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FUTURES MARKETS: THE MATURITY EFFECT ON RISK AND RETURN

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FUTURES MARKETS:
THE MATURITY EFFECT ON RISK AND RETURN

by

MARK G. CASTELINO

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Faculty in Business in partial fulfillment
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1981

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Abstract

FUTURES MARKETS:

THE MATURITY EFFECT ON RISK AND RETURN

by

Mark G. Castelino

Adviser: Professor Jack Clark Francis

The volatility of changes in futures prices increases as contract maturity is approached. Holders of futures contracts are thus bearing increasing levels of risk on a per day or per week basis, closer to maturity than further away from it. This proposition was first rigorously derived in a seminal paper put forth by Paul A. Samuelson in 1965. This dissertation follows through on the theoretical and empirical implications of that proposition.

The first question posed in this dissertation is on the behavior of returns to holders of futures contracts. If hedgers are net short and speculators are net long long, risk aversion on the part of market participants would necessarily imply that returns should increase commensurately over the life of the same contract. The existence of returns to holders of futures contracts, in turn, necessarily implies that futures prices must be biased estimates of expected spot prices at maturity.

If a systematic maturity effect exists for volatility as well as for returns, should there exist a risk-return ratio that is independent of the time left to maturity? Should this risk-return ratio vary from commodity to commodity? These are some of the questions on futures price behavior that are answered both theoretically as well as empirically in this dissertation.

The next set of propositions put forth and analysed is on the behavior of the basis. If futures price changes become increasingly volatile as contract maturity is approached, then the volatility of changes in the basis must necessarily decline as the maturity of the selected contract is approached. A corollary that follows is that a basis with respect to a nearer future must necessarily be less volatile than one with respect to a more distant one. These propositions are interesting because they imply that if risk reduction is the leitmotiv for hedging, then the nearer future is the preferred choice with which to achieve the objective than a more distant one.

Finally the behavior of spreads are analysed. The behavior of a spread follows directly from both, futures price as well as basis behavior. Two propositions on spread volatility are derived. The first is that the volatility of changes in a spread declines as the maturity of the nearer contract is approached. The second is that the volatility of changes in a spread is a function of its length. In other words, a one month spread is less volatile than a two month spread or a six month spread. Spread behavior thus closely follows basis behavior. This is not surprising, as a spread is simply a basis, where both positions taken in the market are in futures contracts rather than one one in futures and one in spot.

The various propositions from futures price volatility to spread volatility are empirically tested for a wide cross-section of commodities from the seasonal and storable type such as wheat to the storable and industrial type such as copper. The propositions are supported overwhelmingly in almost every case. There seems to exist thus, a very strong case for a maturity effect for the securities in futures markets.

Preface

This dissertation evolved from a germ of thought implanted by Paul Samuelson more than a decade ago on how futures prices should behave as the maturity of the contract is approached. The logic of his proposition was so compelling to me that I could not resist putting it to test. Furthermore, if he was correct, then a wealth of propositions should almost naturally follow, and if so, I have myself a dissertation. This is almost exactly what happened.

The completion of this dissertation was facilitated with the enormous help and support of three individuals, to whom I will probably remain forever indebted -- Professors Jack Clark Francis, Ashok Vora and my wife Leonie. Jack, as chairman of my dissertation committee is particularly responsible for often pulling me back from the brink of despair. Often I walked into his office feeling absolutely hopeless, but always came out totally rejuvenated. The wealth of knowledge and information he posessed and the the respect for a balance between theory and empiricism can never be overvalued. Ashok, besides being a member of my dissertation committee is also a personal friend. I can state with absolute certainty that this dissertation would never have been completed when it was, but for him. There was no time of day or night when he was unavailable to me for consultation. The many hours he gave me, filled with invaluable advice is reflected in almost every page of this manuscript. There is no way I can repay him for almost literally, the renting of his mind. As for my wife Leonie, enough cannot be said. Her help and encouragement started from the very germination of the idea to the the last typed word. It was never a question of bearing up with me during my travail, but always, how best should we get this job done. It was truly a team effort throughout.

Finally I wish to thank Roger Mesznik, who despite the enormous load of completing his own dissertation, always found the time to read my manuscripts and come up with very valuable suggestions. I also wish to thank Professors Gerald Pogue and Frank Fabozzi as members of my dissertation committee, the Finance department and the Computer Center of Baruch College of the City University of New York.

TABLE OF CONTENTS

LIST OF TABLES

PART I: INTRODUCTION

Chapter 1: Introduction

CHAPTER 1

INTRODUCTION

> "The theorem is so general that I must confess to having oscillated over the years in my own mind between regarding it as trivially obvious (and almost trivially vacuous) and regarding it as remarkably sweeping. Such perhaps is the characteristic of basic results. And actually the empirical question of the applicability of the model to economic reality must be kept distinct from the logical problem of what is the model's implied content."
>
> Paul Samuelson (1965, p.45)

The above statement was made just after proving the martingale theorem - a necessary step on the way to showing that futures prices must become increasingly volatile as contract maturity is approached.

Do futures prices indeed become more volatile as contract maturity is approached? Consider the following example as a test of the reasonability of the Samuelson proposition. Two contracts for the delivery of wheat exists. The first contract matures in one month and the second in six months. In pricing these two contracts, the market should take into account the fact that the second contract has much more uncertainty to be resolved before maturity than the first contract. Allow one day to pass. The amount of uncertainty that will be resolved for the first contract will exceed that for the second contract in relative terms. How should prices respond to the amount of uncertainty that has just been resolved? Clearly, the response of the price of the nearer contract must be stronger than for the more distant contract simply to reflect the fact that a much larger proportion of total uncertainty has been resolved for it. By extending the logic, a contract maturing in a week must respond far more strongly to new information than a contract that has a month to mature. It thus seems that even without appealing to the rigorous proof provided by Samuelson, common sense would dictate that futures prices must become increasingly volatile as contract maturity is approached.

If futures prices do show increasing volatility as maturity is neared, numerous questions and propositions come to mind immediately. The first of them being on the behavior of returns to holders of futures contracts. In a market composed of risk averse utility of terminal wealth maximisers, should returns follow a pattern that compensate holders of futures contracts for the increasing risk being borne? If the returns do indeed follow a similar pattern, then does a risk-return pattern emerge that is constant through time, in the sense of the ratio of risk to return being a fixed constant? Could we call this a market price of risk?

Still other propositions seem to flow from the increasing volatility hypothesis. For instance, what kind of behavior is implied for the basis? If futures prices do become increasingly volatile, then the basis must necessarily show a declining volatility over the life of the same contract. This is a very powerful result because it implies a constantly changing level of risk being borne by hedgers. Specifically, a nearer contract provides a less risky hedge than a more distant contract. If risk-reduction then is the leitmotiv for hedging, then clearly a nearer contract is the preferred choice with which to achieve the objective than a more distant one.

What is implied for spread behavior? How should a one-month spread behave as compared to a two-month spread or a six-month spread? How should a spread behave as the maturity of the nearer contract is approached? It can be shown that a one-month spread must necessarily be less volatile than a two-month spread or a six-month spread. Furthermore, the behavior of a spread is dependent on the time during which it is monitored. Thus a six-month spread should behave differently when monitored one month before maturity as compared to two or six months from maturity.

The focus of this dissertation will be on the effect of time left to maturity on the behavior of futures prices, the basis and spreads. It will attempt to answer the numerous questions raised earlier. Propositions will be put forth and empirically tested for several commodities ranging from the seasonal ones such as wheat and corn to the industrial type, such as copper. It is hoped that the research accomplished by this dissertation will foster a greater understanding of the behavior of futures markets and contribute to the ever increasing efficiency with which they operate.

In pursuit of my objectives, this dissertation will evolve in four parts. In Part II, the literature on the behavior of futures prices will be critically surveyed. Part III will be devoted to developing and empirically testing the various propositions that will be raised on the behavior of futures prices, the basis and spreads. In Part IV, I will summarize the important results obtained and conclude with suggestions as to where future researchers in this area should probe into.

PART II: REVIEW OF THE LITERATURE

Chapter 2: Markets: Cash, Forward and Futures

Chapter 3: Futures Trading: Hedging and Speculation

Chapter 4: Futures Prices and Risk Premia

Chapter 5: Price Behavior and Market Efficiency

CHAPTER 2

MARKETS: CASH, FORWARD AND FUTURES

The fundamental role of a market is to facilitate the distribution of goods and services through the efficient discovery of prices. Efficient discovery is taken to mean, the speed with which pertinent information relating to supply and demand is reflected in prices. In a free market economy prices serve as signals through which resources are allocated and economic activity is coordinated.

The achievement of an ideal market - a perfectly competitive one - is conditioned upon easy access to it by a broad spectrum of potential market makers. If broad access exists, then the probability of distorting influences affecting price is reduced to a minimum. Thus, prices could be expected to remain at levels that would not return abnormal profits. Any institution that promotes increasing participation in the process of price formation contributes to the achievement of the ultimate ideal of a perfectly competitive market. The institution of futures markets attempt to do precisely that.

Of the three principal types of markets in existence today, namely, the cash, forward and futures markets, the futures market represents the highest state in the evolutionary process toward the achievement of a perfectly competitive one. This can be seen by comparing the three markets.

Cash Markets

Cash markets are markets where titles to specific goods and services are exchanged. Transactions in these markets are unique in the sense that prices formed here are heavily influenced by the identities of the parties involved in the transactions. The enormous amount of subjectivity involved in cash market transactions make prices formed in these markets less objective and meaningful. Too many elements that have little or nothing to do with underlying supply and demand are allowed to influence price.

Forward Markets

Forward markets are a minor extension of cash markets. The only real difference is that instead of immediate delivery, delivery is made after a defined passage of time. The problem of uniqueness of price in these markets is similar to that of the cash markets. Thus, prices formed here have little meaning to parties not involved in the transaction.

Futures Markets

The major contribution of futures markets over cash and forward markets has been the ability to provide highly competitive central market prices that have near universal applicability. The ability of these markets to bridge that major gap has depended on numerous factors, some of which are - trading in highly standardized contracts, creation of a clearing house that eliminates the need for identifying specific buyers and sellers, trading by open outcry on the exchange floor and the attraction of professional traders who bring an increasing amount of information to bear upon price.

CHAPTER 3

FUTURES TRADING: HEDGING AND SPECULATION

Futures trading in organized exchanges consists of the deferred sale and purchase of commodities through the medium of highly standardized futures contracts. These contracts provide for the delivery of the commodity in specified amounts at specified locations at a defined future date. The design of the futures contract provides its seller with a wide latitude as to the grade of the commodity to be delivered as well as the location at which to deliver. This has resulted in a small percentage of contracts being settled by ultimate delivery.[1] Such a result was intended, because futures trading did not develop for the purpose of buying and selling stocks of a commodity at a later date, but rather, to provide central market prices together with a mechanism where risks inherent in uncertain future prices could be transferred from one party to another.

The risks associated with price level changes in the spot commodity are of such a type[2] that they cannot be

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1. Less than one percent of the open interest results in delivery for the major commodities. Rarely ever does delivery exceed ten percent for others.

2. The principle of insurance is based on the law of large numbers which requires the underlying events to be independent of each other. Since this independence requirement is highly unlikely to be satisfied where the insurance of say crop damage is concerned, no insurance company would provide insurance for it.

covered by regular insurance. Therefore, the existence of speculators (those who are willing to carry these risks) is imperative for the proper functioning of these markets. The lack of this speculative element has often been claimed to be one of the major reasons for the failure of numerous markets.[3]

The risk transfer mechanism in its simplest form is as follows. A trader, wishing to neutralize a "buying risk" in the future, will simply buy a futures contract for delivery of the commodity at the pre-arranged (futures) price, while a trader wishing to neutralize a "selling risk" can do so by simply selling a futures contract calling for delivery at the futures price. Since neither trader really wishes to take receipt or make delivery of the underlying commodity, they will presumably undertake an offsetting transaction. Namely, the seller will buy, and the buyer will sell an equivalent amount of futures contracts at some time in the future, but prior to the maturity of the contracts. It should be noted that such actions on the part of traders does not entirely eliminate "price level" risks associated with the commodity under concern, but simply reduce them. In standard terminology, they exchange "price level risk" for "basis risk" - the latter being considerably

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3. See Gray (1966), Working (1970).

smaller than the former. If elimination of price level risk were desired, it could be achieved simply by making or taking delivery of the commodity at contract maturity. However, since it was presumed, and correctly so, that such was not the intent for the use of futures markets, this residual or basis risk will always remain.

A trader who exchanges price level risk for basis risk is commonly referred to as a hedger. The use of futures markets as a medium for hedging has long been recognized. The types of and the motivations for hedging, however, have changed over time. No longer is the simplistic risk avoidance view the dominant one for using the markets. Hedging nowadays is done for different reasons in different circumstances.[4/] No matter what the motivations for hedging are, the act itself necessitates a flow of risk from the hedger to the other major participant - the speculator. Speculating in commodity futures is the holding of a net short or net long position in the market for the purpose of monetary gain, and not for the purposes incidental to operating a producing, merchandising or processing business.

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4. See Working (1953)

CHAPTER 4

FUTURES PRICES AND RISK PREMIA

The fact that hedging facilitates a transfer of risk from the hedger to the speculator cannot be denied. Whether a compensation scheme does or should accompany that transfer is a widely disputed issue that remains unresolved to this day.

Proponents who argue for the existence of risk premia (Keynes (1930), Hicks (1953), Cootner (1960), Houthakker (1957)), essentially base their case on the insurance analogy. The insurance principle rests heavily on the premise of risk aversion on the part of hedgers, who use the markets for the purpose of reducing or eliminating price level risk. If risk is to be reduced, they argue, then hedgers must pay speculators who exist to facilitate this risk reduction. This line of reasoning necessarily implies that futures prices must be biased estimates of expected spot prices at maturity. Whether the bias is upward or downward would depend upon whether hedgers as a group are net short or net long. If they are net short, prices should be downward biased, and if net long then prices should be upward biased. The elimination of this bias as contract maturity is approached would then be the mechanism through which speculators are rewarded for their risk bearing function.

The major proponent for the non-existence of risk premia has been Telser (1958, 1960). Telser's argument rests on the premise that open competition among speculators and free entry into both the long and the short side of the market must necessarily bid speculative profits to zero. Thus, at all times futures prices should be unbiased estimates of expected spot prices. A different line of reasoning leading to the same conclusion has been put forth by Black (1976). Black argues from a general equilibrium setting. He states that futures markets as an institution does not provide a unique mechanism for transferring risk. The corporate form of organization through the public sale of equity can in a highly efficient manner accomplish what futures markets are claimed to do - namely, risk transfer. If such is the case then futures markets are a superfluous institution. Participants in them, especially speculators should not expect to be compensated for bearing risks that can be diversified away. Still another line of reasoning is used by Kaldor (1940). The thrust of Kaldor's position stems from the existence of a "convenience yield", which accrues only to hedgers and not to speculators. Over and above this convenience yield, Kaldor argues that the heterogeneity of expectations among speculators may cause transactions between themselves on either side of the market to be of such a scale as to swamp out any risk premia that may otherwise exist. This is very much like the

Telser argument.

Finally, there is the Hardy (1923) contention. Hardy argues for the existence of a compensation scheme where speculators pay hedgers for the privilege of participating in futures markets. The Hardy hypothesis is based on the presumption that gambling is not a socially acceptable activity. Since futures markets according to him are like gambling casinos, speculators pay for the privilege of gambling in a socially acceptable manner.

Empirical research on the existence or non-existence of risk premia has yielded inconclusive results. Researchers on both sides of the issue claim to have found support for their positions for various commodities. The interested reader may refer to scholarly articles by Houthakker (1957), Telser (1958, 1960), Cootner (1960), Gray (1960, 1961) and Rockwell (1967).

CHAPTER 5

PRICE BEHAVIOR AND MARKET EFFICIENCY

In analysing the behavior of commodity futures prices two important distinctions must be accounted for:

1. The nature of the commodity being analysed (the commodity effect).
2. The effect of time to maturity on prices (the maturity effect).

This dissertation concerns itself with the "maturity effect". It will thus be extensively treated in the later chapters. The "commodity effect" is now looked at in some detail.

Commodity Effect

Commodities can basically be broken down into two categories. The first category includes all those commodities that can essentially be carried to maturity of any futures contract. The second includes those which by and large cannot. The bulk of research to date has dealt mainly with the commodities of the first type - the storable commodities. Examples of such are wheat, corn and cotton. Metals, both precious and non-precious, as well as the more recent currencies and financial instruments also clearly qualify for this category within the broad division. All perishabile commodities such as livestock and eggs are examples of commodities of the second type - the non-storable commodities.

The importance of this division is the implication the storable property has for the relationship between spot and futures prices. The ability to carry a commodity to maturity against the requirements of a futures contract will necessarily link spot and futures prices into a sort of <u>functional relationship</u>, which must then become a key consideration in the use of such markets by its users. The nature of this relationship has been explored in the literature in the "carrying charge" and "price of storage" theories of Working (1949) and Brennan (1958). The inability to carry commodities of the second type to contract maturity cannot impose a similar sort of functional relationship between spot and futures prices.

The most fascinating aspect of commodities, the spot and futures prices of which bear a functional relationship to each other, is the response of the entire spectrum of these prices to the advent of new information. At any moment in time, the position in which futures prices stand in relation to each other and in relation to the spot price, reflects both existing supply and demand together with expectations of the same in the future. The arrival of new information into the market, irrespective of the specific time that information pertains to, must have an effect on the entire constellation of prices. To quote Working:[5]

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[5] See Working (1949) p. 1255

> " ..... prices quoted at any one time, in a futures market for two different delivery dates, stand in a relation which in general <u>does not</u> reflect expectations regarding events that may occur between the two delivery dates."

The underlying reason for such a phenomenon is that in a market for a storable commodity, arbitrage can be relied upon to maintain inter-temporal price spreads which are dependent heavily upon the level of existing supplies. Samuelson (1973) makes a very similar argument in a discussion as to what can reasonably be expected to be arbitraged out in a perfectly competitive market for a storable commodity.

For the identical reason that arbitrage can be relied upon to maintain inter-temporal price spreads for a storable commodity, it cannot be relied upon to accomplish the same for a non-storable commodity. Prices quoted at any time for two different delivery dates for a non-storable commodity can bear no relationship to each other, if supplies supplies cannot be carried from the nearer maturity date to the further one. If such is the case, any information affecting supply or demand for any one of the two dates will alter the inter-temporal price spread in a manner that has little or nothing to do with the level of existing supplies. The linkage between the two prices, if there is any at all, will be a weak one. The reason for this is that in reality almost every commodity has some degree of storability.

The role played by futures markets for the storable and non-storable commodities is thus different. Briefly, for storable commodities, they play an allocative role - allocative in the sense that prices are signalling devices calling for allocation of existing supplies between current and future consumption. For the non-storable commodities, they play a price forecasting role - providing the best estimate of future price from projected supply and demand conditions.

Stochastic Behavior

In 1953, Maurice Kendall(1953) produced evidence that purported to show that a time series of speculative price behavior looked like white noise. Economists at that time were outraged at such a suggestion, because it seemed to imply that pure chance rather than economic law governed the wanderings of price. It was not until the Samuelson (1965) paper that a reconciliation could take place between belief in economic law and the quasi-random behavior of speculative price.

Do commodity futures prices perform a random walk? Working (1958) developed a theory of price movements where he showed that the behavior of market participants would lead to price changes that would be nearly random. The model developed showed how price changes should take place gradually rather than in discrete jumps. Such behavior was premised on the different speeds with which traders respond to new information affecting price.

Samuelson (1973) in a more formal development of the theory used the martingale theorem to show that futures prices do not follow a random walk, but rather display a behavior which is quasi-random and influenced by the time left to maturity of the contract.

Empirical research on the behavior of futures prices has been extensive. (See Houthakker (1957), Smidt (1965), Praetz (1975), Larson (1960), Stevenson and Bear (1970), Rutledge (1976), Dusak (1979) and Segall (1956)). The research has generally followed two lines of approach. The first approach is typified by the work of Houthakker, Smidt and Praetz. The answers sought here were whether any systematic behavior, typically the existence of trends and serial correlation was exhibited by futures prices. The second approach, exemplified by the work of Stevenson and Bear and Larson sought to examine specific properties of the underlying distributions of futures prices. The results obtained have not been conclusive. For instance, the search for non-random behavior has been able to yield positive results in only a few specific cases, which are not generally applicable. Results on the distributional characteristics of futures price changes indicates a tendency to be leptokurtotic, suggesting the non-stationarity property proposed by Samuelson. The work of Rutledge, Dusak and Segall on the nature of the leptokurtotic distributions has been unable to strongly confirm whether the distributions are a result of increasingly volatility over the life of the futures contract.

PART III: THE MATURITY EFFECT

CHAPTER 6

INTRODUCTION: THE MATURITY EFFECT

The behavior of futures prices over the life of a futures contract is of considerable interest to both, students as well as practioners of futures markets. Empirical studies (Bear (1979), Stevenson and Bear (1970), Larson (1960)) have shown that futures returns do not belong to the family of stable symmetric distributions. One possible reason for such a phenomenon is that the first differences of futures prices are not covariance stationary over the life of futures contract. Samuelson (1965) in a seminal paper described the influence of time left to maturity on the volatility of changes in futures prices. Specifically, he showed that as contract maturity is approached, changes in futures prices should display increasing volatility. Empirical confirmation of such a phenomenon has been attempted by Segall (1956), Rutledge (1976) and Dusak (1979). The results obtained have been mixed. However, they showed a tendency to support rather than refute the Samuelson proposition.

If the volatility of changes in futures prices does indeed increase over the life of a futures contract, then important implications can be drawn for equilibrium returns as well as hedging behavior. For instance, if volatility increases as maturity decreases, then equilibrium should require that returns increase commensurately.

Where hedging behavior is concerned, increasing volatility of futures price changes necessarily translates into decreasing volatility of changes in the basis. This decreasing basis volatility implies that hedging in a nearer future is a less risky proposition than hedging in a more distant one. Very similar behavior exists for changes in spreads. A one-month spread must necessarily be less volatile than a two-month spread or a six-month spread. Finally, if a systematic risk-return structure exists for futures prices, then it can be shown that futures prices must necessarily be biased estimates of spot prices expected at maturity.

The focus of Chapter 7 is to develop and test the theoretical implications of the effect of time to maturity on the volatility of changes in futures prices. The implications will be analysed. Various theoretical propositions will be developed. From these propositions testable hypotheses will be drawn. The chapter will conclude by describing the empirical tests together with the results obtained from the various tests. Chapters 8 and 9 will follow an identical pattern. Chapter 8 will cover basis behavior while spread behavior will be covered in Chapter 9 .

CHAPTER 7

THE MATURITY EFFECT ON FUTURES PRICES

Theoretical Development

In order to show the effect of maturity on changes in futures prices, the process generating spot prices will be the one selected by Samuelson, with one modification. The modification imposed allows for the intervention of market forces on the spot commodity as follows.[1]

$$\tilde{S}(t) = aS(t-1) + b\Delta\tilde{M}(t) + \tilde{u}_t \quad \dots\dots\dots\dots\dots\dots\dots \quad 7.1$$

where $\tilde{S}(t)$: Spot price at time "t"

$/a/$: damping factor < 1

$\Delta\tilde{M}(t)$: Change in the market index

b : Sensitivity coefficient for commodity $\gtreqless 0$

$$E(\tilde{u}_t) = 0; \quad E(\tilde{u}_t,\tilde{u}_{t+n}) = 0; \quad E(\tilde{u}_t{}^2) = \sigma_u^2$$

$$t \neq n$$

---

1. The first order damped autoregressive scheme allowing for market intervention has been selected because there is reason to believe that more than just the past history of spot price can affect future spot prices. Since market efficiency precludes the inclusion of antincipated effects, only the unanticipated effects need be included. Furthermore, it should be noted that adding either a deterministic component or increasing the order of the scheme, will not affect the main results, so long as the coefficients of the lagged variables, $/a_i/ < 1$.

Suppose furthermore the market index is generated by a first order autoregressive process as follows.

$$\tilde{M}(t) = cM(t-1) + \tilde{e}_t$$

where $/c/$: damping factor < 1

$$E(\tilde{e}_t) = 0; \quad E(\tilde{e}_t, \tilde{e}_{t+n}) = 0; \quad E(\tilde{e}_t{}^2) = \sigma_e^2$$

$$t \neq n$$

Then the unanticipated change in the market index is given by

$$\triangle \tilde{M}(t) = \tilde{M}(t) - cM(t-1) = \tilde{e}_t$$

Substituting in 7.1, the generating process for spot prices reduces to:

$$\tilde{S}(t) = aS(t-1) + b\tilde{e}_t + \tilde{u}_t \quad \ldots\ldots\ldots\ldots\ldots\ldots 7.2$$

where $E(\tilde{e}_t . \tilde{u}_t) = 0$

By successively substituting for $\tilde{S}(t+1)$, $\tilde{S}(t+2)$, etc., it can be shown that:

$$\tilde{S}(T) = a^{T-t}S(t) + a^{T-t-1}\tilde{u}_{t+1} + a^{T-t-1}\tilde{u}_{t+2} \quad \ldots. + \tilde{u}_T$$
$$+ b(a^{T-t-1}\tilde{e}_{t+1} + a^{T-t-2}\tilde{e}_{t+2} \quad \ldots + \tilde{e}_T)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots 7.3$$

The expected spot price and the variance of spot price at time T as viewed from t is given by:

$$E_t(\tilde{S}(T)) = a^{T-t}S(t)$$

$$Var_t(\tilde{S}(T)) = \frac{a^{2(T-t)}-1}{a^2-1} \left(\sigma_u^2 + b^2\sigma_e^2\right) \quad \ldots. 7.4$$

Equation 7.4 suggests that the expectation of the spot price at time T as viewed from time t will tend towards zero or its normal value (if there is a deterministic or seasonal component to 7.1) as T goes to infinity. Thus, the process guarantees that any perturbation of price above or below its normal value is expected to die out with the passage of time. However, the variance of the spot price to be realized at time T will converge to $(\sigma_u^2 + b^2\sigma_e^2)/(1-a^2)$, as T goes to infinity.[2]

Consider now a futures contract calling for the delivery of the spot commodity at time period T. Assuming no margin requirements, the equilibrium futures price is given by[3]

$$F(t,T) = E_t(S(T)) + \lambda \mathrm{Var}_t(\tilde{S}(T)) \qquad \ldots\ldots\ldots\ 7.5$$

where λ = Risk adjustment factor.

---

2. The damped autoregressive process does not permit the spot price to wander anywhere with the passage of time. It obeys an ergodic probability distribution that tends to pull it back to its normal value with any perturbation away from it.

3. The issue as to whether $\lambda \gtreqless 0$ is hotly disputed, with competing theories vying for what its sign should be

$\lambda > 0$ .... Hardy (1923)
$\lambda < 0$ .... Keynes (1930), Hicks (1953)
$\lambda \lesseqgtr 0$ .... Cootner (1960)
$\lambda \equiv 0$ .... Telser (1958)

Substituting for $E_t(\widetilde{S}(T))$ and $Var_t(\widetilde{S}(T))$ from 7.3

$$F(t,T) = a^{T-t}S(t) + \lambda \frac{(a^{2(T-t)}-1)}{(a^2-1)}(\sigma_u^2+b^2\sigma_e^2) \quad \ldots\ldots \quad 7.6$$

Now consider the price of the futures contract at t+1, as viewed from time t, $\widetilde{F}(t+1,T)$

$$\widetilde{F}(t+1,T) = a^{T-t-1}\widetilde{S}(t+1) + \lambda \frac{(a^{2(T-t-1)}-1)}{(a^2-1)}(\sigma_u^2+b^2\sigma_e^2)$$

Substituting for $\widetilde{S}(t+1)$

$$\widetilde{F}(t+1,T) = a^{T-t}S(t) + a^{T-t-1}\widetilde{u}_{t+1} + b.a^{T-t-1}\widetilde{e}_{t+1}$$

$$+ \lambda\,(a^{2(T-t-1)}-1)(\sigma_u^2+b^2\sigma_e^2) \quad \ldots \quad 7.7$$

Subtracting 7.6 from 7.7

$$\triangle\widetilde{F}(t,T) = a^{T-t-1}\widetilde{u}_{t+1} + b.a^{T-t-1}\widetilde{e}_{T+1}$$

$$- \lambda\,(a^{2(T-t-1)})(\sigma_u^2+b^2\sigma_e^2) \quad \ldots\ldots 7.8$$

Thus:

$$E_t(\triangle\widetilde{F}(t,T)) = -\lambda\, a^{2(T-t-1)}(\sigma_u^2+b^2\sigma e^2) \quad \ldots\ldots\ldots\ldots \quad 7.9$$

$$Var_t(\triangle\widetilde{F}(t,T)) = a^{2(T-t-1)}(\sigma_u^2+b^2\sigma_e^2) \quad \ldots\ldots\ldots\ldots\ldots \quad 7.10$$

Differentiating 7.10 w.r.t. t, the first fundamental result is obtained.

$$\frac{d}{dt}\mathrm{Var}_t(\Delta\tilde{F}(t,T)) = -2a^{2(T-t-1)}(\sigma_u^2 + b^2\sigma_e^2)\ln(a) \quad \ldots\ldots \quad 7.11$$

$$> 0 \quad \ldots\ldots \quad \text{since } /a/ < 1$$

<u>The volatility of changes in futures prices increases systematically over the life of the contract</u>.

Observing 7.9 and 7.10, the only difference between the two expressions is $-\lambda$, thus:

$$\frac{E_t(\Delta\tilde{F}(t,T))}{\mathrm{Var}_t(\Delta\tilde{F}(t,T))} = \frac{\frac{d}{dt}E_t(\Delta\tilde{F}(t,T))}{\frac{d}{dt}\mathrm{Var}_t(\Delta\tilde{F}(t,T))} = -\lambda \quad \ldots\ldots \quad 7.12$$

Thus,

<u>futures returns bear a fixed relationship to the risk associated with those returns</u>.

The extent of the relationship is an empirical question, the result of which will reveal much about the nature of the specific market itself.

<u>Implications of $\lambda \gtreqless 0$</u>

If $\lambda = 0$. then futures prices are unbiased estimates of expected spot prices at maturity. Empirical tests confirming this result would support the Telser[4/] proposition that on an average, speculators neither gain nor lose

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4. See Telser (1958)

in futures markets[5]. According to Gray (1960 A), a $\lambda = 0$, is a revelation of a highly efficient, liquid and mature futures market. In such a market, the costs of hedging have been reduced to the bare minimum of commission costs and interest charges on margin requirements. Most new markets according to Gray (1960 B) tend to start off with a $\lambda < 0$, where an excess of hedging and a lack of adequate speculation imposes a downward bias on futures prices. This causes a steady upward drift in futures prices as contract maturity approaches. As markets mature by an increase in speculative interest, increased competition among speculators bid prices up sufficiently to reduce premiums to zero, making futures prices unbiased estimates of expected spot prices.

If $\lambda > 0$, then futures prices are upward biased estimates of expected spot prices. Market equilibrium would then demand that futures prices gradually decline over the life of the contract. The rate of the decline would depend on the extent of the biasedness (size of λ), as well as the time left to maturity. A $\lambda > 0$ supports Hardy's contention that futures markets are simply gambling casinos

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[5] Cootner (1960) argues that a $\lambda = 0$ is not a strong enough test to prove the Telser proposition. For instance, if $\lambda > 0$ for half a year and $\lambda < 0$ for the remainder for an average value of $\lambda = 0$, speculators could still gain, if they were long in futures for the first half and short for the second respectively.

of the socially acceptable type. Speculators in such markets are willing to pay a fee (reflected in $F(t,T)-E_t(S(T))$, in order to participate. Markets that are highly speculative (as measured by Working's (1960) speculative index), are likely to show a $\lambda > 0$. In such markets, according to Working, the volume of open interest held by speculators far exceeds the minimum needed to carry the hedging interests. This is not a satisfactory state of affairs as prices in such markets tend to be unstable as they are dominated by speculators of the non-professional type.

A $\lambda < 0$ is the classic Keynes-Hicks type of market, where hedgers pay speculators for carrying price level risks. The premium paid by hedgers to speculators manifests itself through gradually rising futures prices over the life of the contract. Such markets thus imply the existence of an additional cost to hedgers over and above commission charges and margin requirement costs. Gray (1960 B) calls this additional cost the price of biasedness. According to him a $\lambda < 0$ is a reflection of a thin market, caused primarily by a lack of adequate speculation. If this bias is not removed, he warns, the market is likely to die prematurely, because no hedger has an infinite tolerance to consistent losses on the futures portion of his hedge.

A $/\lambda/ > 0$ indicates price biasedness. This could readily support Cootner's proposition that speculators are long in 'up' markets and short in 'down' markets. But then the reverse could also be true. One thing is clear however, a $/\lambda/ > 0$ indicates that prices are indeed biased. According to Gray's definition, these markets have yet to reach maturity.

Testable Hypotheses

Numerous testable hypotheses emerge from the preceeding theory. Hypotheses on volatility, returns, as well as risk-return relationships. The various hypotheses are stated below.

Hypothesis 1

The volatility of changes in futures prices increases systematically over the life of a futures contract

$$\frac{d}{dt} \mathrm{Var}_t(\Delta\tilde{F}(t,T)) > 0$$

as t approaches T.

Hypothesis 2

Expected changes in futures prices increase systematically over the life of a futures contract.

$$\frac{d}{dt} E_t(\Delta\tilde{F}(t,T)) > 0$$

as t approaches T.

Hypothesis 3

A constant relationship exists between risk and return.

$$\frac{E_t(\Delta\tilde{F}(t,T))}{\mathrm{Var}_t(\Delta\tilde{F}(t,T))} = -\lambda$$

If $\lambda < 0$ .... Keynes-Hicks

$\lambda > 0$ .... Hardy

$\lambda = 0$ .... Telser

Empirical Tests

The three hypotheses stated will be tested separately, with the results correspondingly displayed.

Hypothesis 1: $$\frac{d}{dt} \mathrm{Var}_t(\Delta\tilde{F}(t,T)) > 0$$

as t approaches T

The difficulty with attempting to test the proposition of increasing volatility of changes in futures prices[6] lies in sample data collection. Theoretically each observation comes from a distribution with a different variance. Aggregation of data by week or by month will thus introduce bias.

Far more serious than this aggregation bias is the existence of a tremendous amount of noise that intervenes as a contract approaches maturity. The inability to suppress this noise will make the detection of the pure maturity effect on futures prices well nigh impossible. Rutledge (1976) attempted to account partially for this noise by partitioning the data to allow for movements in the spot price of the commodity. Dusak (1979) attemtpted an aggregation process by using contract data from several

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[6] The natural logarithm of price relatives is used as a proxy for price change.

years. By increasing the sample size, it was hoped the maturity effect would reveal itself. The failure of both techniques to reveal significant results, probably followed from their inability to account for the distorting influences introduced by "month effects" and "year effects".[7] These effects influence measured volatility to such an extent, that they make detection of the pure maturity effect difficult.

The key to a successful test of the maturity effect then lies in controlling for the two above mentioned effects. These effects operate as follows. Observation of the variance of daily returns measured over any month reveals the month effect. For example, if one observes the variance of daily returns in Jaunary for a wheat contract maturing in December, as compared to that in April for a contract maturing in the following March, the variances differ significantly from each other, even though both are measurements made 11 months from maturity. It seems to matter in which month the measurements are actually made. This month effect becomes overwhelming around the harvest months for a seasonal commodity. Thus for any single contract in wheat, for instance, the measured variances in

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7. See Samuelson (1965, p. 45). The underlying reason for these effects are non-stationarity in the spot generating process.

July, August and September are invariably higher than measured variance in other months. the year effect reveals itself through observation of variances in say January for a December contract in two different years. For data from 1960-1971, lower variances in the earlier years as compared to the later years were observed.

The test procedure used will control for these **two** distorting effects, thus allowing for the pure maturity effect to reveal itself.

<u>Test Procedure</u>

the following test assumes the existence of futures contracts calling for delivery of the commodity in each month of the year.[8] Thus, at any time there are twelve contracts outstanding, including the one calling for immediate delivery.

Let

$V_{ij}(\triangle\tilde{F})$ Variance of futures returns[9] in month "i" for contract "j" months from maturity. $(i = 1,12;\ j = 1,11)$

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8. For most commodities, contracts for delivery in every month do not exist. The test procedure used thus introduces bias. However, the bias is very slight and operates in a direction that would indicate a weaker maturity effect than the one that would be obtained, if either all contract months or symmetrically spaced contract months were available. available. Observations within the delivery month were eliminated in order to avoid distortions from 'short squeezes', a well known event of no interest to this inquiry.

9. The returns are daily returns. The measured variance must be looked upon as an average daily variance, since theoretically it changes every day of the month.

$$\bar{V}_i(\Delta\tilde{F}) = (\prod_{j=1}^{11} V_{ij}(\Delta\tilde{F}))^{1/11}$$

Average[10] variance of futures returns for contracts maturing within 1 through 11 months, as measured in month i.

$$v_{ij}(\Delta\tilde{F}) = V_{ij}/\bar{V}_i = v_k(\Delta\tilde{F}) \qquad k = 1,11$$

Standardized variance of futures returns k months from maturity.

Test Variable $v_k(\Delta\tilde{F})$

The variable v_k measures the variance of returns from which the month and year effects have been removed. For instance, v_1 is a November variance for a December contract in 1965, just as it is a February variance for a March contract in 1970.

For concreteness, consider wheat - which has contracts maturing in March, May, July, September and December. In any year, five observations for v_1 would

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[10] When less than 11 contracts are outstanding, the geometric average variance of the outstanding contracts was used. The use of a geometric rather than an arithmetic average for standardization is necessitated by equation 7.10 It can be shown that the geometric average variance of n contracts of different maturity equals the variance of a contract with a maturity equal to the arithmetic average maturity of the n contracts.

be obtained:

February observations for the March contract
April observations for the May contract
June observations for the July contract
August observations for the September contract
November observations for the December contract.

Furthermore, if 12 years of data are used, 60 observations would be obtained, each coming from measurements made in February, April, June, August and November as described above.

The first test performed is on the homogeneity of the groups from which v_k is composed. Staying with the wheat example, tests are performed to determine the homogeneity of $v_{1,r}$: r=2,4,6,8,11 where r represents the calendar month in which the variances are measured. The null hypothesis here is that the groups are homogenous. It should not be rejected.

If the groups that make up each v_k are homogenous, then tests for the homogeneity of v_k, k=1,11 are performed. This is simply an across group test as compared to the within group test which was performed earlier. If a maturity effect exists, then the null hypothesis of the homogeneity of v_k should be rejected.

The final test performed is to detect the existence of a systematic maturity effect. The test consists of a regression of the natural logarithm of standardized variance versus the number of months left to maturity. The test is developed from equation 7.10 as follows.

Dividing equation 7.10 by the geometric average variance of the outstanding contracts at time t

$$\frac{Var_t(\triangle\tilde{F}(t,T))}{\bar{V}ar_t(\triangle\tilde{F}(t,T))} = \frac{Var_t(\triangle\tilde{F}(t,T))}{Var_t(\triangle\tilde{F}(t,T_{avg}))}$$

$$= \frac{a^{2(T-t-1)}}{a^{2(T_{avg}-t-1)}}$$

$$= a^{2(T-T_{avg})} \qquad \ldots\ldots\ldots\ldots \; 7.13$$

Using the notation introduced in the standardization procedure

$$v_k(\triangle\tilde{F}) = a^{2(k-k_{avg})} \qquad \ldots\ldots\ldots\ldots \; 7.14$$

where k = Number of months to maturity.

k_{avg} = average maturity of outstanding contracts.

Taking the natural logarithm of 7.14

$$\ln(v_k(\triangle\tilde{F})) = 2(k-k_{avg})\ln(a) \qquad \ldots\ldots\ldots\ldots \; 7.15$$

The regression is then run as follows

$$\ln(v_k(\triangle\tilde{F})) = A + B.K \qquad \ldots\ldots\ldots\ldots \; 7.16$$

where $K = k-k_{avg}$

The null hypothesis is that

$A = 0$

$B = 2\ln(a) < 0$

Results

Data Base

The data base used was the futures price series from the Dunn and Hargitt Commodity tape.

The years selected were 1960-1971. The commodities selected were:

i) Wheat (Chicago Board of Trade)
ii) Corn (Chicago Board of Trade)
iii) Soybeans (Chicago Board of Trade)
iv) Soymeal (Chicago Board of Trade)
v) Soyoil (Chicago Board of Trade)
vi) Copper (COMEX)

Table 7.1 reveals the existence of a "Maturity Effect". For every commodity from wheat to copper, the hypothesis that the variances of futures returns are homogenous is rejected at the 1% level of significance.[11] The increase in variance as distance from maturity declines is seen to be the sharpest very close to contract maturity (1 and 2 months) for almost all commodities. The theoretical development did predict this behavior. However,

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[11] F tests were performed to determine the homogeneity of the groups that make up each v_k. At the 1% level, the hypothesis that the groups were homogenous could not be rejected in 58 of 66 cases.

it is also possible that delivery considerations will begin to manifest themselves in the month before maturity causing greater volatility than would otherwise be expected. The higher than expected variance for corn 11 months from maturity could be attributed to the thinness of the market in terms of the low open interest and volume of trade. The fact that the size of the variance changes direction sometimes, for instance, as in copper 4 to 5 months from maturity could result from a misspecification of the spot generating process. Samuelson[12/] has shown that if higher order terms are included in the process, it is possible that the variance can transiently change direction for a few periods near maturity.

Table 7.2 reveals the existence of a systematic maturity effect. The proposition that futures prices become increasingly volatile as maturity is approached is confirmed by the negative slope coefficient, which is highly significant as indicated by the "t" statistic. The intercept is not significantly different from zero as predicted by the theory. The large value of the "F" ratio is further confirmation of the strong maturity effect. The low R^2 on the other hand is indicative that much more than just time left to maturity is responsible for futures price volatility.

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12. Samuelson (1976)

The implicit value of "a" being less than 1 simply confirms the specification of the spot generating process.

TABLE 7.1

Standardized Variance of Futures Returns by Number of Months to Maturity $(v_k(\Delta F))$

Commodity	# of Months to Maturity											F	DF*
	1	2	3	4	5	6	7	8	9	10	11		
Wheat	1.4153	1.2515	1.0752	1.0093	0.9994	0.9813	0.9717	0.9444	0.9486	0.9624	0.9498	11.479	10:642
Corn	1.1794	1.0621	1.0807	1.0288	1.0316	1.0179	1.0032	0.9434	0.9920	0.9609	1.0052	3.716	10:550
Soybeans	1.3956	1.2254	1.1602	1.1834	1.1116	1.0490	0.9885	0.9528	0.8981	0.8687	0.8611	16.570	10:873
Soymeal	2.0219	1.6246	1.4162	1.1769	1.0234	0.9510	0.8720	0.7836	0.7646	0.7561	0.8037	57.135	10:921
Soyoil	1.2815	1.2668	1.1815	1.1542	1.0671	0.9947	0.9403	0.9336	0.9154	0.8224	0.7867	30.640	10:915
Copper	1.2661	1.2289	1.1112	1.0433	1.0876	1.0195	0.9657	0.9582	0.9130	0.8827	0.8810	10.866	10:747

* DF: Degrees of Freedom. Because of erroneous or incomplete data, some observations had to be discarded.

TABLE 7.2

Regression Results of Variance of Futures Returns versus Number of Months to Maturity

$$\ln(v_k) = A + BK$$

Commodity	$\hat{A}^*$	$\hat{B}^*$	a^{**}	F	R^2	DF
Wheat	-0.000000 (0.000)	-0.030949 (9.211)	0.9846	84.807	0.11526	651
Corn	-0.000000 (0.000)	-0.015566 (4.926)	0.9922	24.336	0.04245	549
Soybeans	-0.001576 (0.032)	-0.047290 (13.628)	0.9766	185.891	0.17407	882
Soymeal	-0.001117 (0.026)	-0.101127 (24.605)	0.9506	606.606	0.39477	930
Soyoil	-0.000001 (0.000)	-0.051667 (19.065)	0.9745	362.360	0.28169	924
Copper	-0.000000 (0.000)	-0.034207 (12.622)	0.9830	159.388	0.17412	756

* Quantities in parentheses are 't' statistics.

** Implicit autoregressive coefficient.

Hypothesis 2: $\frac{d}{dt} E_t(\Delta\tilde{F}(t,T)) > 0$

as t approaches T

In testing hypothesis 2, which is a test to determine whether a systematic maturity effect exists for returns, a similar standardization procedure is used to remove the noise generated by month effects and year effects. After these effects are removed by the procedure, two of the three tests performed on price volatility are also performed on returns.

Test Procedure

Let

$\Delta F(i,j)$: Futures returns[13] in month "i" for a contract "j" months from maturity.

$i = 1,12$

$j = 1,11$

$$\overline{\Delta F}(i) = (\prod_{j=1}^{11}(1+\Delta F(1,j))^{1/11} - 1$$

... Average futures returns for contracts maturing within 1 through 11 months as measured in month "i".

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[13] Returns computed are annualized.

$$\triangle f(i,j) = \frac{1 + \triangle F(i,j)}{1 + \triangle F(i)} - 1 = \triangle f(k)$$

... standardized futures returns k months from maturity.

$R(k) = 1 + f(k)$

... test variable for the standardized futures returns k months from maturity.

The test variable $R(k)$ posseses properties similar to $v_k(\triangle F)$, the standardized variance of futures returns. It represents futures returns from which all observation month and year effects have been removed.

The first test performed is to determine whether the groups that make up each $R(k)$ are homogenous. For instance, for returns one month from maturity for wheat, five groups make up each $R(1)$, The groups are composed of returns measured in February, April,June, August, and November for the March, May, July, September, and December contracts respectively. The null hypothesis here is that the groups are homogenous. It should not be rejected.

If the groups that make up each $R(k)$ are homogenous, a similar test is performed to determine whether the groups $R(k)$, $k=1,11$ are homogenous. This is simply an across group test as compared to the within group test performed earlier. If a maturity effect exists for returns, then the null hypothesis that the groups are homogenous should not be rejected.

Results

F tests were performed to determine the homogeneity of the groups that make up each R(k). At the 1% level, the hypothesis that the groups were homogenous could not be rejected. This result was obtained for every R(k) and every commodity from wheat to copper. These tests thus confirm the appropriateness of the standardization procedure.

Another series of F tests were next performed to determine the homogeneity of R(k), k=1,11. Table 7.3 reveals the results of these tests. It can be seen that unlike the results for return volatility, a maturity effect for returns cannot be discerned for every commodity except copper.

It should be noted that the standardization procedure used to normalize returns for the tests caused the returns to effectively lose their sign. Thus no statement can be made from these tests as to whether the average returns obtained are positive, negative or zero. Tests of Hypothesis 3 will permit one to discern the sign of the returns as well as provide evidence of a risk-return relationship.

TABLE 7.3

STANDARDIZED FUTURES RETURNS BY NUMBER OF MONTHS TO MATURITY (R(k))

Commodity	Number of months to maturity											F	DF*
	1	2	3	4	5	6	7	8	9	10	11		
Wheat	1.0007	1.0270	0.9926	1.0030	1.0196	0.9951	1.0133	1.0204	1.0104	1.0490	1.0350	0.488	10:652
Corn	0.9979	1.0191	0.9720	0.9667	0.9870	0.9940	1.0274	1.0003	1.0413	1.0582	1.0468	2.145	10:550
Soybeans	1.0175	1.0111	1.0081	1.0044	1.0164	1.0005	1.0375	1.0080	1.0185	1.0541	0.9887	0.797	10:883
Soymeal	1.1262	1.0665	1.0314	1.0220	1.0199	1.0270	1.1378	1.0080	1.0274	1.0142	1.0533	0.885	10:934
Soyoil	1.1255	1.0998	1.0138	1.0102	0.9972	1.0233	1.0534	1.0495	0.9895	1.0009	1.0697	1.727	10:929
Copper	1.4807	1.2703	1.1032	1.0508	1.0191	0.9918	0.9995	0.9699	1.0138	0.9883	0.9914	2.420**	10:760

Note

* Degrees of Freedom

** Significant at the 1% level

Hypothesis 3: $\frac{E_t(\Delta\tilde{F}(t,T))}{Var_t(\Delta\tilde{F}(t,T))} = -\lambda$

Hypothesis 3 specifies the existence of a constant risk-return relationship over the life of a futures contract. The single most important implication that could be drawn from the results of this test is the extent, if any, of the biasedness of futures prices,

Test Procedure

Several tests were performed on Hypothesis 3. Unlike the tests of Hypotheses 1 and 2, no standardization of variables is required for the tests performed here. The reason for this can be seen by noting equations 7.9 and 7.10. Rewriting 7.9 and 7.10

$$E_t(\Delta\tilde{F}(t,T)) = -\lambda a^{2(T-t-1)}(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots 7.9$$

$$Var_t(\Delta\tilde{F}(t,T)) = a^{2(T-t-1)}(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots 7.10$$

The underlying reason for normalizing variances and returns in tests of Hypothesis 1 and 2 was the non-staionarity of the spot variance $(\sigma_u^2+b^2\sigma_e^2)$. The standardization procedure used, eliminated the spot variance from the test variables. The test variable used for Hypothesis 3 automatically eliminates the spot variance and is thus free of the previously noted distorting influences of month effects and year effects.

The test variable M(k) is obtained as follows: Divide 7.9 by 7.10.

$$\frac{E_t(\Delta\widetilde{F}(t,T))}{Var_t(\Delta\widetilde{F}(t,T))} = \frac{-\lambda\, a^{2(T-t-1)}(\sigma_u^2 + b^2\sigma_e^2)}{a^{2(T-t-1)}(\sigma_u^2 + b^2\sigma_e^2)}$$

$$= -\lambda$$

$$= \frac{\Delta\widetilde{F}(k)}{Var(\Delta\widetilde{F}(k))} = M(k)$$

Test Variable M(k)

The test variable M(k) represents the return-risk ratio, k months from maturity.

Three tests are performed on M(k). The first test is an F test to determine the homogeneity of M(k), k=1,11. The null hypothesis is that the 11 groups are homogenous. It should not be rejected.

The second and third tests performed are to determine the distributional characteristics of M(k). The first of these two obtains the characteristics of each M(k) k=1,11. The second aggregates all the observations and obtains the distributional characteristics of the combined set of observations. The purpose of these last two tests is to determine whether M(k) which is the estimate of $-\lambda$ is significant.

Results

The test results on the homogeneity of M(k), k=1,11 is revealed in Table 7.4. At the 1% level, the hypothesis that the groups are homogenous cannot be rejected for every single commodity. This result suggests that aggregation of the observations is permissable since they all come from the same distribution, whose characteristics are revealed in the second and third tests. Of particular note here is the behavior of M(k) for copper. Every M(k) being positive for copper suggests a price biasedness on the downward side.

The results of the first test obviates the need to perform tests to determine the distributional characteristics of each M(k). The tests were performed nonetheless. The results, however, did not reveal anything of consequence that is not revealed in the third test, which aggregates all the observations of M(k), k=1,11 before obtaining its distributional characteristics.

Table 7.5 reveals the distributional characteristics of M(k) for all the commodities from wheat to copper Except for wheat which has a rather high skewness and kurtosis, every other commodity has a distribution which is approximately normal. If such is the case the "t" statistic is meaningful for the significance of the mean. The table reveals the estimate for $-\lambda$ which is simply the negative of the mean values displayed in the table.

Thus,

Wheat $\lambda = 0.002$ $t = .0.10$
Corn $\lambda = 0.057$ $t = 2.85^{*}$
Soybean $\lambda = 0.046$ $t = 1.84$
Soymeal $\lambda = -0.034$ $t = -2.615$
Soyoil $\lambda = 0.033$ $t = 2.75^{*}$
Copper $\lambda = -0.094$ $t = -5.53^{*}$

Note: * Significant at the 1% level

These results indicate that a significant bias, however small, exists for corn, soymeal, soyoil and copper. For corn and soyoil, futures prices are upward biased estimates of expected spot prices, while for soymeal and copper they are downward biased estimates of expected spot prices. For wheat and soybean futures prices are unbiased estimates of expected spot prices.

The results obviously seem to support every risk premia theorist from Keynes to Hardy. In fact, it is coincidental that the extent of the support is even. Keynes is supported with soymeal and copper, Telser with wheat and soybean and Hardy with corn and soyoil. Even though the support on the issue of risk premia is divided, one thing seems evident from the results, and that is, the markets are very mature in Gray's (1960) sense. The values of λ are so small that as a practical matter it would not be incorrect to suggest that the price biasedness costs of hedging are almost non-existent. In other words, hedgers are paying almost no premium to speculators who carry the price level risks for them. These markets are thus ideally suited for hedging.

TABLE 7.4

RETURN-RISK RATIO BY NUMBER OF MONTHS TO MATURITY (M(k))

Commodity	Number of months to maturity											F	DF*
	1	2	3	4	5	6	7	8	9	10	11		
Wheat	-0.047	-0.048	-0.042	-0.041	0.010	0.027	0.011	0.081	-0.025	-0.012	0.088	0.560	10:630
Corn	-0.023	-0.029	-0.147	0.004	-0.100	-0.109	-0.016	-0.130	-0.064	-0.056	0.063	0.920	10:585
Soybeans	-0.037	-0.157	-0.055	-0.154	0.006	-0.155	0.052	-0.095	0.077	-0.011	0.029	1.109	10:814
Soymeal	-0.034	0.020	0.021	-0.012	0.048	-0.014	0.060	0.073	0.071	0.048	0.136	1.082	10:847
Soyoil	-0.056	-0.030	-0.059	-0.026	-0.071	-0.057	-0.068	0.007	-0.027	0.015	0.053	0.847	10:874
Copper	0.037	0.137	0.073	0.104	0.107	0.090	0.077	0.095	0.063	0.139	0.114	0.300	10:711

Note

* Degrees of Freedom

TABLE 7.5

DISTRIBUTIONAL STATISTICS FOR M(k) AFTER AGGREGATION

Commodity Desc. Statistic	Wheat	Corn	Soybean	Soymeal	Soyoil	Copper
Mean	-0.002	-0.057	-0.046	0.034	-0.033	0.094
Std. Error	0.020	0.020	0.025	0.013	0.012	0.017
"t" Statistic	-0.10	-2.85	-1.84	2.61	-2.75	5.53
Skewness	1.412	0.346	-0.364	-0.013	-0.749	0.094
Kurtosis	7.968	2.660	3.841	3.144	1.520	3.833
Minimum	-1.737	-2.279	-3.455	-1.568	-1.625	-2.007
Maximum	3.633	2.382	3.722	1.858	0.951	1.889
No. of Obs.	641	596	825	858	845	722

CHAPTER 8

The Maturity Effect on the Basis

Theoretical Development

The behavior of the basis from the time the hedge is placed until the time it is lifted is of considerable interest to the hedger. The very essence of hedging involves an exchange of risk between the hedger and the speculator -- an exchange of price level risk for basis risk. In the placing of a hedge, a hedger is confronted with a choice of several contracts. The selection of the appropriate contract is no trivial matter, since it involves the incurring of a risk pattern or structure that is functionally dependent on the time left to maturity of the selected contract.

This chapter is devoted to analysing that risk structure. It will be shown that if changes in futures prices become increasingly volatile as contract maturity is approached, then the volatility of changes in the basis must necessarily become less volatile as the maturity of the same contract is approached. After theoretically deriving this proposition of declining volatility for changes in the basis, it will be empirically tested and the results will be displayed.

If the proposition is supported empirically, then the implications for hedging are very important. It would mean that hedging in a nearer contract has less risk per day or per week than hedging in a more distant contract. Furthermore, if risk reduction is the leitmotiv for hedging (Working (1953) has argued that this may not be the case) then clearly the choice of the contract in which to place the hedge in must always be the nearer one.

Let

$B(t,T)$: Basis at time "t" for a contract maturing at time "T".

$\Delta\tilde{B}(t,T)$: Change in the basis between "t" and "t+1"
= (Opening Basis) - (Closing Basis)
= $\Delta\tilde{S}(t) - \Delta\tilde{F}(t,T)$

The behavior of the volatility of changes in the basis can be shown very easily by first substituting for $\Delta\tilde{S}(t)$ and $\Delta\tilde{F}(t,T)$ in the definition of the changes in the basis $\Delta\tilde{B}(t,T)$

Using 7.2 to obtain $\Delta\tilde{S}(t)$ and 7.8 for $\Delta\tilde{F}(t,T)$

$$\begin{aligned}\Delta\tilde{B}(t,T) &= \Delta\tilde{S}(t) - \Delta\tilde{F}(t,T) \\ &= (a-1)S(t) + (1-a^{T-t-1})(\tilde{u}_{t+1} + b\tilde{e}_{t+1}) \quad \ldots 8.1\end{aligned}$$

Thus,

$$E_t(\Delta\tilde{B}(t,T)) = (a-1)S(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8.2$$

and

$$Var_t(\Delta\tilde{B}(t,T)) = (1-a^{T-t-1})^2(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots\ldots\ldots 8.3$$

Differentiating 8.3 w.r.t. t

$$\frac{d}{dt}\,(\mathrm{Var}_t(\Delta\tilde{B}(t,T))) = 2(1-a^{T-t-1})(\sigma_u^2 + b^2\sigma_e^2)\ln(a)$$

$$< 0 \quad \ldots\ldots\ldots\ldots\ldots \quad 8.4$$

since $\ln(a) > 0$

Thus, <u>the volatility of changes in the basis declines as contract maturity is approached.</u>

The reason for this result will make itself evident when one realizes that as contract maturity is approached, futures prices turn slowly into spot. The arrival of new information then is far more likely to affect futures and spot prices in the same manner closer to maturity than further away from it.

A corollary that follows from the above result is that the basis with respect to the nearer future must fluctuate less than one with respect to a more distant future.

$$\mathrm{Var}_t(\Delta\tilde{B}(t,T_1)) < \mathrm{Var}_t(\Delta\tilde{B}(t,T_2)) < \ldots\ldots$$

$$\ldots\ldots < \mathrm{Var}_t(\Delta\tilde{B}(t,T_n)) \quad \ldots\ldots\ldots \quad 8.5$$

where

$$T_1 < T_2 < T_3 < \ldots\ldots\ldots\ldots < T_n$$

Thus, if risk reduction is the sole purpose of hedging, a nearer future is the preferred choice with which to accomplish the objective than a more distant one.

Testable Hypothesis

The volatility of changes in the basis declines as contract maturity is approached.

$$\frac{d}{dt} \mathrm{Var}_t(\Delta B(t,T)) < 0$$

as t approaches T

Empirical Tests

Testing for a maturity effect on basis change volatility faces the same problems as for futures price volatility. Namely, the volatility changes every day and therefore bias is introduced through the process of aggregation. Furthermore, there also exists here the problem of non-stationarity of the spot generating process which manifests itself through month effects and year effects. Therefore, in order to account for these distorting influences, a standardization procedure similar to the one used on futures prices is necessary. After the new variables are created, they are used to test the hypothesis of declining volatility.

Test Procedure

The standardized variances of changes in the basis are created as follows. Raw variances of daily changes[1] in the basis are measured over each observation month for all contracts in the commodity that are maturing in twelve months or less. Rewriting equation 8.3 for changes in the basis for a contract maturing in T_n,

$$\mathrm{Var}_t(\Delta\tilde{B}(t,T_n)) = (1-a^{Tn-t-1})^2(\sigma_u^2 + b^2\sigma_e^2) \ldots\ldots \quad 8.6$$

Clearly, one can observe from 8.6 that the non-stationarity of the spot variance $(\sigma_u^2 + b^2\sigma_e^2)$ will affect measured basis change volatility. Therefore, in order to remove that effect, the raw variances are normalized by dividing them by the variance of spot price changes in the measured month. Thus, dividing 8.6 by $\mathrm{Var}_t(\Delta\tilde{S}(t))$.

$$\frac{\mathrm{Var}_t(\Delta\tilde{B}(t,T_n))}{\mathrm{Var}_t(\Delta\tilde{S}(t))} = (1 - a^{T-t-1})^2 \quad \ldots\ldots\ldots \quad 8.7$$

$$\text{since} \quad \mathrm{Var}_t(\Delta\tilde{S}(t)) = \sigma_u^2 + b^2\sigma_e^2$$

The left hand side of equation 8.7 is now the normalized variance of changes in the basis from which the month and

---

[1] Change in the basis is measured as follows:

$$\Delta B(t,T) = \ln\left(\frac{S(t+1)}{S(t)}\right) - \ln\left(\frac{F(t+1,T)}{F(t,T)}\right)$$

year effects have been removed. The variance is now identified solely by the number of months left to maturity. This can be seen by rewriting 8.7 to obtain the test variable $v_k(\triangle B)$.

$$v_k(\triangle B) = (1-a^k)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 8.8$$

where,

$$v_k(\triangle B) = \frac{Var_t(\triangle\tilde{B}(t,T_n))}{Var_t(\triangle\tilde{S}(t))}$$

$= \text{Normalized variance}$

$k = T-t-1$

$= \text{Number of months left to maturity.}$

Test Variable v_k

The test variable v_k measures the variance of changes in the basis k months from maturity. It is a variable that controls for the non-stationarity of the spot generating process. By doing so, the procedures used, allows the sample space to be extended as follows. In any calendar year five observations for v_1 would be obtained for wheat -- February observations for the March contract, April observations for the May contract, June observations for the July contract, August observations for the September contract and finally the November observations for the December contract. Thus, if twelve years of data are used, 60 observations for v_1 would be obtained. The observations derived from February, April, June, August and November as explained above.

Three tests are performed on v_k, since any single v_k comes from several groups. For instance, in the case of wheat, v_1 is a February observation for March contract, just as much as it is an April observation for a May contract. The first test performed is to determine whether the groups (five for wheat) that make up each v_k are homogenous. The null hypothesis stating that they are should not be rejected.

The second test performed is to determine the homogeneity of v_k, k=1,11. Since, the theory suggests that the volatility of changes in the basis is not constant over the life of the contract, the null hypothesis that they are homogenous should be rejected.

The final test is to determine whether there is a systematic decline in the variability of the basis as contract maturity is approached. If the variability of the basis does decline as predicted by the theory, then

$$v_k < v_{k+i}$$

or

$$v_k - v_{k+i} < 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.9$$

where $k = 1,10$
$i = 1,11-k$

Here, difference in mean tests are performed to determine the direction of change of variability. These tests taken together are analagous to a regression of basis volatility versus number of months to maturity. Since the theoretical result (see equation 8.8) cannot be reduced easily to a form where a regression can be run, it is necessary to perform the difference in mean tests to empirically support or reject the proposition of declining variability with distance from maturity.

Results

Data Base

The data base used was the futures price series from the Dunn and Hargitt Commodity tape.

The years selected were 1960-1971. The commodities selected were:

1) Wheat (Chicago Board of Trade)
2) Corn (Chicago Board of Trade)
3) Soybeans (Chicago Board of Trade)
4) Soymeal (chicago Board of Trade)
5) Soyoil (Chicago Board of Trade)

The effect of distance from maturity on hte variability of changes in the basis is revealed in Table 8.1. It can be seen that for every commodity other than soymeal, the hypothesis that the variance of changes in the basis is independent of distance from maturity can be rejected at the 1% level. The behavior of soymeal as well as corn seems to differ significantly from wheat, soybeans and soyoil. The distinguishing feature being the much higher variances of the former as compared to the latter. One possible explanation for such a phenomenon is that basis change variability for soymeal and corn when compared to spot price variability is much larger than for either wheat, soybeans and soyoil. In fact, for some months (9 through 11 for corn and 6 through 11 for soymeal) basis variability exceeds spot price variability. This is highly unusual because it seems to imply that hedging in distant contracts for those commodities is more risky than not hedging at all. The empirical result seems to challenge Working's (1948) contention that prices (both spot and futures) for storable commodities are inextricably tied together by time arbitrage. Working's hypothesis implies that basis variability must necessarily be less than spot variability for storable commodities. Another possibility, however, exists which could explain the anomalous behavior of both soymeal and corn. If the spot market for both commodities are thin or inactive, then the standardization procedure would overestimate basis variability. This possibility seems more likely as preliminary observation of spot variances for corn and soymeal seemed small when compared with either wheat, soybeans or soyoil.

F-tests were next performed to determine whether the groups that make up each v_k were homogenous. At the 1% level, the hypothesis that the groups were homogenous could not be rejected for every v_k and every commodity.

Tables 8.2A through 8.2E reveal the existence of a systematic risk pattern for basis variability by distance from maturity. The almost consistent negative "t" values for the differences in the mean variances is solid evidence that the variability of the changes in the basis declines as maturity is approached. Once again, corn and soymeal seem to behave erratically. The erratic behavior though is more pronounced when comparisons of two distant months are made. The explanation given earlier together with the results obtained earlier on the variability of futures prices could explain the observed behavior.

TABLE 8.1

STANDARDIZED VARIANCES OF CHANGES IN THE BASIS BY DISTANCE FROM MATURITY ($v_k(\Delta B)$)

COMMODITY	# of months to maturity (k)											F	DF*
	1	2	3	4	5	6	7	8	9	10	11		
Wheat	0.4413	0.4905	0.5731	0.5586	0.6337	0.6127	0.6721	0.6628	0.7149	0.7990	0.7826	2.858	657
Corn	0.7024	0.7617	0.8094	0.8736	0.8150	0.9320	0.9523	0.8938	1.0362	0.9351	1.1037	5.010	612
Soybeans	0.3744	0.4626	0.5047	0.5605	0.5895	0.6234	0.6445	0.7110	0.7354	0.7219	0.7575	7.574	881
Soymeal	0.8827	0.9257	0.9011	0.9752	0.9866	1.0220	1.0199	1.0148	0.9780	0.9911	1.0520	1.510	954
Soyoil	0.3119	0.3259	0.4058	0.4246	0.4769	0.4670	0.5279	0.5380	0.5418	0.5605	0.5628	7.361	964

Note: * Degrees of Freedom

TABLES 8.2A - 8.2E

"t" STATISTICS FOR THE TEST OF DIFFERENCES IN MEANS FOR VARIANCES v_k AND v_{k+i}

WHERE k=1,10 AND i=1,11-k

Table 8.2A

WHEAT

k \ k+i	2	3	4	5	6	7	8	9	10	11
1		-1.68	-1.99	-2.76	-3.19	-2.86	-3.09	-3.34	-2.56	-3.41
2			-3.06	-2.00	-5.24	-2.06	-3.53	-3.55	-1.74	-3.64
3				-2.24	-1.44*	-2.91	-1.47	-2.64	-3.07	-1.99
4					-3.27	-1.55*	-4.18	-1.91	-2.92	-3.62
5						-0.92**	-1.60*	-2.73	-0.55**	-2.30
6							-3.04	-1.74	-3.85	-1.91
7								-2.21	+1.64***	-3.47
8									-3.14	-0.01**
9										-2.95
10										

Note: * Not significant at the 5% level.
** Not significant at the 10% level.
*** "t" > 0.

Table 8.2B CORN

k+1 k	2	3	4	5	6	7	8	9	10	11
1		-2.00	-4.72	-1.55*	-3.00	-4.95	-4.18	-5.40	-5.00	-4.37
2			-2.58	-3.68	-3.10	-4.69	-5.78	-3.98	-5.88	-2.94
3				-1.94	-2.17	-2.63	-1.99	-3.21	-3.38	-2.52
4					-2.30	-3.05	-3.17	-3.39	-4.86	-1.81
5						-1.85	-1.90	-1.70*	-2.20	-0.99**
6							-3.58	-1.70*	-3.38	-2.45
7								-1.77	-0.60**	-1.16**
8									-2.33	-0.71**
9										-1.01**
10										

Note: * Not significant at the 5% level.
** Not significant at the 10% level.
*** "t" > 0.

Table 8.2C SOYBEANS

k+1 k	2	3	4	5	6	7	8	9	10	11
1	-4.35	-5.13	-4.40	-7.65	-5.64	-8.24	-6.04	-8.73	-4.94	-7.74
2		-3.36	-4.74	-2.94	-5.72	-4.73	-6.71	-5.26	-6.59	-3.07
3			-2.40	-5.26	-4.89	-5.61	-4.15	-6.16	-4.05	-5.43
4				-1.82	-3.30	-3.02	-4.33	-3.32	-4.34	-2.01
5					-0.08^{**}	-3.45	-0.74^{**}	-4.34	-0.35^{**}	-4.04
6						-2.68	-3.33	-4.23	-3.23	-3.00
7							-2.38	-2.76	-3.33	-2.56
8								-1.63^{*}	-2.05	-1.79
9									-1.28^{**}	-1.24^{*}
10										-0.62^{**}

Note: * Not significant at the 5% level.
** Not significant at the 10% level.
*** "t" > 0.

Table 8.2D SOYMEAL

k \ k+1	2	3	4	5	6	7	8	9	10	11
1	-0.32**	-2.02	-1.93	-2.30	-2.11	-3.91	-1.87	-2.35	-1.90	-3.67
2		-0.76**	-1.49*	-0.89**	-1.82	-2.95	-0.50**	-2.73	-1.58*	-2.59
3			-2.52	-1.86	-3.40	-2.39	-2.25	-3.14	-0.92**	-3.00
4				-0.89**	-2.04	-2.26	-1.95	-1.63*	-1.12**	-1.49*
5					-0.16**	-1.96	-0.54**	-1.74	-1.63*	-1.91
6						-0.91**	-1.65*	-0.38**	+0.48***	-0.78**
7							-0.96**	-1.00**	-1.56*	-1.83
8								-0.98**	-0.38**	+0.49***
9									-2.14	-1.56*
10										-1.68*

Note:
* Not significant at the 5% level.
** Not significant at the 10% level.
*** "t" > 0.

Table 8.2E SOYOIL

k+i k	2	3	4	5	6	7	8	9	10	11
1	-3.63	-2.59	-5.45	-5.82	-8.97	-5.01	-10.37	-6.67	-7.67	-6.83
2		-1.93	-4.78	-4.51	-5.51	-6.91	-7.97	-8.15	-4.67	-5.65
3			-3.11	-1.89	-1.85	-5.58	-3.81	-4.87	-3.32	-5.77
4				-1.57	-4.05	-3.95	-5.63	-5.43	-3.00	-4.47
5					-4.61	-2.67	-4.03	-4.15	-3.03	-5.14
6						-2.70	-3.72	-4.40	-3.48	-4.13
7							-4.44	-1.76	-1.72	-2.62
8								-2.22	-1.07**	-3.31
9									-0.74**	-1.53*
10										-1.62*

Note: * Not significant at the 5% level.
** Not significant at the 10% level.
*** "t" > 0.

CHAPTER 9

The Maturity Effect On The Spread

The effect of time to maturity on the volatility of changes in the spread parallels that for the basis. A spread is simply a basis where both, the long as well as the short positions taken are in futures contracts. Given the effect of increasing volatility of changes in futures prices and the declining volatility of changes in the basis, it is rather straightforward to show that the volatility of changes in the spread

1. increases as the maturity of the nearer
cont contract is approached.
2. must be positively related to the length of
th the spread. A one month spread is less volatile than a two month or a six month spread.

Consider the following two futures contracts maturing at time T_1 and time T_2 respectively.

$$F(t,T_1),\ F(t,T_2)$$

where,

$$T_2 - T_1 = x > 0$$

If an investor takes equal and opposite positions in these two contracts, then the investor has a position in a spread of length x. Of concern here, is the behavior of this spread as T_1 is approached, as well as its behavior as the length x is increased.

Let SPD(t,x) be a spread of length x,where

$$SPD(t,x) = F(t,T_1) - F(t,T_2) \quad \ldots\ldots\ldots\ldots\ldots\ 9.1$$

Therefore, the change in the spread between t and t+1, $\Delta\widetilde{SPD}(t,x)$ is given by

$$\Delta\widetilde{SPD}(t,x) = \Delta\widetilde{F}(t,T_1) - \Delta\widetilde{F}(t,T_2)$$

Substituting for $\Delta\widetilde{F}(t,T_1)$ and $\Delta F(t,T_2)$ and simplifying,

$$\Delta\widetilde{SPD}(t,x) = a^{T_1-t-1}(1-a^x)(\tilde{u}_{t+1} + b\tilde{e}_{t+1}) - \lambda a^{2(T_1-t-1)}(1-a^{2x})(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots\ldots\ldots\ 9.2$$

Thus,

$$E_t(\Delta\widetilde{SPD}(t,x)) = -\lambda\, a^{2(T_1-t-1)}(1-a^{2x})(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots\ldots\ldots\ldots\ 9.3$$

$$Var_t(\Delta\widetilde{SPD}(t,x) = a^{2(T_1-t-1)}(1-a^x)^2(\sigma_u^2 + b^2\sigma_e^2) \quad \ldots\ldots\ldots\ldots\ 9.4$$

Differentiate 9.4 partially with respect to t,

$$\frac{\partial}{\partial t} Var_t(\Delta\widetilde{SPD}(t,x)) = -2a^{2(T_1-t-1)}(1-a^x)^2(\sigma_u^2 + b^2\sigma_e^2)\ln(a) \quad \ldots\ldots\ldots\ldots\ 9.5$$

$$> 0 \quad \text{since} \quad /a/ < 1$$

Thus, <u>the volatility of changes in a spread of given length increases as the maturity of the nearer contract is approached.</u>

Next, differentiate 9.4 partially with respect to x,

$$\frac{\partial}{\partial x} \mathrm{Var}_t(\Delta \widetilde{\widehat{SPD}}(t,x)) =$$

$$-2a^{2(T_1-t-1)}(1-a^x)(\sigma_u^2 + b^2\sigma_e^2)\ln(a)$$

$$> 0 \qquad \ldots\ldots\ldots\ldots \; 9.6$$

Thus, <u>the volatility of changes in a spread increases with its length.</u>

Testable Hypotheses

Hypothesis 1

The volatility of changes in a spread increases as the maturity of the nearer contract is approached.

$$\frac{\partial}{\partial t} \operatorname{Var}_t(\Delta \widetilde{SPD}(t,x)) > 0$$

$$\text{as } t \text{ approaches } T$$

$$\text{where } x = T_2 - T_1$$

Hypothesis 2

The volatility of changes in a spread increases with the length of the spread.

$$\frac{\partial}{\partial x} \operatorname{Var}_t(\Delta \widetilde{SPD}(t,x)) > 0$$

Empirical Test

Of the two hypotheses that are testable, only Hypothesis 2 will be tested in this dissertation. Hypothesis 1 is not testable here because of insufficient data. In testing for spread behavior, every observation that exists between two contract months has to be discarded. For instance, if a July-December spread is being analysed, only observations from January to June can be used. Thus, in each year only one-half of the price data is useful for this spread. Furthermore, the need to standardize the changes in the spread to account for non-stationarity in the spot variance, further reduces the amount of useful observations to the point where tests of significance cannot be made. Thus, only Hypothesis 2 is tested, and that too, only for contracts maturing within July through December.

The behavior of the following spreads are analysed.

Wheat & Corn:	July - September	(x=2)
	- December	(x=5)
Soybean:	July - August	(x=1)
	- September	(x=2)
	- November	(x=4)
Soymeal & Soyoil:	July - August	(x=1)
	- September	(x=2)
	- October	(x=3)
	- December	(x=5)
Copper:	July - September	(x=2)
	- October	(x=3)
	- December	(x=5)

Test Procedure

From equation 9.4 it can be seen that the variance of changes in the spread of length x is dependent upon the time at which the spread is measured. Two effects operate on measured volatility. The first is clearly the effect of time left to maturity. Thus, the variance of a July - September spread can be expected to different when measured in January as compared to May. The second effect is due to the non-stationarity of the spot variance. If two spreads of different length are compared, then these two effects must be accounted for.

The following procedure is used to account for the above mentioned effects. Consider the variance of changes in a spread of length x $(x=T_2-T_1)$

$$\mathrm{Var}_t(\triangle \widetilde{\widehat{SPD}}(t,x)) = a^{2(T_1-t-1)}(1-a^x)^2(\sigma_u^2 + b^2\sigma_e^2) \qquad \ldots\ldots\ldots\ldots 9.7$$

Next, obtain the geometric average variance of changes in futures prices for the two contracts that make up the spread.

$$\overline{\mathrm{Var}}_t(\triangle\widetilde{F}) = a^{2(T_1-t-1)} \cdot a^x \cdot (\sigma_u^2 + b^2\sigma_e^2) \quad \ldots\ldots \quad 9.8$$

Divide 9.7 by 9.8 to obtain

$$\frac{\mathrm{Var}_t(\triangle\widetilde{\widehat{SPD}}(t,x)}{\mathrm{Var}_t(\triangle\widetilde{F})} = (1-a^x)^2 = v(x) \quad \ldots\ldots\ldots \quad 9.9$$

Test Variable v(x)

The test variable v(x) is the standardized variance of changes in the spread from which both, the maturity effect, as well as, the month and year effects have been removed.

In order to prove the validity of Hypothesis 2, it is necessary to show that

$$v(x_1) < v(x_2) < v(x_3) < \cdots\cdots\cdots < v(x_n)$$

where $x_1 < x_2 < x_3 < \cdots\cdots\cdots < x_n$

Thus, for wheat or corn, it is necessary to show that

$$\underset{\substack{\text{July - Sept}\\ \text{spread}}}{v(2)} < \underset{\substack{\text{July - Dec}\\ \text{spread}}}{v(5)}$$

or for Soymeal and Soyoil

$$\underset{\substack{\text{July-Aug}\\ \text{spread}}}{v(1)} < \underset{\substack{\text{July-Sept}\\ \text{spread}}}{v(2)} < \underset{\substack{\text{July-Oct}\\ \text{spread}}}{v(3)} < \underset{\substack{\text{July-Dec}\\ \text{spread}}}{v(5)}$$

The first test performed on $v(x)$ is to show that the groups $v(x_1)$, $v(x_2)\ldots\ldots v(x_n)$ are not homogenous. An F-test is used to provide evidence of non-homogeneity of the groups.

If the non-homogeneity is confirmed, then a t-test is performed to show that

$$\bar{v}(x_1) < \bar{v}(x_2) < \cdots\cdots\cdots\cdots < \bar{v}(x_n)$$

Results

Table 9.1 reveals the results of the F-tests to determine the homogeneity of the groups $v(x_i)$, $i=1,n$. At the 1% level the null hypothesis that the groups are homogenous is rejected for every commodity from wheat to copper.

The evidence is overwhelming that the volatility of changes in a spread is a function of its length.

The size of the measured variances for the different commodities is also very revealing. In a well arbitraged market, one would expect low variances of changes in a spread. This is because in such a market well informed traders would be expected to discount new information immediately into the entire spectrum of prices. Thus, if new information enters the market for a commodity that is storable, the effect of this information on the spread should be extremely small. This should show up in a low measured volatility. The evidence is thus no surprise. Wheat is probably the most mature and efficient futures market. Next would rank the corn and soybean markets among the non-processed agricultural commodities. The relative variances of the three commodities is exactly what one might have expected -- the lowest for wheat and the highest for soybeans.

The extremely low values for copper, even lower than wheat, is also no surprise. Copper is a non-seasonal commodity. The futures market for copper is seldom, if ever, subject to inversion. The kinds of information that would affect the price of any single futures contract in copper, would almost certainly affect the price of every other futures contract in a similar manner. The same cannot be said with the same level of certainty for a seasonal commodity -- storable or not. In other words, the spread for a seasonal commodity is affected by a far greater variety of information than a spread for a non-seasonal commodity. The lower variances for copper as compared to that of wheat then should come as no surprise.

The results of the t-tests to determine the effect of the length of the spread on volatility is revealed in Table 9.2. Table 9.2 contains the "t"values for the difference in means of $v(x_i)$ and $v(x_j)$, $j > i$. The "t" values computed underestimate their true values because $v(x_i)$ and $v(x_j)$ are positively correlated (the correlation coefficient is one in theory). Despite this underestimation, all but 3 of the t-values are significant at the 1% level. The evidence thus provides overwhelming support for the hypothesis that the volatility of a spread increases with the length of the spread.

TABLE 9.1

VARIANCE OF CHANGES IN THE SPREAD BY SPREAD LENGTH (v(x))

Commodity	length of spread						
	1	2	3	4	5	F	DF*
Wheat	-	0.0973	-	-	0.1386	7.448	1:174
Corn	-	0.2946	-	-	0.5725	16.327	1:141
Soybean	0.1339	0.4881	-	0.7605	-	45.517	2:204
Soymeal	0.0941	0.4635	0.8081	-	0.9616	50.639	1:231
Soyoil	0.1143	0.2201	0.3991	-	0.6807	12.587	3:222
Copper	-	0.0504	0.0814	-	0.1202	6.072	2:131

Note: * Degrees of Freedom

TABLE 9.2

"t" STATISTICS FOR THE DIFFERENCES IN MEANS FOR THE VARIANCES $v(x_i)$ AND $v(x_j)$, $j>i$

WHEAT

i \ j	5
2	-2.545

SOYMEAL

i \ j	2	3	5
1	-9.350	-11.518	-10.874
2		- 4.632	- 5.605
3			- 1.602*

COPPER

i \ j	2	3
1	-2.052***	-3.250
2		-1.686

CORN

i \ j	5
2	-4.033

SOYOIL

i \ j	2	3	5
1	-4.525	-6.521	-4.233
2		-3.829	-3.415
3			-2.014***

SOYBEAN

i \ j	2	3
1	7.443	9.470
2		7.443

Note: If no asterisk appears, t statistic is significant at 1% level.
* Significant at 10% level
** Significant at 5% level
*** Significant at 2.5% level.

PART IV: SUMMARY AND CONCLUSIONS

Chapter 10: Summary and Conclusions

CHAPTER 10

SUMMARY AND CONCLUSION

The research carried out in this dissertation was focused on the price behavior of futures contracts as maturity is approached. It was the compelling logic of the Samuelson proposition on futures price volatility, together with the inability of previous researchers to empirically confirm it, that motivated the efforts that were undertaken here. This chapter is devoted to summarizing the content of the research accomplished and the results achieved. It will conclude with suggestions to further research that needs to be done, both in futures markets, as well as forward contract markets. It is hoped that the work accomplished here, as well as the future efforts it will motivate will contribute to the ever increasing efficiency with which these markets operate.

Summary

Futures prices become increasingly volatile as contract maturity is approached. This is the Samuelson proposition. In order to prove it, Samuelson used the Martingale theorem together with the assumption of uniformity through time of the distribution of changes in spot prices. Several possible stochastic generating processes for spot prices were then applied to the model to generate the result. The process selected in this dissertation is the one referred to by Samuelson as obeying economic law -

the first order damped autoregressive process.

In order to permit general market forces to affect commodity prices, the first order damped autoregressive process was extended to include a market factor. The resultant process was thus made up of two components. The first componenent included influences that were commodity specific. The second included those influences that would affect all commodities in general. This process thus allowed for an "own effect" and a "market effect". Given this process generating spot prices, the price of a futures contract was developed through the axiom of mathematically expected price formation.

The issue as to whether futures prices are biased or unbiased estimates of expected spot prices was left open. The theoretical model developed, however, was consistent with both sides of the issue. The resolution of it was left to empiricism.

The crux of this dissertation was to discern the existence of a maturity effect on the behavior of futures prices, the basis and the spread. The several propositions developed are listed below.

Proposition 1

The volatility of changes in futures prices increase over the life of a futures contract.

Proposition 2

If the volatility of changes in futures prices do increase over the life of a futures contract, should futures returns follow a similar pattern?

Proposition 3

If a systematic maturity effect exists both for risk (Proposition 1), as well as for returns (Proposition 2), should there exist a Risk-Return pattern that is independent of time left to maturity?

Proposition 4

The volatility of changes in the basis declines over the life of a selected futures contract. The basis with respect to a nearer future must then necessarily be less risky than one with respect to a more distant future.

Proposition 5

The volatility of changes in a spread increases as the maturity of the nearer contract is approached.

Proposition 6

The volatility of changes in a spread is an increasing function of the length of a spread.

Proposition 2 and 3 were posed as questions because they depended critically upon the issue of biasedness of futures prices. For instance, if futures prices show no bias, then clearly no maturity effect can exist for return. If they do, then a maturity effect should exist. This is thus an empirical question.

Every proposition stated above, except for Proposition 5 was empirically tested for a wide cross-section of storable commodities. The commodities and markets selected were those that were in existence for a long period of time and generally considered to be mature and efficient markets. The following are the commodities and the markets selected.

1. Wheat (Chicago Board of Trade)
2. Corn (Chicago Board of Trade)
3. Soybeans (Chicago Board of Trade)
4. Soymeal (Chicago Board of Trade)
5. Soyoil (Chicago Board of Trade)
6. Copper (Commodity Exchange of New York)

The years selected were 1959-1971. The choice of years was selected purely on the basis of the existence of complete data.

The results of the empirical testing were extremely revealing for every proposition tested. A summary of the major findings is presented below.

1. Proposition 1 was very strongly supported. For every single commodity from wheat to copper, a systematic maturity effect was found to exist for the volatility of changes in futures prices. The existence of a significant increase in price change volatility as contract maturity is approached was confirmed. It was further discerned that even though a strong maturity effect existed,

it was able to explain only a small percentage of the total volatility.

2. The existence of a maturity effect for returns could only barely be discerned for copper. No effect existed for every other commodity. The effect on returns for copper was found to be similar to the effect on volatility. Namely, increasing returns as contract maturity is approached.

3. Evidence of price biasedness was divided evenly. No bias was found for wheat and soybean, upward bias was found for corn and soyoil, and downward bias for copper and soymeal. However, the extent of the results was indicative that the markets were very mature and efficient -- a near perfect market for hedging.

4. The results of the maturity effect on the basis were as strong as those obtained for futures prices. Volatility of changes in the basis was seen to decline for every commodity tested. Unusually high variances for changes in the basis were obtained for corn and soymeal. This could be traced to standardization procedure which would overestimate basis change variances, if the measured variances of changes in the spot price were low.

5. The evidence uncovered for spread behavior, once again provided strong support for the proposition that the volatility of changes in a spread is positively related to the length of the spread. For every commodity from wheat to copper, the evidence indicated very clearly that a longer spread was more volatile than a shorter spread. More interestingly, spread change volatility seemed to be the lower for the older markets (wheat and soybean), and higher for the newer markets (soymeal and soyoil). Copper displayed the lowest volatility as one might expect because of its non-seasonal nature.

Conclusion

Time to maturity does influence the behavior of futures prices, the basis and spreads. There exists a maturity structure for risk wherein the risk of holding futures contracts is more on a per day or per week basis for a contract nearer to maturity than one further away from it. No such structure seems to exist on average for returns. The biasedness of futures prices, if any does exist is very small for mature markets. This makes such markets excellent hedgers' markets because hedging costs are extremely low.

The existence of a maturity effect, though highly significant, could only explain a small proportion of the variation in futures prices. Much of the total variation thus still remains to be explained. Further research in this area should focus on the unexplained variation.

The inability of the study to discern the existence of a maturity effect for returns could be the result of the assumption that hedging is always net short, and therefore, speculating is net long. If such is not the case, then it is possible that a maturity effect does exist without the tests being able to detect it. If data could be obtained on the breakdown of the open interest between hedgers and speculators, the existence of the maturity effect for returns could possibly be discerned. This could be another area for future researchers to probe.

Finally, the difference between futures market and forward markets is minor, if the forward markets are mature and active. The existence of the maturity effect on prices in active forward markets is almost certain to exist. Almost all work done in forward contract markets presumes stationarity of forward price changes. This assumption needs to be challenged in the light of the research accomplished here on futures prices.

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