3-Month Rates

Change in $F_{t,t+1}$	$\beta_1$	T-Stat	$R^2$	Cochrane-Orcutt	
				$\beta_{CO}$	T-Stat
F(t+1,t+2)	.50	12.84	.45	.21	5.1
F(t+2,t+3)	.50	12.84	.45	.21	5.1
F(t+3,t+4)	.50	14.24	.50	.21	5.6
F(t+4,t+5)	.41	10.63	.36	.10	2.7
F(t+5,t+6)	.35	9.92	.33	.08	1.9
F(t+6,t+7)	.28	7.23	.21	-.03	-1.1

Note that the coefficients of the change in short-rates, while highly significant, decline monotonically. This is consistent with Meiselman's original results using forecast errors as a regressor. To gain further insight into the effect of short-rates on forward rates, it is incorporated into Meiselman's test of regressing the change in forward rates against the past forecast error of forward rates noted above. The model then is as follows:

$$(6) \Delta F(t,t+1,t+2) = \alpha + \beta_1 \text{Pol}(t,t+1) + \beta_2 \text{Error} + \epsilon(t)$$

Here, Error refers to the prediction error of one-year forward rates, and Pol refers to the change in short-rates. As noted above Meiselman found the Error variable to be highly significant in explaining forward rates. By incorporating short-rates into the regression, it is hoped to get a clearer picture of their impact on forward rates. The motivation for this test is that by controlling for the explanatory power of past forecast errors, which are assumed to be highly significant, to what extent do short-rates account for the change in information available to investors. These results are reported in Table III.

Table III

Ex Post Forecast of One-Year Interest Rates

Change in $F_t$	$\beta_1$	$\beta_{22}$	$R^2$	Cochrane-Orcutt	
	(t)	(t)		$\beta_1$	T-Stat
F(t+1,t+2)	-0.03 (-0.76)	.74 (18.76)	.86	-0.07	-1.51
F(t+2,t+3)	-.20 (-4.27)	.80 (25.58)	.79	-0.03	-0.54
F(t+3,t+4)	-.11 (-2.54)	.68 (16.24)	.79	-0.05	-1.11
F(t+4,t+5)	-.25 (-5.21)	.74 (15.94)	.72	-0.07	-1.69
F(t+5,t+6)	-.23 (-5.01)	.67 (14.63)	.68	-0.12	-2.43
F(t+6,t+7)	-.33 (-5.96)	.68 (13.00)	.57	-0.18	-3.69

As noted above, Meiselman concluded that expectations are revised on the basis of past prediction errors. This is confirmed above in Table III. In Table II, it is shown that the change in short-rates by itself is highly significant in accounting for the revision of expectations as measured by the change in forward rates. By incorporating both variables into the analysis, it is hoped to be able to evaluate the true impact of short-rates on expectations. In Table II, the coefficients decline monotonically but remain positive as the time period of the forward rate evaluated increases. In the multiregression of Table III, the coefficients are uniformly negative as well as monotonically declining. Moreover, the significance of the impact of the change in short-rates, as measured by the T-Statistics, increases as the forward rate in question lengthens.

The key issue involved is whether the impact on long-rates of changes in short-rates involves an overreaction, and therefore by extension evidence of irrational markets instead of the rational revision of expectations. By examining the impact on forward rates of changes in short-rates, a clear consistent pattern emerges. It is shown in Tables II and III that, moreover, this pattern is highly significant. The examination of the actual pattern is deferred to the following chapter. However, at this point, it would be helpful to examine the change in expectations with a focus on ascertaining whether the revision process is efficient.

#### Section IV. Forward Rates and Changes in Expectations

In Chapter IV, Forward Rates along with unanticipated inflation were used as predictors of future interest rates. The point there was to examine if forward rates were unbiased if used in conjunction with price shocks. The same analysis will be employed here with the exception that changes in expectations, as defined in (4) will replace unexpected inflation. A regression is thus run on the following equation:

$$(6) \quad S(t, t+\tau) = \alpha + \beta_1 F(t, t+\tau) + \beta_2 \delta \text{Exp}(t+\tau) + \epsilon(t)$$

where  $\text{Exp}$  denotes the change in Expectations as defined in the left hand side of (4).

The focus here is on examining the coefficient of  $\alpha$  and  $\beta_1$ , which in the absence of risk should equal to zero

and unity respectively. The results from the various regressions are reported in Table III.

Table III

Ex Post Forecast of One-Year Interest Rates

Ex Post Rate	$\alpha$	$\beta_1$ (t)	$\beta_{exp}$ (t)	Cochrane-Orcutt		
				$R^2$	$\beta_{exp}$	T-Stat
S(t+1,t+2)	.36	1.49 (24.42)	.89 (27.45)	.83	.40	4.58
S(t+2,t+3)	.21	1.48 (28.39)	.90 (21.02)	.81	.44	4.33
S(t+3,t+4)	.22	1.36 (27.50)	.87 (19.09)	.81	.53	5.29
S(t+4,t+5)	-1.03	1.34 (22.53)	.98 (14.76)	.75	.34	2.66
S(t+5,t+6)	-1.05	1.23 (18.69)	.98 (11.55)	.70	.73	4.35
S(t+6,t+7)	-0.66	1.19 (16.97)	.93 ( 8.87)	.68	.37	2.48

Several observations can be made from the above results. First, a high degree of forecast precision can be made by incorporating the expectations variable along with forward rates as predictors. Secondly, the coefficients of forward rates are all of the right sign and highly significant. Moreover, they remain significant even when corrected for autocorrelation. It should be noted that there remains an upward bias to forward rates as well as an inconsistent bias to  $\alpha$ . This can be probably be due to the fact that the risk premium remains unaccounted for.

It is also interesting to note that the significance of the forward rate coefficient relative to that of the change in expectations increases with the time horizon of the forward rate. The inference being that the longer

forward rates, being more accurate predictors per Chapter II, are less dependent on expectation revisions. Thus, forward rates, when combined with changes in available information as measured by changes in Federal Reserve policy, are shown to be accurate, unbiased predictors of future spot rates.

#### Section V. Summary.

In this chapter, the issue of policy changes as a determinant of expectations was examined. The underlying rationale for the analysis being that policy changes should be incorporated as part of the information set of bond market participants. Consequently, they can be utilized to help determine the efficiency of prices that are set in that market. This includes the efficiency then of forward rates. For purposes of this chapter, 3-month Treasury Bills were used as a proxy for the policy variable.

The empirical results can be summarized as follows:

- (1) Changes in short-rates lead to more than proportional changes in long-rates. This is seen to be evidence that changes in policy lead to changes in expectations under the Rational Expectations Hypothesis.

- (2) Changes in policy lead to changes in individual forward rates that vary inversely with the time at which the forward rate will be realized. This will be presented as evidence of mean-reversion to be presented in the following chapter.
- (3) The changes in expectations resulting from changes in policy, combined with forward rates, are accurate predictors of future spot rates.

## Chapter VI.

### The Mean-Reversion Model

#### Section I. Introduction

In previous chapters, allusions have been made to the idea that interest rates, as well as their expectations tend to follow a mean-reversion process. For purposes of this paper, this will refer to the tendency of interest rates to tend toward some long-term average. While admittedly there has been some upward drift in recent decades to this tendency, it does not change the thrust of the overall analysis. In this chapter, this idea will be explored more fully.

The model to be followed in this paper for this process is as follows: Interest rates, being positive functions of inflation, among other variables, react to various shocks. These can be externally generated (e.g. oil price shocks) or internal (unanticipated policy changes). These stimuli occur almost continuously, and consequently interest rate forecasts, as measured by forward rates are prone to error. However, there is a tendency for policy authorities to gradually offset these shocks to the interest rate environment over time. Since policy authorities tend to act in this fashion, it is reasonable to conclude expectations would be conditioned on such behavior.

If this model can be shown to be consistent with the data, then this could help explain some of the anomalies that have been pointed out in the past concerning forward interest rates. For example, the relatively poor forecasting ability of forward interest rates over short-term time horizons compared to longer-term forecasts. Moreover, by forming an expectations generating model, the Rational Expectations Hypothesis can be placed on more solid theoretical underpinnings by countering a typical criticism of the Hypothesis. "The final weakness of the model is that it gives no account of how expectations of future rates are formed. This omission presents particular difficulties for empirical work, since it is impossible to test ... unless we have observations of the expected short rates, or know by what observable variables they are determined, or at least can hypothesize some model of their determination."<sup>1</sup>

It has been pointed out previously that Meiselman modeled forward rates as being a function of past forecast errors. The purpose of this chapter is to use a mean-reversion model of expectations formation to lend theoretical support to the test results that have been previously reported. This is not a novel idea, but rather a recurrent notion running through the literature from Lutz (1940)<sup>2</sup> to Malkiel (1962)<sup>3</sup> and Fama and Bliss (1987)<sup>4</sup>. In

these models, the market's forecast of future interest rate trends are generally determined by comparing current interest rates with past levels. For example, if interest rates are high by historical standards, the bond markets, as reflected by implied forward rates, should have a tendency to predict lower future rates. Thus for example, in 1974, and 1980-81 when rates were at historically high levels, the yield curve was inverted. That is, the market was auguring for lower future rates.

The Chapter begins with a more thorough examination of the previous chapter's results. This will be followed by alternative tests of the tendency for both spot and forward rates to follow a cyclical pattern.

### Section II. Examining Previous Results

As noted in the previous chapter, Meiselman (1962) presented evidence that changes in forward rates are highly correlated with past forecast errors of forward rates. At this point, a closer examination of his results will be taken.

As mentioned previously, changes in forward rates were modeled to be a function of past forecast errors of forward rates. The the regression coefficients of past forecast errors ranged from 0.7 for the change in one-year forward rates to 0.2 for the change in the eight-year

forward rate.<sup>5</sup> The coefficients for the intermediate forward rates declined monotonically.<sup>6</sup> As summarized by Meiselman, "...the regression coefficient and hence the sensitivity of forward rates to forecasting errors also tends to vary dependably and inversely with the maturity of the dependent variable."<sup>7</sup> The inference that can be taken from these results is that the near-term forward rates are revised much more substantially with the receipt of new information than do longer-term forward rates. An analysis of the magnitude of the coefficients was not pursued by Meiselman.

This is taken, however, to be empirical support for the notion of the mean-reverting tendency of interest rate expectations. The lower coefficient values indicate that the longer forward rates are less sensitive to new information as it becomes available. Rather, they are determined more by the long-run tendency of the underlying variable. The assumption here is that interest rates fluctuate cyclically about a long-term average. It appears from Meiselman's results then that the longer-term forward rates are revised less due to the market expecting the shock that caused the revision to be at least partly offset in the future by policy makers.

Similar results were reported in the previous chapter with

a different information variable: policy changes. Policy changes were shown to have impacted short-term forward rates much more substantially than longer-term forward rates. Coefficients there ranged from 0.5 for changes in one-year forward rates to 0.28 for 6-year forwards. The analysis that combined Meiselman's error variable with policy changes indicated that policy changes implied progressively negative coefficients for future short rates. Taken as a whole, the analysis decisively indicates that the market "expects" interest rate shocks to be offset at least in part the farther into the future the analysis is extended.

As has been suggested, this is consistent with the notion that the market anticipates counter-cyclical policies to be undertaken. This in turn would suggest that forecast errors would tend to be canceled out through time. This is supported by the evidence presented in Chapter II where the forecast accuracy of forward rates increased with the forecast horizon. In such a model, shocks to the financial markets lead to prediction errors of forward rates. It can also be argued that the policy makers themselves intentionally induce such shocks in pursuit of its policy goals. However, if it is true that counter-cyclical policies tend to dominate policy-maker thinking, then they will act to offset these surprises over time.

Consequently, interest rates would tend to return to some historical average over a multi-year period, albeit with perhaps some upward drift. As a result, the forecast accuracy of longer forward rates would increase relative to that of short-term forwards under such a model. This is consistent with the evidence presented in Chapter II.

### Section III. Alternative Tests.

Such a mean-reversion model could be modeled as follows:

$$(1) F(t, t+\tau, t+\tau+1) = S(t, t+1) + \beta \{ S(t, t+1) - S^* \} + \pi + \epsilon(t)$$

where  $S^*$  refers to some long-term average of one-year rates.  $\pi$  refers to a risk premium and  $\beta$  is presumably negative and greater than  $-1$ . Subtracting  $S_{(t, t+1)}$  from both sides of (1) yields

$$(2) F_{(t, t+\tau, t+\tau+1)} - S_{(t, t+1)} = \pi + \beta \{ S_{(t, t+1)} - S^* \} + u_t$$

As an approximation, the average one-year interest rate for the 1970-1987 period is substituted for  $S^*$ .

Regressing the spread between forward and spot rates against the difference between current rates and their average yield the following results:

Table I

#### Difference Between Forward and Spot Rate vs Deviation

##### From Average Rate

<u>Term Spread</u>	<u><math>\beta</math></u>	<u>T-stat</u>
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.13	-5.48
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.23	-8.50
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.23	-8.75
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.28	-8.98
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.31	-9.72
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.35	-9.95

The results tend to confirm that market expectations tend to be formed through a mean reversion process. Not only are the coefficients of the right sign and significant, but the significance increases proportionately with the time horizon of the forward rates. This increase in significance with time tends to validate the notion that the market expects policy authorities to act gradually to offset shocks to the interest-rate environment.

The analysis is extended to allow for an upward drift to  $S^*$ . This is done by regressing the one-year rate against time. Fitted values are then calculated from the regression parameters to obtain the  $S^*$  with upward drift. These results are reported in Table II.

Table II

Forward-Spot Spread vs Deviation From Moving Average Rate

<u>Term Spread</u>	<u><math>\beta</math></u>	<u>T-stat</u>
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.21	-8.73
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.31	-11.25
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.33	-12.45
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.40	-13.01
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.44	-15.00
$F(t, t+\tau, t+\tau+1) - S(t, t+1)$	-0.50	-15.60

These results reinforce the previous ones. The regression coefficients are decidedly more negative as well as being more significant. Not only do these results lend support to the results of Chapter II that forecast accuracy increases with time, but adds empirical content to casual assertions made by supporters of the Expectations Hypothesis, such as Lutz and Malkiel.

Meiselman (1962) also observed "If actual rates are higher than had been anticipated, then the market may systematically revise upward expectations of what short-term rates in the future are likely to be. Similarly, if actual rates are lower than had been anticipated, then the market may also systematically revise downward expectations of future short-term rates."<sup>5</sup> The results here extend this conclusion. Forward rates tend to be revised in the same direction as the forecast error as asserted by Meiselman. However, relative to current spot rates, forward rates are also determined by the level of rates relative to their long-term average. In particular, forward rates are negatively related to their position relative to their long-term average.

#### Section IV. Summary

One of the criticisms that have been leveled at the Expectations Hypothesis is that it does not provide for a theory of expectations formation. The purpose of this chapter has been to address this issue. The major conclusion that has been reached is that expectations of future rates, as embodied by forward rates are determined by the level of rates relative to historical levels. Empirical tests were reported help support that conclusion.

In addition, test results of Meiselman, as well as those of the previous chapter were examined as to the pattern of expectations revision. The major conclusion reached from this analysis was that near-term forward rates are much more sensitive to the receipt of new information in the form of shocks to the financial environment. Such a conclusion helps provide a tractable theory as to the pattern of forecast accuracy examined in Chapters II and III. It would appear from the test results that bond market participants generally anticipate policy authorities to react by pursuing counter-cyclical policies that offset these financial shocks. Rational markets in turn dictate that these expectations be based on the actual tendency of policy makers. It is then to the extent that policy smooths out short-run financial surprises that determine the forecast accuracy of longer-term forward rates relative to short-term ones.

## Chapter VII.

### Summary

#### Section I. Major Conclusions.

Under the Expectations Hypothesis of the Term Structure, implied forward interest rates are held to be unbiased predictors of future spot interest rates. This purpose of this paper has been to evaluate this notion from the perspective of the market efficiency of forward rates. That is, to what extent is information available to lenders and borrowers incorporated into the current spectrum of interest rates. The paper has taken the stance that the validity of the Expectations Hypothesis is not solely contingent upon its forecast power but rather that the forecast error can be explained by the addition of new information available to bond market participants.

In Chapters II and III, empirical tests were performed which established the forecast accuracy of forward rates when risk premia are taken into account. Moreover, along the lines of Fama and Bliss (1987), the forecast accuracy was shown to increase as the forecast horizon lengthened in time. This predictive accuracy when adjusted for risk was shown in these two chapters was shown to be greater than has been traditionally held. While predictive power is important in order to establish the validity of the Expectations Hypothesis, it is not the sole determinant.

It is equally important to be able to justify the forecast errors within the context as to whether the error could have been plausibly avoided by using available information at the time the forecast was made. Forecast error was analyzed in Chapters IV and V. Both of these chapters revolved around the notion that information received after the time the forward rate was determined can account for most of the forecast error. In Chapter IV this incremental information was posited to be unexpected inflation. In Chapter V, policy changes subsequent to the determination of the forward rate was examined as a source of forecast error. In addition, this variable was used to compare the Expectations Hypothesis with competing theories. The overall thrust of these two chapters was to validate the Rational Expectations Hypothesis by accounting for the forecast error of forward rates by these information variables.

In Chapter VI forward rates were modeled as being formed according to a mean-reversion process. The key idea within this chapter was to give a conceptual framework to support the results of chapters II and III. By modeling forward rates, and hence future interest rates, as having long-term tendencies, it gives theoretical support as to why the forecast accuracy of forward rates improves with the forecast horizon. Bond markets are seen as

anticipating countercyclical policy making actions. This in turn is used to give theoretical support to why forecast accuracy increases with time, and consequently helps reconcile this counter-intuitive notion with the data.

### Section II. Policy Considerations.

The Federal Reserve has been known to attempt to adjust the term structure in its conduct of monetary policy. For example, in 1961-62 "Operation Twist" attempted to raise short-term rates and simultaneously lower long-rates. The analysis of this paper suggests that this is not a prudent course of action. The mere shifting of the supply and demand for securities along different points of the term structure by policy authorities will not be successful in altering long-rates. The analysis in Chapter V suggests that expectations are paramount in the determination of long-rates. The mere shifting of the supply and demand for securities of particular maturity securities by the policy authorities would presumably lead to offsetting transactions in other bond holder portfolios according to their expectations of the course of future rates. Simply rebalancing the Fed's portfolio of securities would not materially affect the money supply and hence the expectations of the future course of interest rates.

Moreover, it would seem counterproductive on the part of

policy authorities to pursue policies that tend to be premised on the "surprising" of market participants. Shocks are seen to increase the uncertainty inherent in the interest rate environment, and hence lead to higher risk premia in longer-term interest rates. Thus the volatile interest rate environment of the 1970's is seen as a determinant of the high real interest rates of the 1980's. Whether this suggests the reliance on interest rate or money stock targeting on the part of the Federal Reserve would then hinge upon which makes policy more predictable over the long term.

### Section III. Topics for Future Research.

In terms of future research, several points would appear to be fertile areas for further investigation. First would be a more rigorous accounting of risk premia. The assumption that such premia do not change substantially over short-time intervals served well as an approximation in this paper, but it would undoubtedly improve the results if such terms could be accurately modeled explicitly. The ARCH-M methodology also enhanced the analysis but possibly could be improved with a more refined modeling of the causal factors which determine the risk preferences of bondholders. For example, the analysis in this paper used past shocks as the sole determinant of current risk premia. Perhaps other variables should also have been employed as a determinant of time-varying risk

premia such as the level of rates, adjusted duration, etc..

Secondly, a more refined notion of policy makers means of "fooling" the financial markets would be warranted. It is generally accepted that financial markets closely monitor policy authorities for clues as to their future course of action. However, what markets expect and how policy makers react to such expectations has not been rigorously analyzed in this paper. The underlying assumption of Chapter V was that the most recent changes in policy come as a surprise to participants. Obviously this is a simplistic assumption. It would aid the analysis developed in that chapter to develop a means with which to determine the extent to which policy changes were unanticipated.

Finally, it would seem important to more fully account for the forecast error of forward rates. While, the results recorded in this paper seem to have determined the sources of most of this error, a comprehensive explanation of the sources of forecast error and forecast bias would seem to be warranted.

## APPENDIX A

Calculation of Implied Forward Rates

Obtaining a consistent series of yields is a major problem that plagues empirical work in the area of the term structure of interest rates. Among the problems that have been faced by researchers has been to first of all to gather a consistent series of yields over time of the yield curve. The data used in this paper has been obtained from the series found in Federal Reserve Bulletin, which uses an average of daily yields to maturity for each time period to estimate the monthly yield. Such a consistent series can be obtained monthly since 1970. Unfortunately, this data has gaps that require some interpolation. For example, the Fed's series includes yields for 3 month, 6 month, 1, 2, 3, 5, and 7 year yields. To obtain an estimate of 4 and 6 year yields, a linear interpolation was used between the 3 & 5 year and 5 & 7 year yields respectively. That is, the 4 year yield was assumed to lie halfway between the 3 and 5 year, while the 6 year was estimated at the midpoint of the 5 & 7 year yields.

Another problem to be resolved was that the 3 and 6 month yields are quoted on a bank discount basis while the annual yields were quoted on a coupon basis. Since the discount yields underestimate the true yields, they were adjusted in the following manner:

$$r = (365 * d) / (360 - (d * t))$$

where r = equivalent bond yield

d = discount yield

t = time to expiration ( = 90 for 3  
month and 180 for 6 month)

In calculating the implied forward yield, it is necessary to first "strip" the bond. That is convert it to a one period yield since the quoted yield is really a weighted average of all the intervening yields. This was done iteratively. For example, to obtain the two year spot yield, assuming the bond is par value, is equivalent to solving the following equation for s(2):

$$1 = \{r(2)/(1+r(1))\} + \{(1+r(2)) / (1 + s(2)) * 2\}$$

where r(t) denotes the quoted yield for time t

s(t) denotes the one period "spot" yield

The two year spot yield is then used to calculate the three period yield, etc.

Once having calculated the spot yields, the forward yields are then calculated. The implied forward yields (denoted f(t)) for any twelve month time period are estimated as a component of a geometric average. For example, the one-year forward rate going one year into the future is calculated as follows:

$$s(2) = \{(1+s(1)) (1+f(1))\} * .5$$

In general,

$$s(t) = \{(1+s(1))(1+f(1))(1+f(2))\dots(1+f(t-1))\} * 1/t.$$

The subsequent t+1 maturity bond is equal to:

$$s(t+1) = \{(1+s(1))(1+f(1))(1+f(2))\dots(1+f(t))\} * 1/(t+1).$$

Dividing the second equation by the first and rearranging then

$$f(t) = \{[(1+s(t+1)) * t+1] / [(1+s(t)) * t]\} - 1.$$

## APPENDIX B

The Relationship Between Long and Short Interest Rates

Under the Expectations Hypothesis, it is often asserted that long-term rates are determined by a geometric average of current and expected short-term rates. However, this only holds true for zero coupon bonds where there is no reinvestment of interest. Under such a formulation, each individual short-rates is equally weighted. For coupon bonds, which are the bonds used in the analyses of this paper, reinvestment income accounts for a major portion of a long-term bonds compounded return. Hence, the near-term short-term expected interest rates should receive a heavier weight in the bond's determination than expected rates farther out in the investment horizon. Hence, a model taken from Shiller (1979) is derived below to incorporate such a weighting scheme in the formulation of long rates.

The formula for the price of an period coupon bond is

$$(1) P = \sum Cpn / (\pi(1+E_{\tau}[r(\tau)])) + [(Par Value) / (\pi(1+E[r(n)]))]$$

where  $E_{\tau} [r(\tau)]$  represents the expected short-rate at  $\tau$

Assuming a par value bond with \$1 face value

$$(2) 1 = \sum Cpn / (\pi(1+E_{\tau}[r(\tau)])) + [(1+r(n)) / (\pi(1+E_{\tau}[r(\tau)]))]$$

This can be rewritten as:

$$(3)$$

$$1 = \sum \{ E[r(\tau)] / (\pi (1+E_{\tau}[r(\tau)])) \} + \\ \{ (1+r(n)) / (1+(\pi(1+E_{\tau}[r(\tau)])) \}$$

If  $R$  is defined as the yield-to-maturity of the par value bond and is equal to the coupon rate. (3) can be rewritten as:

$$1 = \sum \{E[r(\tau)] / (1+R)^\tau\} + \{(1+r(n)) / (1+R)^n\}$$

Setting  $g = 1 / 1+R$ ,

$$(4) \quad 1 = \sum g^\tau * E[r(\tau)] + g^n$$

$$(5) \quad 1 - g^n = \sum g^\tau * E[r(\tau)]$$

Dividing by the left side of (5)

$$(6) \quad 1 = \{ \sum g^\tau * E[r(\tau)] \} / (1 - g^n)$$

Multiplying by  $R$ :

$$(7) \quad R = (R * \{ \sum g^\tau * E[r(\tau)] \}) / (1 - g^n)$$

$$(8) \quad R = (1 - g) \{ \sum g^\tau * E[r(\tau)] \} / (1 - g^n)$$

## Appendix C

The Macauley Duration

Frederick Macauley <sup>1</sup> developed the notion of the Macauley duration in order to have a simple measure of the price volatility of a bond. This was useful because a bond price sensitivity to changes in interest rates is directly related to its maturity and inversely related by the level of its coupon payments. The measure is calculated as follows:

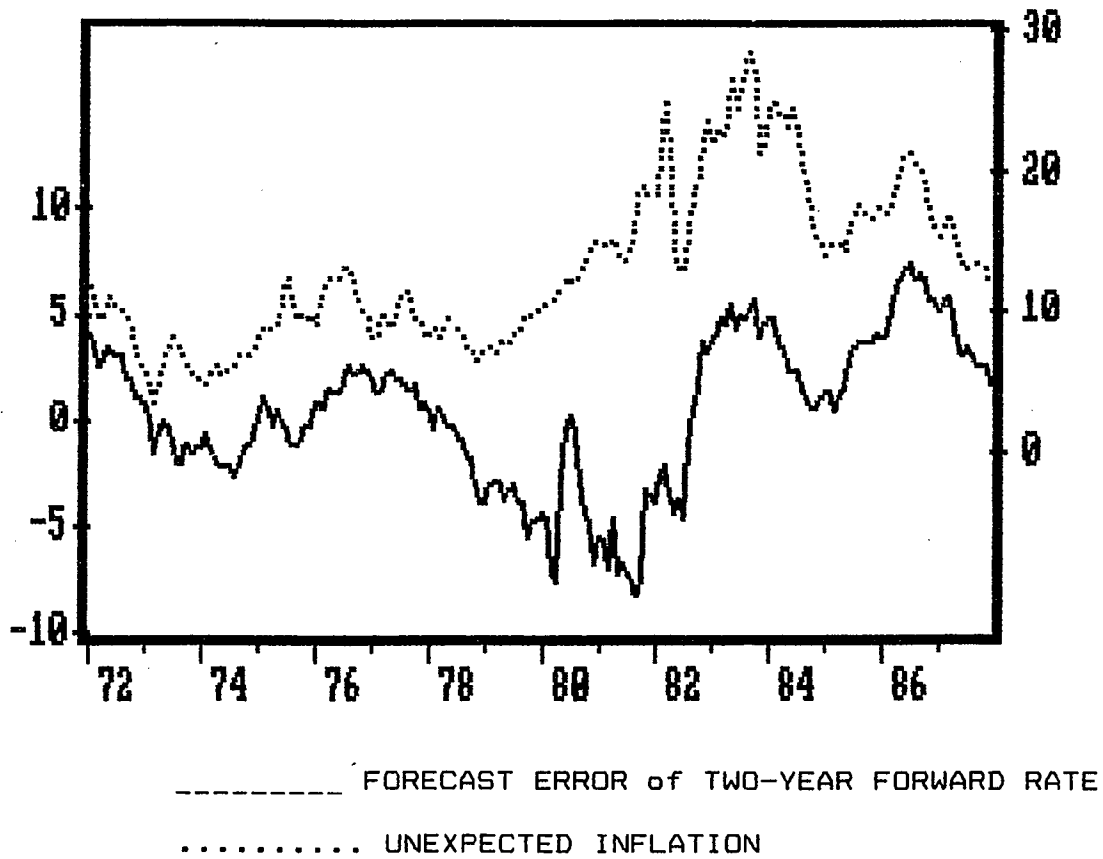
$$\text{Macauley Duration} = \frac{\sum \{CF(t) / (1+i)^t\}}{\sum \{CF / (1+i)^t\}}$$

where CF = bond's cash flows including coupons and  
principal

t = time at which cash flow is realized

i = market yield on the bond.

## APPENDIX D

The Relationship Between Forecast Error  
and Unexpected Inflation

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