Concepts of Infinity in Chinese Mathematics

by

Yibao Xu

A dissertation submitted to the Faculty in the Ph.D. Program in History
in partial fulfillment of the requirements for the degree of Doctor of Philosophy,
The City University of New York.

2005

UMI Number: 3187462

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This manuscript has been read and accepted for the Graduate Faculty in History in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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ABSTRACT

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by

Yibao Xu

Adviser: Distinguished Professor Joseph W. Dauben

This dissertation investigates the concept of infinity in Chinese mathematics by focusing on a series of case studies devoted to specific works or ways in which the concept has appeared and been used throughout the history of mathematics in China. The conventional view holds that ancient Chinese classic texts before 221 BCE introduced mathematical concepts of the infinite. By questioning the validity of employing the Greek concept of pure geometric magnitudes, which has no counterpart in ancient Chinese mathematics, and by analyzing the possible contexts in which these Chinese texts allegedly used concepts of infinity, I conclude that the conventional interpretations are untenable. The second major aspect regarding the appearance of the infinite in ancient Chinese mathematics concerns the phrase "*yi mian ming zhi*" in the *Jiuzhang Suanshu*, which has been interpreted as evidence for a Chinese concept of irrational number. My conclusion is that this interpretation of "*mian*" is also invalid.

The remainder of the thesis is devoted to introductions of Western concepts of infinity to China. I identify the original source for the first Chinese translation (1859) of Books VII through XV of Euclid's *Elements*, and also discuss how the concept of incommensurability was introduced through an analysis of the translation of certain definitions and propositions from Book X. The introduction of infinitary concepts at the

core of the calculus is dealt with primarily through a discussion of the translations of the calculus textbook by Elias Loomis in 1851, and by analyzing the article on "Fluxions" by William Wallace in the *Encyclopedia Britannica* (1855). The story of the introduction of transfinite set theory and mathematical logic is told mainly through a discussion of Bertrand Russell's lectures on mathematical logic in China and Chinese translations of his two books.

The final analysis of concepts of infinity in modern Chinese mathematics treated in this dissertation focuses on translations of Karl Marx's *Mathematical Manuscripts.* These were especially significant for debates in China over foundations of the calculus during the Cultural Revolution (1966–1976), and played a central role in influencing the introduction of Abraham Robinson's controversial nonstandard analysis in the 1970s.

To My Beloved

Delia and Jonathan

For the Bad and Good Times We Have Shared

ACKNOWLEDGMENTS

This dissertation was successfully defended on March 30, 2005. Now it is time to record my indebtedness accumulated over the past ten years.

First of all, a special word of appreciation to Professor Joseph W. Dauben. It is he who accompanied me to the Embassy of the United States in Beijing on June 26, 1995, and helped me to get my visa. It is also he who has mentored me ever since. Without his very kind help and valuable advice, my life would be totally different than what it is, and of course, this dissertation would not exist.

I want to thank Professors Philip Dawson, Dolores Greenberg, James R. Jacob, Kathleen McCarthy, David Nasaw, Richard G. Powers, Stuart E. Prall and Nancy Siraisi for teaching me the history of science, U.S. history, and European history, respectively.

I also want to thanks my fellow students Sarah Covington, LIN Yongjun 林勇軍, Christopher Nelson, Laura O'Keefe, George Ouwendijk, Carole L. Sussman, TU Hong 涂洪, and Benjamin Zibit for their friendship and help in various ways.

I am grateful to Professor LI Di 李迪, my former mentor at the Institute for the History of Science, Inner Mongolia Normal University in Huhehot, China, for his moral support and for teaching me continuously through his latest publications on history of Chinese mathematics. I am equally grateful to Professor LIU Dun 劉鈍, Director of the Institute for the History of Natural Science of the Chinese Academy of Sciences; Professor LI Banghe 李邦河 of the Chinese Academy of Mathematics and Systems Sciences; Professor LIN Xiashui 林夏水 of the Chinese Academy of Social Sciences; Professor GUO Shirong 郭世榮 of the Department of History of Science, Inner Mongolia Normal University; Professor FENG Lisheng 馮立升 of Qinghua University;

Mr. LI Chuanji 黎傳紀, senior editor at the Editorial Office of Jiangxi Local History in Nanchang, Ms. LI Zhijin 黎知謹 of the National Library of China in Beijing, and Miss WANG Jue 王玨, a graduate of Beijing University and now a graduate student at the Graduate School of the University of Wisconsin at Madison, for providing useful information and needed materials.

I am most pleased to acknowledge my deep gratitude to Professor Karen H. Parshall of the University of Virginia for inviting me to participate several national and international conferences, and for her help in shaping my academic career; to Professor HORNG Wannsheng 洪萬生 of the National Normal University in Taipei for providing me an opportunity to visit Taiwan for the first time, to Professor Thomas H. C. Lee 李弘祺 of the City College of New York for giving me the second opportunity to take part in conferences held in Taiwan, and for his support of my career; to the late Professor Mariano Hormigón and Dr. Elena Ausejo of the University of Zaragoza for inviting me to a conference in Spain in July of 1999, and to Dr. José-Antonio Cervera for showing me the beautiful scenery of Spain during my first trip there.

Looking back on the very difficult first years in New York, I want to thank Mr. YU Cujin 余促進 and Dr. HU Mingjie 胡明傑 for their crucial financial help. I am also grateful to Ms. Esther M. Owens, of the SEEK Department of John Jay College, for her kindness and for offering me a tutoring position in her Department on which two-and-a-half-year of my life depended.

Above all, I am very grateful to my wife, Delia. In order to let me concentrate on my studying and finish this dissertation sooner, she has worked hard both at home and at various workplaces, and has also shouldered the financial burden of the family to a very

large extend. Her sacrifice is one of the major factors that have contributed to the completion of my studies at the Graduate Center. I am equally grateful to my son, Jonathan, for the happiness he has brought into my life and for giving me yet another reason to appreciate the meaningfulness of this dissertation.

Last but not the least, I render thanks to the China Times Cultural Foundation for granting me a "Young Scholars Award for the year 2000" and to the Bentley Historical Library of The University of Michigan at Ann Arbor for awarding me a "Mark C. Stevens Researcher Travel Fellowship for the year 2000." Needless to say, neither the Foundation nor the Library should be responsible for any views presented in this dissertation.

Yibao Xu 徐義保
Flushing, New York, May 2, 2005
September 10, 2005

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NOTES ON STYLE

Throughout this dissertation BCE (Before Common Era) and CE (Common Era) are used instead the conventional BC and AD. Moreover, translations are done by the author of this dissertation himself unless otherwise acknowledged or noted.

With the exception of "Confucius," "Mencius," and a few others, all Chinese names and words are transliterated according to the pinyin romanization system for mandarin Chinese. Moreover, Chinese names are given according to their usual Chinese order, namely family names (in capitals) appear before the given names. All Chinese characters to be found in this dissertation are in traditional rather than simplified form. For instance, LIU is the surname of LIU Hui 劉徽, and Hui his given name.

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Introduction

Infinity is one of the most pivotal concepts in the history of Western ideas. It has different, sometimes related, meanings in cosmology, mathematics, ontology, philosophy, and theology. In mathematics infinity includes three aspects: namely, the infinitely large, the infinitely small infinitesimals, and various infinite processes.

As extant historical documents testify, ancient Greek mathematicians and philosophers alike confronted the problems of the infinite mathematically, largely in response to the paradoxes of motion devised by Zeno of Elea and by the discovery of incommensurable magnitudes made by the Pythagoreans as early as the fifth century BCE.[1] Due to concerns over rigor in mathematics, the Greeks avoided infinity as much as they could, regarding it as a logically paradoxical concept. One way to avoid the dilemma posed by the problems of infinity was proposed by Aristotle, who differentiated infinity as potential and actual. Aristotle admits that only the potential infinite exists.[2] Aristotle's contemporary, about twenty years his senior, Eudoxus of Cnidos, extended the Pythagorean theory of proportions to accommodate incommensurable magnitudes and invented an indirect method of limits that Gregory of Saint Vincent later in the seventeenth century coined as "the method of exhaustion."[3]

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[1] Zeno of Elea flourished in the mid-fifth century BCE. Only four of his four hundred paradoxes, namely the Archilles, the Arrow, the Dichotomy, and the Stadium, survived. The essence of these paradoxes from a mathematical point of view is infinite bisection. See Lee 1936; Owen 1957; and Salmon 1970. As for the time of the discovery of incommensurable magnitudes, according to Wilbur R. Knorr, it is sometime between 430 and 410 BCE; see Knorr 1975, 36-40.

[2] Aristotle *Physics* 207a33-b34.

[3] Ernest J. Dijksterhuis strongly disliked the term "method of exhaustion." He said: "We will avoid the widely used term 'exhaustion method'; for a mode of reasoning which has arisen from the conception of the inexhaustibility of the infinite, this is about the worst name that could have been devised" Dijksterhuis 1957, 130. Wilbur R. Knorr and Geoffrey E. R. Lloyd also point out that this term is misleading. See Knorr 1982, 123; Lloyd 1996a, 20; 1996b, 149.

Greek mathematicians since Eudoxus, notably Archimedes, exploited the method of exhaustion to calculate areas and volumes. Archimedes, however, also treated a line as composed of points, a plane as composed of lines, and a solid as composed of planes, and used the very notion of indivisible as a means to aid him in finding find new results as recorded in *The Method*.[4]

Archimedes' use of indivisible geometric magnitudes later was adopted by Johannes Kepler, who used the notion of infinitesimally small geometric quantities, or infinitesimals, to investigate the volumes of wine casks and other geometrical shapes. Kepler's infinitesimal approach in turn inspired Bonaventura Cavalieri, who took the method of indivisibles and developed a principle which was later named after him. By using infinitesimals and his principle, Cavalieri was able to find the areas and volumes of various geometric figures effectively and rapidly.[5]

During the so-called "century of anticipation" as Carl Boyer has called it, mathematicians like Evangelista Torricelli, Gilles Roberval, Pierre Fermat, Blaise Pascal, John Wallis, James Gregory, and Isaac Barrow, among others, made great contributions to the final formulation of the differential and integral calculus.[6] But its was Isaac Newton and Gottfried Wilhelm Leibniz who were credited for the creation of the subject because they independently generalized earlier infinitesimal techniques and developed a powerful and systematic theory for dealing with the tangent of a curve and summation of the area under the curve.

---

[4] See Heath 1921, 19-20; 27-34; 34-56; and 85-91.

[5] Boyer 1959, 108-124; Baron 1969, 108-135.

[6] See Boyer 1959, 96-186; Baron 1969, 149-252; Edwards 1979, 98-188.

Neither Newton's fluxions nor Leibniz's differentials, however, were given rigid and solid foundations. Newton's treatment of infinitesimals both as zero and non-zero was criticized by Bishop Berkeley, who denounced fluxions as "the ghost of departed quantities." Likewise, Leibniz's differentials of higher order were criticized by Bernard Nieuwentijdt as fictions.[7]

In order to put the calculus on a rigorous base, Bernhard Bolzano used the limit concept to define the derivative of a continuous function, and treated the infinitesimal as a variable rather than a fixed number. A similar approach was also taken at the same time by the French mathematician Augustin-Louis Cauchy. However, it was through the lectures delivered and textbooks written by Cauchy that limits became basic in the calculus. An even more secure foundation for the calculus, however, was provided by Karl Weierstraß, who constructed a purely formal arithmetic basis for analysis by defining irrational numbers independently of the concept of limit.[8]

Though Weierstraß used infinity in defining irrational numbers, it was Georg Cantor who turned it into a crucial concept in mathematics. Cantor proved that infinity not only was actual, but also had different degrees of magnitude, i.e. all infinites are not equal, and there is fact an endless hierarchy of infinite numbers. Cantor's theory of transfinite sets has had a profound impact upon the subsequent development of mathematics, and upon the philosophy of mathematics as well. Cantor, however, rejected the existence of infinitesimals in part because of "the character of his transfinite

[7] Boyer 1959, 224-229 and 248.

[8] Boyer 1959, 267-298.

numbers." [9] But the fact that infinitesimals do actually exist and can be consistently defined rigorously as mathematical entities was later proved by Abraham Robinson using model theory and the tools of mathematical logic. Thus was become what Robinson called nonstandard analysis.[10]

The development of Western mathematics from ancient times down to the present can be said to have been largely due to the evolution of the concept of infinity. Accordingly, in this thesis I have several questions about this very concept in *Chinese* mathematics: is there any such concept of infinity in ancient Chinese mathematical texts? If so, in what contexts do such concepts of infinity appear? In what ways may a comparison of the concepts in Chinese mathematics and those in Greek mathematics be drawn? What roles did concepts of infinity play in the development of Chinese mathematics? How were Western concepts of infinity introduced into China? What sorts of impact did the concepts introduced have upon the later development of Chinese mathematics?

Seven years ago, in raising such questions in my thesis proposal, I presented my thesis committee at the Graduate Center with the following tentative title: "*Concepts of Infinity in Chinese Mathematics*: *A History from Ancient Times to the Present*." In my proposal, I wrote as follows:

> This dissertation has three principal objectives: firstly, to investigate whether the Chinese had the same or similar concepts of the infinite as those in the West by examining Chinese indigenous ways of approaching and resolving questions which involved similar concepts; secondly, to trace the transmission of the theories of incommensurability in Euclid's *Elements* and modern European mathematical theories involving infinity such as the differential and integral calculus and mathematical logic into China, examining the ways in which the

[9] Dauben [1979]1990, 233.

[10] Dauben 1995, 346.

> Chinese adopted Western concepts, methods, and theories; and finally, to discuss how both politics and ideology deeply influenced debates over foundations of the calculus during the Great Proletarian Cultural Revolution (1966–1976), involving discussions of Karl Marx's *Mathematical Manuscripts* and the introduction of nonstandard analysis. Through the examination of these problems, this dissertation intends to offer a systematic study of concepts of infinity in China.

When I began my research and actually started to draft the thesis, I soon realized that the proposed topic was far too ambitious for a dissertation. I faced the same dilemma as Tristram Shandy experienced when he had covered only two days of his autobiography after two years of writing. Like Tristram, I wondered whether I could ever complete this dissertation about the infinite in Chinese mathematics in a finite amount of time. And I worried whether I could finish this thesis within the time frame allowed by the Graduate Center. To do so, I have adopted the case-study approach, and in the dissertation which follows, I have chosen to focus on a number of major aspects of the mathematical infinite in Chinese mathematics.

In Part I: Chinese Indigenous Concepts of Infinity, I discuss whether some fragments in Chinese Pre-Qin classics, especially the *Mo Jing* and *Zhuang Zi*, widely held to include mathematical concepts of infinity, actually do entail such concepts or not, and if so, what sorts of infinitary concepts these might be (Chapter one). I consider in particular possible meanings of the term: "*yi mian ming zhi* 以面命之" in the *Jiuzhang Suanshu* 九章算術 (*Nine Chapters of Mathematical Procedures*), and analyze whether the term can be interpreted as an evidence for Chinese concept of irrational number in particular (Chapter two).

Part II: Western Concepts of Infinity in Transition, is comprised of three chapters, four through six. After providing an outline of the introduction of Euclid's *Elements* to China, Chapter four devotes three sections to identifying the original source of the last

nine books of the *Elements* which the translators used for preparing the Chinese version. Based on the newly-identified original source, this chapter goes on to discuss how the concept of incommensurability was introduced into China through an analysis of the translation of certain definitions and propositions from Book X of Euclid's *Elements*.

Chapter five concerns the introduction to China of the differential and integral calculus. The first section describes the background and presents evidences about what books on the calculus were actually available in China in the mid eighteenth century. Section two goes on to discuss two Chinese translations of the calculus by focusing the original sources, authors, and translators. The last section discusses the translations of some definitions and theorems related to concepts of infinity, especially infinitesimals. Different treatments of these concepts in the two Chinese translations, one done in 1859 the other in 1874, are also compared.

The last chapter of Part II is devoted to the introduction of mathematical logic in China. Primarily through a discussion of Bertrand Russell's lectures on mathematical logic and Chinese translations of his two books: *Introduction to Mathematical Philosophy* and *Mathematica Principia* (only partially translated), this chapter tells how Russell's mathematical logic was brought into China. It is true that Russell's logic did not include the whole of mathematical logic in the first half of the twentieth century, and by the 1940s it no longer represented the main trends in logic. But from the 1910s on, an increasing number of Chinese students went to Europe or came to America to study mathematics. Some of them studied mathematical logic directly from their foreign professors or learned it through books in English, French or German. When they returned to China, they used the original books for their own reference and even for teaching their

students. It is indeed a challenge to present a full picture of the introduction of a mathematical subject in modern times when the existent historical record is fragmentary. Nevertheless, despite the difficulties, the last section of this chapter is devoted to a discussion of Chinese understanding of concepts of infinity in mathematical logic and the original contributions made by Chinese mathematicians and philosophers to the enrichment of this very concept.

Part III, the Politics of Infinity, discusses how both politics and ideology deeply influenced the Chinese translations of Karl Marx's *Mathematical Manuscripts* and nation-wide discussions on Marx's views on the foundations of the calculus, as well as the introduction of nonstandard analysis during the Cultural Revolution. Chapter seven provides the needed background with an outline of the essence of Marx's philosophy of mathematics, including MAO Zedong's writings on this subject. It also describes the politics of mathematics research and education in the Communist China.

After describing the contents and history of Marx's *Mathematical Manuscripts*, as well as their earlier introduction to China, Chapter eight gives a detailed account of the contexts and motivations for two separate translations of the *Manuscripts* by groups of scholars primarily at Fudan University in Shanghai, and Beijing University in Beijing, in the 1970s. The nation-wide discussion of Marx's *Manuscripts* and its value for the foundation of the calculus, and the ways in which nonstandard analysis was introduced into China, are briefly discussed in the concluding chapter.

This dissertation is about concepts of infinity in Chinese mathematics. But to cover the entire subject would mean having to remain a student at the Graduate Center forever, indefinitely, perhaps. Although a number of aspects of the concept of the infinite

in Chinese mathematics have not yet been dealt with here, I nevertheless believe that this dissertation has achieved its goal as a “partial fulfillment of the requirements for the degree of Doctor of Philosophy.”

Part I: Chinese Indigenous Concepts of Infinity

Chapter 2 Concepts of Infinity in the Pre-Qin Dynasty

> If we think of it [infinity], belief in the existence of something infinite or unlimited would seem to arise out of five considerations: (1) that of time: for it is infinite; (2) that of the division of magnitudes: for mathematicians also use the notion of the infinite; (3) if coming to be and passing away do not give out, this can only be true on the assumption that the source from which things come to be is infinite; (4) the limited must always come up to some limit, whence it follows that there can be *no* limit if one thing must always come up to some other thing as its limit; (5) above all and most decisive—there is what constitutes the difficulty that all alike feel, namely that number and mathematical magnitudes and what is outside the universe appear to be infinite because they never give out in *thought*.
>
> -Aristotle[1]

> Current studies of the *Mo Jing* 墨經 (*The Mohist Canon*) cannot find support [for their arguments or explanations] from any documents or texts surviving from ancient times. Adding or removing words [in the original texts] by one's sparkling wit is very much like an able magician who turns a thing into a donkey or a chicken with a magic spell.
>
> -CHEN Yinke 陳寅恪[2]

The human mind has long pondered a number of fundamental questions regarding humanity in general and the universe in which we live: What is the universe? How did it come into being? Where and how did life begin? Where did human beings come from? What are the things in the world made of? Some of these questions inevitably lead, eventually, to such questions as whether the universe is finite or infinite, whether time had a beginning or will ever come to an end, and whether or not matter is infinitely

---

[1] Aristotle *Physics* III, 4: 203^{b}15- 30, quoted from Heath 1949, 102-3.

[2] Translated from CHEN Yinke's evaluation, dated June 11, 1930, of the first volume of *Zhongguo zhexue shi* 中國哲學史 (*History of Chinese Philosophy*) by FENG Youlan; see FENG Youlan 1990, 1194. CHEN's given name here is pronounced "Yinke" rather than the conventional "Yinque." According to ZHAO Yuanren, CHEN himself pronounced his own given name "Yinke"; see ZHAO and YANG 1980, 26. See also Zhongguo Dabaike Quanshu Zhongguo Lishi, vol. I, 1992, 96a-97a.

divisible. To these questions, different ancient civilizations provided their own religious, philosophical, or scientific speculations or answers. Chinese philosophers in the pre-Qin dynasty (before 221 BCE), like their Greek counterparts, were also conscious, although to a far less degree, of the problem of infinity, particularly as it related to space and time. Before discussing the role of the infinite in Chinese mathematics, it will be useful and in fact necessary to discuss Chinese philosophers' speculations about infinity in order to have a better understanding of the intellectual environment in which Chinese *literati*, including mathematicians, worked.

This chapter first discusses Chinese, in particular Daoist thinking about space and time, and analyzes to what extent Daoist writers can be said to have thought, if at all, of infinite space and time. The second part of this chapter is devoted to a discussion of various fragments in the *Mo Jing* 墨經 and *Zhuang Zi* 莊子, which have often been interpreted historically by modern scholars as treating or considering concepts of mathematical infinity. After having examined these fragments in their relevant historical contexts, this chapter then goes on to provide some new interpretations of their likely meaning.

2.1 Space and Time: Finite or Infinite?

As early as the fifth century BCE, in particular during the Warring-states period (480-221 BCE), different Chinese philosophical schools were formed one after another. Among these schools, Confucianism, Daoism, Moism, Legalism, and the School of Names were the most prominent. The dominant schools of the Warring-states period, however, were Confucianism and Moism. Confucianism focused on human morality and social affairs,

advocating benevolence (*ren* 仁), and promoted a social hierarchy through moral codes. Moism also focused on human society, but unlike Confucianism, it took a different approach, promoting righteousness (*yi* 義), emphasizing utilitarian objectives, and taking universal love as its primary doctrine. Moism, moreover, opposed Confucianism by condemning ritual ceremonies, music, extravagant funerals, and fatalism—all important aspects of Confucian doctrine. The minor school of the Legalists, however, rejected both Confucianism and Moism. To the Legalists, Confucian moral standards and Mohist religious sanctions were unimportant, if not useless, to rulers. The Legalists sought to ensure absolute power for rulers by advocating the promulgation of universally valid and brutal penal laws. The interests of the Legalists sided with the ruling class, which though a small fraction, was nevertheless an important part of human society. The main concern of the School of Names was the art of debate, which placed particular stress upon the relation between artificial names and the actuality of things in the physical world. Not only were their interests less directed towards the immediate needs of human society than were the aims of the Confucianists, Mohists, and Legalists, but members of the school paid little heed to nature, either.

Comparing the schools mentioned briefly above, Daoism stands out for its close relation to nature. Its doctrine of *wu wei* 無爲 demanded that no action be taken against the laws of nature, in other words, nature should be left to take its own due course, and thus Daoists were encouraged to observe things in nature and to ponder deep questions about them. One of the fundamental concepts of Daoism is *dao* 道 (the way). What was *dao*? According to *Lao Zi* 老子 or the *Daode Jing* 道德經, a book attributed to LI Er 李耳, or Lao Dan 老聃, the father of the Daoist school, *dao* was indescribable, formless,

invisible, and yet was taken to be *the* origin of the universe and of all things, including all life therein.[3] The *Dao* was said to be vigorous yet unborn. It existed before, is working now, and will last forever. The *Dao* is eternal. The beginning sentence of the transmitted *Lao Zi* says that: "The Dao that can be told of is not the eternal Dao."[4] In chapter twenty-five, *dao* is explained as follows:

> There was something undifferentiated and yet complete,
> Which existed before heaven and earth.
> Soundless and formless, it depends on nothing and does not change.
> It operates everywhere and is free from danger.
> It may be considered the mother of the universe.
> I do not know its name; I call it *Dao*.
> If forced to give it a name, I shall call it Great.
>
> 有物混成，先天地生。寂兮寥兮，獨立不改，周行而不殆，可以為天下母。
> 吾不知其名，字之曰"道"，強為之名曰"大"。[5]

The *Dao* not only existed before the universe, but also was responsible for its creation. "Dao generated the One." Chapter forty-two of the book then continues: "The One produced the Two. The Two made the Three, and from the Three derived all things in the universe."[6] What are the One, the Two, and the Three? *Lao Zi* provides no

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[3] Little is known of LI Er's life. The scattered historical records about him are often contradictory. It is generally accepted among scholars that he was a contemporary of Confucius, who is said to have paid him a visit to ask about the rites, and that he was the court archivist of the kings of Zhou and authored the book that later bore his name. For discussions of LI Er's life and a possible dating of *Lao Zi*, see FENG Youlan 1952, 170-170; LAU 1963, 8-12, and CHEN Rongjie [Wing-tsit Chan] 1963, 136-38.

[4] Quoted from CHEN Rongjie 1963, 139. According to CHEN Guying, the word "*dao* 道" appears seventy-three times in *Lao Zi*. It has different meanings within different contexts. Its basic meanings are among the following: metaphysical being, natural law, and criteria or guiding principles for human society. See CHEN Guying [1970] 2000, 1-45, esp. 2, 12. Here we focus on the first meaning given above, as metaphysical being.

[5] Quoted from Chen Rongjie 1963, 152. The Chinese text is quoted from *SBBY* vol. 130, 1: 13b-14a. The text in the *SBBY* has no punctuation, which is provided by the author for quotations cited throughout this dissertation.

[6] Quoted with minor change from Chen Rongjie 1963, 160. The Chinese texts are: 道生一，一生二，二生三，三生萬物。See *SBBY* vol. 130, 2: 5a.

explanation. *Huainan Zi* 淮南子, a book compiled by scholars under the patronage of the Prince of Huainan, LIU An 劉安, who died in 122 BCE, offers the following suggestions:

> *Dao* begins as the One, which is unborn. The One divides into *yin* and *yang*. The mixtures of *yin* and *yang* generate all kinds of things. Therefore, there are these saying: One produces Two, and Two generates Three, and from Three evolves ten-thousand things.
>
> 道 (日規) 始於一，一而不生，故分為陰陽，陰陽合和而萬物生，故曰一生二，二生三，三生萬物。[7]

Here the One is interpreted as the original primordial force, *dao* itself; the Two as *yin* and *yang*; and the Three as the blending of *yin* and *yang*. This interpretation by the former Han *literati* may capture the essence of the true meanings of *Lao Zi*. Nevertheless, this does not help to explain how the formless *dao*, or *yin* and *yang*, could have produced solid material things.

Presumably aware of the metaphysical difficulty of taking either something formless or something tangible as the one origin of all things, the author of *Lao Zi* theorized that the *dao* was both "*wu* 無" and "*you* 有." "*Wu*" literally means "nothing," "does not have," or "non existence," and "*you*" connotes "something," "does have," or "existence." The philosophical meaning of this pair of opposites in Daoism was close, but not identical, to that of being and nonbeing in Greek philosophy.[8] For example, Parmenides maintains that being is what "is," and nonbeing is what "is not;" and what "is not" cannot become what "is," namely, being cannot be generated from non-being. The

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[7] Translated form *Huinan Zi*. The Chinese text is quoted from *HNZ* 1897, vol. 1, 216. About LIU An and the *Huinan Zi*, see Major 1993, 1-21.

[8] For discussion of the translation of "*you*" and "*wu*," see A. C. Graham 1959; [1986] 1990, 322-359; and Hansen 1992, 219-222. Because this pair is not equivalent to "being and non-being," we do not translate them as such, but instead leave them untranslated throughout this thesis.

world, Parmenides believes, is static, bounded, self-contained and self-sufficient. Based on such reasoning, he concludes that things cannot change. Similar arguments supposedly motivated him to argue that there can be no motion.[9]

But the Daoist's world is dynamic, and *dao* is *the* ultimate force behind change. It is because of this that *dao* could not be a concrete thing among those things that it generated. Accordingly, the *dao* should be formless and invisible, which might best be called "*wu*." On the other hand, the *dao* was the source from which everything in the material world was believed to come into being. Therefore, the *dao* could not be nothing, it had to be something, which might thus be called "*you*." The *wu* in *Lao Zi* is clearly not meant to be understood as nothingness or nullity. Although it is shapeless, colorless, and soundless, *wu* contains all the subtle essence that *you* has.[10] In this sense it is perhaps more helpful to say that *wu* represents the essence of *dao*, and *you* reveals its function; thus *wu* and *you* may be taken as two complementary aspects of *dao*. For the relation between them, consider the following from *Lao Zi*:

> *Wu* is the name of the beginning state of Heaven and Earth;
> *You* is the name of the mother of all things.
> Therefore, the subtleties of *dao* may be understood from *wu*,
> And the tangible results of *dao* may be seen from *you*.
> The two names are different, but derived from the same origin.
> They both are deep and profound,
> Which are the gateways of all subtleties!
>
> 無，名天地之始；有，名萬物之母。故常無，欲以觀其妙；常有，欲以觀其徼。此兩者，同出而異名，同謂之玄。玄之又玄，眾妙之門。[11]

[9] Guthrie [1965] 1980, 26-49; 57-66.

[10] Chapter fourteen of *Lao Zi* offers further explanations of *wu*; see CHEN Rongjie 1963, 146.

[11] Translated from Chapter one of *Lao Zi*. There are many possible interpretations of the first two sentences due to different grouping of Chinese words. For other alternatives, see FENG Youlan 1952, 178; and CHEN Rongjie 1963, 139. The Chinese text is quoted from *SBBY* vol. 130, 1: 1a-1b.

This passage states that both *wu* and *you* originate from the common source, which is the *dao*, and both are "deep and profound." The relationship between *wu* and *you* is further elaborated in chapter forty of *Lao Zi*, which says: "All things in the universe come from *you* and *wu*."[12] But the original text of this sentence later was changed and it meant: all things in the universe come from *you*, which, in turn, derives from *wu*. Several questions now arise naturally: How does the *dao* generate *wu* and *yu*? How do they together form a tangible object? What are the subtle differences between *wu* and *yu*? And how does *wu* generate *you* if the former is temporally prior to the latter? None of these questions is answered in the transmitted or excavated versions of the *Lao Zi*, but the text itself drew criticism for its concepts of *you* and *wu* from other schools, and also generated further reactions from the followers of the *dao*. These matters will be discussed further in section three of this chapter after considering the *Zhuang Zi*, the most important book on Daoism after *Lao Zi*.[13]

The earliest surviving version of the extant *Zhuang Zi* was edited and commented on by GUO Xiang 郭象 (253?-312), a gifted classic commentator and thinker in the Western Jin dynasty (265-317 CE). Because of the relative lateness of this text, the lack of earlier authentic historical records about the book, and inconsistencies in some of the

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[12] The Chinese text on bamboo strips discovered in a Han tomb excavated at Guodian of Hubei Province in 1993 reads: 天下之物生於有，生於無。This is slightly different from the earliest transmitted version of the *Lao Zi* (天下萬物生於有，有生於無。), and from what is found on silk scrolls unearthed in Changsha, Hunan Province, in 1973 (天下之物生於有，有於無。). Whether or not they all can be interpreted as equivalent in meaning is debatable. Some scholars, for instance, Robert Henricks, hold that they have the same meaning, namely: All things in the universe come form *you*, and *you* from *wu*; *wu* is prior to *you* in the temporal sequence (see Henricks 2000,77-78). But others, for example CHEN Guying, believe the meaning of the sentence in the bamboo-strip version differs dramatically from the other two versions, and means that all things in the universe are derived both from *you* and *wu*; see CHEN Guying 2003, 186-187.

[13] Some scholars, for instance QIAN Mu, argue that *Zhuang Zi* was prior to *Lao Zi*; see QIAN Mu [1957]1998, 25-130. This view, however, is gradually losing currency since the silk-scroll versions were unearthed in 1973, and especially after the Guodian bamboo-strip version was excavated in 1993.

terminologies and texts it includes, the author and dating of the *Zhuang Zi* have been debated since the late Ming and early Qing dynasties. Although there is no consensus about these issues among historians, it is generally agreed that *Zhuang Zi* was the product of different hands over a period ranging from the mid- to late-Warring States (fourth to late third centuries BCE). At one time, the book was actually attributed to ZHUANG Zi, who is now credited only with large parts of the first seven, so-called "inner" chapters. The "outer" (eleven) and another "miscellaneous" (fifteen) chapters were believed to have been written by his followers.[14]

ZHUANG Zi's life, like that of LI Er, is virtually unknown. According to fragments scattered throughout the extant records, Zhou 周 was his given name and he was supposed to have lived between 399 and 295 BCE. He was presumably a minor official. It was said that he turned down an offer to become prime minister in order to retain his freedom. ZHUANG Zhou advanced Daoism beyond *Lao Zi* in many ways, and carried it to new heights.[15]

In cosmogony, *Lao Zi* holds that the universe had a beginning, and all things in the universe came from both *you* and *wu*.[16] ZHUANG Zi, however, does not agree with this, and thinks that the universe, like the *dao* itself, has neither a beginning nor an end,

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[14] There are four major opinions about the authorship of the book: 1. ZHUANG Zi was the sole author; 2. ZHUANG Zi was only responsible for the inner chapters, and the other parts were written by his followers; 3. ZHUANG Zi was only responsible for the outer and miscellany chapters; 4. The entire work was written by ZHUANG Zi in collaboration with his students. As for dating, it is believed to have been written sometime between the late years of ZHUANG Zi and the early years of the Former Han dynasty. For a detailed account of these issues, see MEI Yibao 1964; LIU Xiaogan 1988, 1994; and ZHANG Songhui 1997, esp. 11-13.

[15] See FENG Youlan 1952, 221-45; and Schwartz 1985, 215-37.

[16] Chapter fifty-two of *Lao Zi* says: There was a beginning of the universe which may be called the Mother of the Universe 天下有始，以爲天下母; see *SBBY* vol. 130, 2: 10a.

and in fact it is eternal. The following passage, taken from the *Qi wu lun* 齊物論 (Equality of All Things) section of chapter two of the *Zhuang Zi* explains all this as follows:

> There is a beginning. There is a not yet beginning to be a beginning. There is a not yet beginning to be a not yet beginning to be a beginning. There is a *you*. There is a *wu*. There is a not yet beginning to be *wu*. And there is not yet beginning to be a not yet beginning to be *wu*. Suddenly there is *wu*. But I do not know, when it comes to *wu*, which is really *you* and which is *wu*.
>
> 有始也者，有未始有始也者，有末始有夫未始有始也者。有有也者，有無也者，有未始有無也者，有未始有夫未始有無也者。俄而有無矣，而未知有無之果孰有孰無也。[17]

The importance of this passage lies not only in its discussion of the relation between *you* and *wu*, but also in its implication of the existence of infinite time. If there were a beginning point for the universe, then there would have to have been another point before that beginning point. And there would have to have been yet another point before the beginning of that beginning. As this reasoning goes on *ad infinitum*, it is logical to conclude that time had no beginning.

Zhuang Zi devotes no space to a discussion of time *per se*. Besides the passage just quoted above, there are several more comments on time to be found implicitly in *Zhuang Zi*. For instance, in an outer chapter entitled "*zhi bei you* 知北游 (Knowledge Wandered North)" there is a passage that also discusses whether or not the universe had a beginning. The argument, basically, is that there is always a source from which things come into being. For example, if we are the children of our parents, then they are the

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[17] Quoted from Watson 1968, 43, with "*you*" substituted for "being," and "*wu*" for "non-being." For alternative translations, see CHEN Rongjie 1963, 185-86; and A. C. Graham 1981, 55. The Chinese text is from *SBBY* vol. I, 18a. A passage similar to this also appears in *Huainan Zi*, where the passage was interpreted as seven fixed stages of the generation of the universe, rather than as an indefinite process. For translations of the passage in *Huainan Zi*, see FENG Youlan [1937]1952, 395-96; and CHEN Rongjie 1963, 306-07.

children of their parents. Continuing this line of thought, there would not have been a beginning point for human life. If human beings and the universe are taken as continuously reproducing entities, with no end of predecessors or successors, then they can have had no beginning and no end in time.[18]

The earliest Chinese word for "time" is probably "*zhou* 宙."[19] In *Zhuang Zi*, "*zhou*" is defined as something that has length (長) but no beginning and no end (無本剽).[20] This definition clearly suggests that time is unlimited on both ends. Time, as one of the five considerations raised by Aristotle that serve to generate concepts of the infinite, truly did so in a natural way for such ancient Chinese thinkers as ZHUANG Zi, who clearly introduced a concept of infinity in this temporal sense. But the matter of an unlimited or "infinite" space as Aristotle understood it is not so natural, nor so obvious.

Excavated records and artifacts dating back to the Shang dynasty confirm that the ancient Chinese believed that they lived at the center of a flat earth, and were surrounded by barbarians in the *si fang* 四方 (four quarters, namely, North, South, East and West). They called the world they lived in *liu he* 六合, or *yu* 宇, which was bounded by the four quarters, and the space above and below the earth.[21] The shape of the *liu he*, they thought,

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[18] The Chinese text is: 冉求問于仲尼曰："未有天地可知邪？"仲尼曰："可，古猶今也。……無古無今，無始無終。未有子孫而有子孫，可乎？" See *SBBY* vol. 132, 7: 28b-29a. For an English translation, see Watson 1968, 245. Similar arguments can also be found in a book entitled *Lie Zi* 列子, compiled in the Jin 晉 dynasty. Under a section with the heading *tang wen* 湯問, there is the following: 殷湯問于夏革曰：古初有物乎？夏革曰：古初無物，今惡得物？後之人將謂今之無物，可乎？ (TANG of Yin asked JI of Xia: "Were there things at the beginning of the universe?" JI answered: "If there were no things, how can there be things now? Could a man who will live after us say there are no things now?"). The Chinese text is quoted from *SBBY* vol. 131, 5: 1a.

[19] The other two candidates are "*jiu* 久" and "*shi* 時," which are discussed in *Mo Jing*.

[20] The Chinese text is: 有長而無本剽者，宙也 (《庄子・庚桑楚》). Quoted from *SBBY* vol. 132, 8: 7b.

[21] SHI Jiao 尸佼, who flourished in the seventh century BCE., once said that "上下四方曰宇，往古來今宙." The Chinese text may be translated as follows: The four directions, together with what is above [the earth]

was like the body of a turtle. The round and domed top shell resembled the sky, and the plane bottom of the turtle mirrored the flat earth.[22] This view of the universe was called the *tian yuan di fang* 天圓地方 (spherical sky and square earth), out of which the influential cosmological theory, *gaitian* 蓋天, was born.[23] The major differences between them were: 1) the former held that the earth was flat, the latter that it was in the shape of an up-side-down bowl; and 2) the former believed that the sky and earth were connected on the edges, the latter that the distances between the sky and the earth were fixed, though not everywhere equal.[24] Both cosmological theories held that the universe was finite.[25]

Pre-Qin Chinese philosophers were preoccupied, as already noted above, with human affairs, and gave little thought to what was outside their world. This intellectual inclination was well-summarized by ZHUANG Zi, who said, "What is beyond the world, the sage leaves it as it exists and does not discuss it."[26] ZHUANG Zi, however, went a step further to question whether there was indeed a limit to the extent of the world, a

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and what is below [the earth], are called the *yu* 宇; the bygone past and the coming future are called the *zhou* 宙." See *SBBY* vol. 128, 2: 5a. This also explains the *liu he* 六合, the four "quarters" plus the "above" and "below."

[22] See Allan 1991, 74-111.

[23] The *gaitian* theory was first recorded in *Zhoubi Suanjing* 周髀算經, an astronomical and mathematical book compiled no earlier than 200 BCE. For detailed accounts of this theory, see Needham 1959, 210-26; and Cullen 1996, 50-53; 128-37.

[24] See JIN Zumeng [1991] 1996, 65-76; 165.

[25] Later WANG Chong 王充 (27-ca. 97 CE) developed his own theory based upon the *gaitian* 蓋天 theory; WANG held that both sky and earth were flat and parallel, and separated by an enormous distance. WANG's theory implies that the universe might be indefinitely extended between the sky and earth. For WANG Chong's theory, see ZHENG Wenguang 1978, 47; JIN Zumeng [1991] 1996, 77-87.

[26] The Chinese text is: 六合之外，圣人存而不論。See *SBBY* vol. 132, 1: 19a. The English translation is quoted from CHEN Rongjie 1963, 186.

question already posed by the sages of the Shang dynasty: "Are there any limits to what is above, below, or the four quarters?" asked one sage. "There is always another limitless beyond a limitless," the other replied.[27]

What does "another limitless beyond a limitless" really mean? Assuming that an infinite space cannot have another infinite space outside or beyond itself, can it be said that this statement contradicts itself, or might it be interpreted in another way? *Zhuang Zi* provides no specific arguments related to this unlimited space, although it arises in passing in two further statements, one of which gives a negative answer to the question: "Is there any limit to the four quarters and the above and below?" The other statement says that: "[A person] should be as broad-minded as the four quarters which are unlimited and have no borders among each other."[28] Nevertheless, the *Lie Zi* 列子, a book attributed to a wise man who flourished in the fifth century BCE. (although the extant version is believed to be a compilation from the mid-third century CE.) includes the following passage, a part of which recalls the sentence in *Zhuang Zi* just discussed:[29]

> TANG of Yin asked: "Are there any limits to the above, the below, or the eight directions?" JI [of Xia] replied: "Do not know." As TANG pressed the issue again, JI said: "If there are not, then they are unlimited. If there are, then they must be limited. How could I know? But that which is outside the unlimited is

[27] The present version of *Zhuang Zi* does not contain these two sentences, which some scholars believe were left out accidentally by some copyist in the process of transmission. See CHEN Guying [1975]1999, 1:14-15; and LIU Xiaogang 1988, 218. The amended Chinese text is: 湯問棘曰：上下四方有極乎？棘曰：無極之外，复無極也。Quoted from CHEN Guying [1975]1999, 1:14.

[28] The former is in the outer chapter "*zeyang* 則陽," and its Chinese text is: 君以意在四方上下有窮乎？君曰：無窮。The latter is in the miscellaneous chapter "*quishui* 秋水," and its Chinese text is: 泛泛乎其若四方之無窮，其無所畛域。The texts are quoted from *SBBY* vol. 132, 8: 26a-26b; 6: 10b.

[29] The dating of *Lie Zi*, like that of other Chinese classics, is an open question. It is widely agreed that the transmitted book is a product of the mid-third century CE, but retains some original thought of LIE Zi, who was an historical figure of the fifth century BCE, and was mentioned several times in *Zhuang Zi*. The idea reflected in the following quotation may be prior to that in *Zhuang Zi*. For the dating of *Lie Zi*, see YANG Bojun [1979] 1996, 1-4; 287-348; and ZHANG Yongyan 1991, 189-208.

> still limitless; and that which is within the unexhausted is still inexhaustible. The unlimited is surpassed by another unlimited; and the unexhausted is followed by another unexhausted. From this, I think they are infinite and inexhaustible, and *yet* do not know whether they are [really] finite and limited.
>
> 殷湯曰：然則上下八方有極盡乎？革曰：不知也。湯固問。革曰：無則無極，有則有盡，朕何以知之？然無極之外，复無無極；無盡之中，复無無盡。無極复無無極；無盡复無無盡。朕是以知其無極無盡也，而不知其有極有盡也。[30]

Not surprisingly, there are diverse interpretations of this passage.[31] The above translation reflects the author's own understating of the original text. The reasoning employed in this passage may be as follows: Everything in the world contains something smaller, and at the same time is contained in something larger. Accordingly, the smaller thing in turn contains something smaller, and the larger thing is contained in something larger. This reasoning process can go on indefinitely in both directions. But at some point, the biggest thing is confined or limited by the above, the below, and the eight directions. It is logical to conclude that there exists some thing that is bigger than the biggest thing. Therefore, the world, defined as *liu he*, is unlimited. A thing, or space, beyond that world is still limitless. The same reasoning can apply for smaller things in the opposite direction.

However, the last sentence of the quoted text is problematic; it is not clear whether JI 革 believed the infinite universe existed only conceptually, or also physically. This is similar to the Aristotelian or scholastic distinction between the "potential" infinite and the "actual" infinite. It seems that JI declined to acknowledge the real existence of an actually infinite universe for fear that more difficult questions might then arise. This

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[30] The Chinese text is in a section with the heading "*tang wen* 湯問," quoted from *SBBY* vol. 131, 5: 1b-2a.

[31] See A. C. Graham 1960, 94-95; the preface to the 1990 edition, esp. xiv-xv; CHEN Rongjie 1963, 311-12; and Needham 1969, 222-23.

may be similar to the stance JI took with respect to passages immediately before and after the text quoted above, passages in which he did not reveal his thoughts on such questions as: What is it that is outside the [biggest solid or tangible] matter? What was there before any event had occurred? And is there anything really greater than the heavens and earth?

A. C. Graham, the late English scholar of Chinese philosophy, interpreted the above passage in a very different way. He took his clues from the commentaries by ZHANG Zhan 張湛 of the Eastern Jin dynasty (317 – 420), and conjectured that the arguments for infinity were left out immediately before the words: "*ran wu ji zhi wai* 然無极之外." He interpreted the first "*wu* 無" in "*wu ze wu ji* 無則無极" as "nothing," and suggested that the missing arguments for infinite space might have been based on this presumption: "There is no limit where there is nothing at all; there are limits only where there are bodies to limit." Moreover, he translated the entire sentence "*ran wu ji zhi wai*, *fu wu wu ji*; *wu jin zhi zhong*, *fu wu wu jin* 然無极之外，复無無极；無盡之中，复無無盡," as "there is nothing limitless outside what is limitless, and nothing inexhaustible within what is inexhaustible"; this, Graham said, is a counter argument for infinity.[32]

However, there are good reasons to believe that the above-quoted passage is intact, and that the sentence Graham took as a counter argument for infinity may actually be part of an argument *for* infinity. Consider the conversation between TANG and JI immediately after the quoted text. This conversation begins with a question from TANG about whether there is anything beyond the four seas, which were believed to be the remotest parts of the flat earth. JI answers that what is outside the four seas is just like what is within the proper domain of China. TANG then asks JI to justify his words. JI

[32] See A. C. Graham 1960, 95.

replies that he once went east to a place called Ying, and found out that the people who lived there were just like his own people. He asked people there what lay to the east of their country, and was told that what was there was the same as in Ying. JI says that he also went west to a place named Bin, and found out that people there were also like his own people. He again asked people there what lay to the west of their border, and was told that there was the same as what was in Bin. From this, JI concludes that things or people anywhere are the same in general.[33] He then goes on to say:

> The lesser is always enclosed by a greater, and so on without end. Heaven and earth, which enclose the myriad things, are themselves enclosed. The enclosing of the myriad things never reaches a limit. Likewise, the enclosing of heaven and earth never reaches an end. How do I know that there is not a greater universe outside our own? This is something I do not know.
>
> 故大小相含，無窮極也。含萬物者，亦如含天地。含萬物也，故不窮；含天地也，故無極。朕亦焉知天地之表不有大天地者乎？亦吾所不知也。[34]

The above statements are elaborations of the sentences that Graham arguably misunderstood as counter arguments for infinity. The reasoning employed above, like that in the previously quoted text, is based on the reasoning that the lesser is contained by a larger indefinitely in two directions. Based on such reasoning, inexhaustible things and the unlimited *yu* logically follow.[35] Whether there really exists a solid thing larger than the universe combined with the sky and the earth, JI clearly acknowledges was beyond

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[33] The Chinese text reads: 湯又問曰：四海之外奚有？革曰：猶齊州也。湯曰：汝奚以實之？革曰：朕東行至營，人民猶是也。問營之東，復猶營也。西行至豳，人民猶是也。問豳之西，復猶豳也。朕是以知四海、四荒、四極之不異是也。Quoted from *SBBY* vol. 131, 5: 2a. For English translations of the text, see CHEN Rongjie 1963, 312; and A. C. Graham 1960, 96.

[34] Quoted from CHEN Rongjie 1963, 312. The Chinese text is taken from *SBBY* vol. 131, 5: 2a-2b.

[35] What things are inexhaustible here may not mean that things are infinitely divisible in space. It may most likely mean that things never die out in the time sequence. This point is implied in ZHANG Zhan's commentary; see YANG Bojun [1979] 1996, 149.

his knowledge.[36] Therefore, it follows that the passages discussed here, both from *Zhaung Zi* and *Lie Zi*, do include the concept of the unlimited, or more precisely, the concept of indefinitely large space at least in thought (the potentially infinite).

Scholars have argued that the other two Chinese cosmological theories, namely *hun tian* 渾天 and *xuan ye* 宣夜, also included concepts of spatial infinity.[37] Although some fragmentary descriptions of the *hun tian* theory can be traced back at least to the fourth century BCE, the relatively systematic and detailed descriptions of the theory are only found in *Ling xian* 靈憲 (*Spiritual Constitution of the Universe*) and *Hun tian yi zhu* 渾天儀註 (*A Commentary on the Armillary Sphere*), both by ZHANG Heng 張衡 (78 -139), an astronomer, mathematician and geodesist.[38] According to ZHANG Heng, the *hun tian* theory held that the sky was a sphere, half of which was below the earth and could not be seen, and that the sky and earth were connected by water.[39] This theory (or cosmological model) said nothing about what lay beyond the celestial sphere. ZHANG Heng, however, in the *Ling xian*, considered this issue and wrote as follow:

> What is beyond [the celestial sphere] is unknown and could be known [in the future]. It may be named *yu zhou*. The surface of *yu* is unbounded, and the starting-point of *zhou* is endless.

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[36] ZHANG Zhan 張湛, the above-mentioned editor and commentator on *Lie Zi*, explained that the one which contained both the sky and the earth was *taixu* 太虛, a sort of ether. See *SBBY* vol. 131, 5: 3a.

[37] See Needham 1959, 216-224; HO 1966; ZHENG Yanzu 1976, 298-303.

[38] CHEN Jiujin doubts that ZHANG Heng was the true author of *Huntianyi Zhu*, and argues that it was written by an anonymous person during the second part of the third century CE. See CHEN Jiujin 1981.

[39] Scholars have debated whether the shape of the earth described in ZHANG Heng's account of *hun tian* 渾天 was round or flat. For the debate and the specific arguments supporting each point, see BO 1990 and JIN Zumeng [1991] 1996.

過此而往者，未之或知也。未之或知者，宇宙之謂也。宇之表無極，宙之端無窮。[40]

Some historians believe that the Chinese text quoted above suggests that ZHANG Heng refers to both spatial and temporal concepts of infinity.[41] But whether the original text can be interpreted this way or not is questionable. As mentioned above (footnote 21), in the *Si Zi* 尸子, *yu* 宇 refers to the four directions of the earth, together with what is above and what is below, and *zhou* 宙 is defined as the *bygone* past and the yet-to-come future. The combined words, *yu zhou* 宇宙, in the passage immediately quoted above, are used to define whatever was beyond the celestial sphere and in this author's view, do not have any meaning regarding time as defined in the *Si Zi*. Moreover, whether ZHANG Heng had a clear concept of spatial infinity is also debatable. First, ZHANG Heng acknowledged that whatever lay beyond the celestial sphere was unknown but could be known in the future. This view clearly echoes a passage in the *Zhoubi Suanjing* 周髀算經:

> What is beyond this (the seventh *heng* 衡, the outmost imaginary celestial sphere with its center at a pole erected in Yangcheng 陽城, capital of the Xia dynasty) is unknown but might be known. What [we mean by] might be known is that [we] doubt its intelligibility and understand the difficulty of knowing it.
>
> 過北而往者，未之或知。或知者，或疑其可知，或疑其難知。[42]

[40] The Chinese text is quoted from *Houhan Shu* 後漢書, Zhi 10, Tianwen I, 3216 (Taipei: Dingwen, 1999 second edition).

[41] For instance, BO 1992, 74.

[42] Christopher Cullen holds that the Chinese character, "*bei* 北," should be interpreted as "*wai* 外." But more likely, here "北" is a corruption of "*ci* 此." Moreover, Cullen's translation of this passage differs dramatically from the one provided above. For his translation, see Cullen 1996, 187. Another alternative translation of this passage is kindly provided by Andrea Bréard as follows: "What is beyond this (*heng*), it is not yet possible to know. The possibility of knowing consists (at the moment) either in doubting that this can be known, or in suspecting that this is difficult to know." The Chinese texts are quoted from *TH* 1: 40a.

If this were truly an "unknown" area, how could ZHANG Heng say with any certainty that it was infinite in size? Secondly, the terms *wuji* 無極 and *wuqiong* 無窮 could also mean something very large or indefinitely large, but not necessarily something actually infinite in extent. Accordingly, we may safely say that ZHANG Heng realized that the space beyond the celestial sphere might be infinite, but was unsure as to whether this was so or exactly to what extent there might be anything beyond the "seventh *heng*."

The *xuanye* 宣夜 cosmology is widely held among historians to include a clear-cut concept of spatial infinity. This cosmology was probably conceived in the Warring-states period, but it never enjoyed the status of its rival theories, *gaitian* 蓋天 and *hun tian* 渾天. As a matter of fact, the *xuanye* theory was ignored by ancient Chinese scholars and virtually everything written about it has been lost. What we now know about this is based upon a short passage from the chapters on *tianwen* 天文 (Celestial Phenomena) in the *Jin Shu* 晉書 (*History of the Jin Dynasty*).[43] According to this passage, the *xuan ye* cosmology held that heaven was immensely high and far away from the earth, and it had no definitive form as a material substance. The sun, moon, and five planets were not tied to heaven as both the *gaitian* and *huntian* theories held, but were thought to float, driven by *qi* 氣, a sort of ether. Why some historians believe that the *xuanye* cosmology included a concept of spatial infinity is based, primarily, on two considerations: reference to the immense height of heaven and the ambiguous term, *wuji* 無極, which literately means "endless," "boundless" or "limitless." But this could also mean something big enough for the human eye to see, describing the height of heaven. From the original text it is not clear whether *wuji* was meant to be taken literately or in a metaphorical sense,

[43] For an English translation of this passage, see Needham 1959, 219; and HO 1966, 52a.

but the latter seems more likely. If so, the *xuanye* cosmology does not convey a clear-cut concept of spatial infinity as some historians have claimed, although the concept of "*wuji*" did represent a step forward towards the idea of infinite space.

Chinese philosophers and scholars generally in the Pre-Qin period inevitably raised questions as to whether or not space and time were infinite. But the extant documents suggest that they did so to a much lesser degree than is generally thought, and certainly took the matter much less seriously than did their Greek counterparts, who also pondered such questions.[44] From the consideration of "coming to be" and "passing away," which never ceased, the Chinese realized that time could not have a beginning, nor an end, and hence time must be infinite. From the consideration of one solid thing containing a smaller one, and at the same time being contained by something larger, the Chinese also realized that there might be something material beyond the universe in which they lived. They were thus aware that space too might be infinite, but they also understood the tremendous difficulties in proving or even explaining this idea. Instead they simply tried to ignore the question and did not bother to provide rational or even speculative explanations, unless pressed to do so. What they were interested in and most concerned about were human society and the things that lay between heaven and earth. This intellectual bent was best summarized by YANG Shen 楊慎 (1488 -1559), a scholar of the *Ming* dynasty, who wrote:

> LIU Bowen of this (Ming) dynasty once said: "Does heaven have a limit? [If so, then] what is outside heaven? Is heaven infinite? [If that is the case, consider this:] anything that has a certain form must be limited. The reasons and forces

[44] For comparison of the concepts of infinity between ancient Greece and China, see Lloyd 1996a, 140-164; and 1996b. For those who are interested in the discussion of infinity by scholastic theologians and natural philosophers in the late Middle Ages, see Grant 1981, 105-115; and Grant 1994, esp. 63-185.

> behind these are something sages could not know of."…. Only staying outside a thing can one see its true shape. We live within heaven and earth. How do we know their true nature? Moreover, the learning of the sages focuses on human affairs, so why should we bother with what lies beyond heaven?
>
> 本朝劉伯溫……曰："天有極乎？ 極之外又何物也？天無極乎？凡有形必有極。理也；勢也；是聖人所不能知也。"……蓋處于物之外方見物之真也。吾人固不出天地之外，何以知天地之真面目與？ 且圣賢之學切問近思亦何必求知天外之事耶？[45]

Ancient Chinese thinkers did not, indeed, bother themselves with what might exist beyond heaven. Nevertheless, they encountered the concept of infinity from very much the same considerations that concerned Aristotle, particularly considerations 1, 3, and 4 discussed above (page 10).[46] Some historians have also shown that the Pre-Qin Chinese texts, especially the *Mo Jing* and the last chapter of the *Zhuang Zi*, dealt with paradoxes of the finite and the infinite in terms of mathematical magnitudes (Aristotle's considerations 2 and 5, as above). But such interpretations need to be carefully examined in their contexts, both historical and historiographical. To that end, it is necessary to turn to the *Mo Jing* and *Zhuang Zi*, to see if either ever came to terms with the mathematical concept of infinity, and if so, how and on exactly what terms they did so.

2.2 Do The *Mo Jing* and *Zhuang Zi* Really Include Mathematical Concepts of Infinity?

The *Mo Jing*, also known as *Mo Bian* 墨辯 (*Disputations of the Moists*), refers to chapters 40 through 43 of the *Mo Zi* 墨子 (*The Book of the Moists*), and may also include the following two chapters, 44 and 45 (*Mo Jing* in this dissertation is taken in the narrower sense). The texts of *Mo Jing* have been greatly corrupted in the process of being copied

[45] The Chinese texts are quoted from YANG Shen 1983, 729b-730a. Punctuation is supplied by the author.

from one format and medium of writing to another. All the extant versions of the *Mo Jing* prior to 1783 CE are virtually unintelligible. It is through the hands of such commentators as BI Yuan 畢沅 (1720-1797), ZHANG Huiyan 張惠言 (1761-1802), and especially SUN Yirang 孫詒讓 (1848-1908), that the texts have been collated and edited in such a way as to confer some meaning upon the text, so that passages in the *jing* 經 (Canons, chapters 40 and 41) and *shuo* 說 (Explanations, chapters 42 and 43) chapters have some meaningful interpretations.[47]

The *Mo Jing* traditionally was attributed to MO Di 墨翟, the founder of the Moist School who flourished between the second part of the fifth and the first part of the fourth centuries BCE.[48] This view was challenged by WANG Zhong 汪中 (1745-1794), who in 1780 suggested that the disciples of MO Di, rather than MO Di himself, were responsible for the *Mo Jing*. Since then, scholars, generation after generation, have debated over the authorship and the dating of the texts. Concerning these questions many theories have been advanced, of which the following are among the most popular: 1) At least one of the Canons (chapter 40) was authored by MO Di himself; whereas the other chapter (41), together with the Explanations, are more likely by his immediate students; and 2) The entire *Mo Jing* was composed by MO Di's disciples during the late Warring-states period.

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[46] See note 1.

[47] For a history of the book of the *Mo Jing* itself, see A. C. Graham 1978, 64-72.

[48] For the life and times of Mozi as well as his philosophy, see FENG Youlan 1952, 76-80; Lowe 1992, 24-73; and Hansen 1992, 95-150.

The difference in the presumed dating of when each version may have been composed is about two hundred years.[49]

Due to a lack of solid evidence about the author(s) and when the texts were actually composed, and even more unfortunately, due to the fact that the texts have been seriously corrupted, it is very difficult to determine their original contents, let alone the aims and motivation of the *Mo Jing*. Accordingly, numerous approaches have been taken by modern and contemporary scholars, including some who have even used Euclidean geometry, Western science and formal logic to interpret the text. As a result, the *Mo Jing* has been considered by many scholars as *the* most important pre-Qin Chinese classic on science. For instance, A. C. Graham in his book, *Later Mohist Logic, Ethics and Science*, claims:

> The Mohist researches are a very remarkable exception [in science], and deserve to be classed with similar brief episodes in Greece and mediaeval Europe among the movements which in retrospect look to us like abortive efforts in the direction of modern science.[50]

Graham, like many scholars before and after him, believes that the major contents of the *Mo Jing* are concerned with geometry, optics, and mechanics.[51] Though few surviving fragments can be interpreted geometrically, Graham and other scholars hold that a large body of the Moist texts on geometry must have been lost. Among the relevant extant fragments, the following, more or less, have been widely interpreted as referring to mathematical concepts of the infinite:

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[49] For a survey of the debates over the authorship and the dating of the *Mo Jing*, see TAN 1964, 21-34; and LI, ZHOU, and ZHOU 1989, 307-315.

[50] A. C. Graham 1978, 53.

[51] *Ibid.*, 53-58; 301-316.

Canon I, 41:[52] The *qiong* (exhaustion or limit) is wherever at the next advance there is no room for a measured length.
窮，或有前不容尺也。

Explanation: If somewhere there is no room for a measured length, it is limited; if everywhere there is room for a measured length, it is limitless.[53]
窮：或不容尺，有窮；莫不容尺，無窮也。

Canon I, 55: The *hou* (thickness) is something which has a certain size.
厚，有所大也。

Explanation: The bigness of *hou* is only in contrast with *wu* 無.
厚，惟無所大。

Canon I, 61: The *duan* (extremity) is the foremost *wuhou* 無厚 (thicklessness) which precedes all others.
端，體之無厚而最前者也。[54]

Explanation: There is not any difference between *duan*.
端，是無同也。

Canon II, 60: If bisecting something [solid] again and again, then the process will stop at some point. That is because of *duan*.
非半弗斲，則不動。說在端。

Explanation: If bisecting something [solid] always ahead, then the remainder, which may be treated as a sort of *duan*, cannot be further halved after certain moves. If bisecting it ahead and then behind, then *duan* would be in the center. If every cut must halve [the solid], then the cutting cannot go on [forever].
非：斲半，進前取也。前則中無為半，猶端也。前後取，則端中也。斲必半，毋與非半，不可斲也。

The above-quoted canons and their explanations in the *Mo Jing* resemble several sentences in the last chapter of the *Zhuang Zi*, and are therefore widely interpreted as additional evidence for Chinese concepts—or even definitions—of the actual infinite and

[52] The numbering system of the Canon and its Explanations used in this dissertation is adopted from the *Mobian fawei* 墨辯發微 (*Illuminating the Disputations of the Moists*) by TAN Jiefu (TAN 1964). This system differs somewhat from the one used by A. C. Graham (Graham 1978). The differences arise mainly as a result of different readings and interpretations of the original texts.

[53] The translation of this Canon and its explanation are adopted from A. C. Graham 1978, 294.

[54] In the original text of the *Mo Jing*, *hou* 厚 actually is *xu* 序. This correction is suggested by WANG Niansun 王念孫 in his *Dushu zazhi* 讀書雜志 (*Notes on the Books Read*).

of infinitesimals. Two of these are part of the so-called ten theses (or paradoxes) of HUI Shi 惠施, who was senior to and a friend of ZHUANG Zi. Few authentic records about HUI Shi have survived. Moreover, virtually no contexts are known for any of the theses in the *Zhuang Zi*. As a result, the first two: "The greatest is nothing outside itself, which is called *da yi* 大一; the smallest is nothing inside itself, which is called *xiao yi* 小一 " (thesis I);[55] and "Thicklessness (*wuhou* 無厚) cannot be accumulated, yet its extent may reach a thousand miles" (thesis II),[56] along with the other eight theses, may be interpreted in many different ways, as indeed they have. Another related sentence is to be found in the section on the so-called twenty-one paradoxes of the Dialecticians, which immediately follow the just-mentioned ten theses in *Zhuang Zi*. Although scholars do not agree on whether HUI Shi was one of the Dialecticians who should be credited with this work, or for at least part of the twenty-one paradoxes, the dialectician, GONGSUN Long 公孫龍, a disciple of HUI Shi, was very likely responsible for the following: "[Given] a stick one foot long, if one takes away a half every day, it will not be exhausted for ten-thousand generations" (hereafter referred to as thesis twenty-one).[57]

Many scholars have provided explanations, some lengthy, some brief, for the fragments quoted above from both the *Mo Jing* and *Zhuang Zi*. Only those of QIAN Baocong 錢寶琮, LI Jimin 李繼閔, MEI Rongzhao 梅榮照, ZOU Dahai 鄒大海, and A. C.

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[55] There are various translations for *da yi* 大一, and *xiao yi* 小一. For instance, "greatest unit" and "smallest unit," see FENG Youlan [1937] 1952, 197; "great unit" and "small unit," see CHEN Rongjie 1963, 233; "infinity" and "infinitesimal," see MEI Yibao 1964, 205; "the one of largeness" and "the one of smallness," see Watson 1968, 374; and "macro-one" and "micro-one," see Harbsmeier 1998, 293.

[56] The Chinese texts of the two theses are: 1. 至大無外, 謂之大一; 至小無內, 謂之小一; and 2. 無厚不可積也, 其大千里. Quoted from *SBBY* vol. 132, 10: 20b.

[57]The Chinese text for this paradox is: 一尺之棰, 日取其半, 萬世不竭. Quoted from *SBBY* vol. 132, 10: 22b.

Graham will be outlined here. QIAN in his *History of Chinese Mathematics* holds that the Canon I, 41, should be understood as follows: "using a ruler to measure a distance, if what remains ahead is less than the length of the ruler, then the distance is limited; if the measuring can continue, and there is always room for the ruler, then the distance is unlimited."[58] The *duan* as defined in Canon I, 61, in QIAN's view, is a geometrical point. As for the two terms defined in the first thesis of HUI Shi, QIAN believes that *da yi* refers to "space and time as a whole," and *xiao yi* "a point in space or an instant of time."[59] The second thesis of HUI Shi, QIAN holds, should be interpreted to mean that the accumulation of lines cannot make an area, and the accumulation of areas cannot generate a volume. Accordingly, this interpretation considers lines and areas in the sense of Euclidean geometry, i.e. although dimensionless, they do have some size or extension. As for the twenty-first paradox, QIAN points out that its meaning is very close to Zeno's paradox of dichotomy.[60]

Clearly QIAN Baocong drew upon the Paradoxes of Zeno and Euclidean geometry to interpret the fragments of both the *Mo Jing* and *Zhuang Zi*. LI Jimin, however, goes even further and interprets them in terms not only of Euclidean geometry, but of modern mathematical analysis and set theory as well. He believes that *da yi* means "infinite space," and that both *xiao yi* and *duan* have the same meaning as the Euclidean

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[58] QIAN Baocong [1964] 1981, 19.

[59] *Ibid.*, 20.

[60] *Ibid.,* 20-21. The explanations of the fragments both in the *Mo Jing* and *Zhuang Zi* offered by QIAN Baocong in the 1930s differ from the one outlined above. For instance, for Canon I, 41, QIAN said that it meant that an accumulation of points could make a line, and in turn an accumulation of lines could make an area; he thus took the *da yi* and *xiao yi* to refer to the mathematical infinite and infinitesimal, respectively; see QIAN Baocong [1932] 1998, 182-187.

geometrical point (i.e. dimensionless or without parts). According to LI Jimin, the two terms of *da yi* and *xiao yi* are "the germs of modern ideas of infinity, infinitesimal and transfinite number."[61] He holds that the twenty-first paradox is intended to argue the indivisibility of a line segment, and that it also provides the first example of an infinite series in the history of Chinese mathematics. In addition, LI Jimin thinks that Canon II, 60, is similar in meaning to thesis twenty-one, and the method of cutting forward and backward as stated in the Explanation actually constructs two sets of intervals, in a manner "astonishingly similar to" the Dedekind cut.[62]

Like LI Jimin, MEI Rongzhao also uses modern mathematics (as well as dialectics) to inform his interpretations, though they differ in their details. MEI believes that Canon II, 60, has the opposite meaning from thesis twenty-one. The latter is meant to emphasize "the changing status and result of infinite cutting. Finite cutting…is a change of quantity; infinite cutting…is a change of quality"; while the former is intended to mean that "a finite length can be composed of an infinite number of units, the focus of this thesis (twenty-one) is on the contradiction of being both finite and infinite."[63] From a mathematical point of view, MEI thinks that the latter involves an infinite series, whereas the former embodies the notion of limit. As for the first thesis of HUI Shi, MEI believes that it provides a definition of the infinite. He also argues that the definition of *qiong* in the Canon I, 41, is "very close in essence" to a modern mathematical definition

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[61] LI Jimin [1990] 1992, 444.

[62] *Ibid.*, 448.

[63] MEI 1992, 110.

of the infinite.[64] Moreover, MEI also holds that *duan* means a geometrical point, and that the three paradoxes in *Zhuang Zi* currently in question all involve geometrical points.[65]

Recently ZOU Dahai has published several articles and has also devoted one section of his latest book, *Zhongguo shuxue de xingqi yu xianqin shuxue* 中國數學的興起與先秦數學 (*Chinese Mathematics before the Qin Dynasty*), to concepts of infinity in the Pre-Qin Chinese classics. He cites more research by students of the history of philosophy and the Chinese classics than any other historian of Chinese mathematics has done previously. He has tried to contextualize his discussions as much as possible, but often his conclusions are very similar to concepts both in ancient China and Greece due to his primary interest in drawing comparisons.[66] For Canon I, 41, ZOU holds that the explanation provided by QIAN Baocong (see page 34) is the most reasonable among those he has surveyed. He then "translates" QIAN's explanation into symbols as follows: If $A - nR < R$, then A is finite; If $A - nR \geq R$, then A is infinite (here, A stands for a ray or a line segment; R for the length of the ruler in question; and n for a natural number). Based on this interpretation/translation, ZOU not only reaches the same conclusion as MEI, but argues that Canon I, 41 is equivalent to the Eduoxian Axiom, now better known as the Archimedian Axiom.[67]

As for *duan* in the *Mo Jing*, ZOU holds that it means a geometrical point as well as an atom of some physical material. He believes that the first thesis of HUI Shi

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[64] *Ibid.,* 102.

[65] *Ibid.*, 107-110.

[66] See ZOU Dahai 1995a; 1995b; 1997; 2000; 2001, 179-217; and 2001b.

[67] See ZOU Dahai 1995a, 75; 1997, 19; and 2000, 71. A similar view, that the Canon I, 41 is equivalent to the Archimedian Axiom, can also be found in HUANG Chengui 1987.

undoubtedly defines both the finite and infinite, and that the second thesis involves the idea that spatial indivisibility cannot be accumulated. Thesis twenty-one, according to ZOU, was meant to criticize Canon II, 60, by emphasizing the infinite process of halving.[68]

Graham, unlike the Chinese historians already mentioned, holds that *duan* cannot be interpreted as a geometrical point, all of which are undifferentiated according to Euclidean geometry, but in the *Mo Jing* is taken to be a special point: the starting-point.[69] However, Graham argues that the Chinese term *wuhou* does mean geometrical point in a general sense.[70] The second thesis of HUI Shi, in Graham's view, is meant to express the paradox that "points cannot be accumulated yet a length is the sum of the units into which it is ultimately divisible, which are points."[71] As for thesis twenty-one and Canon II, 60, Graham holds that the halving described in both passages argues for "lengths as infinitely divisible."[72]

Though the detailed explanations provided by QIAN Baocong, LI Jimin, MEI Rongzhao, ZOU Dahai, and A. C. Graham, differ from one another, there is no doubt that they all believe that the *Mo Jing* and *Zhuang Zi* include rich mathematical concepts of infinity. Moreover, all of their explanations are based, more or less, on the concept of geometrical magnitude. Using such concepts as points, lines and areas to interpret the

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[68] ZOU Dahai 2001b, 47-75.

[69] A. C. Graham 1978, 302; 310.

[70] *Ibid.,* 302; 306.

[71] *Ibid.,* 306.

[72] *Ibid.,* 302.

fragments of the *Mo Jing* and *Zhuang Zi*, however, did not begin with QIAN Baocong, and this approach may be traced back at least to ZOU Boqi 鄒伯奇 (1819-1869). Although a traditional scholar, he had a tremendous interest in the newly introduced Western science, especially in optics and mechanics.[73] Like many Chinese scholars of his time, ZOU, on the one hand, was amazed by the contents of Western science, but on the other hand, believed that it could all be found in Chinese literature surviving from ancient times. In one short essay: "Ancient China Already Had All Western Methods (Lun xifa jie gu suoyou 論西法皆古所有)," ZOU wrongly asserts that the "*Mo Jing* had all of Western mathematics," among other claims.[74] This false statement had a direct influence on CHEN Li 陳澧 (1810 – 1882), who was a highly reputed scholar of Confucian learning and happened to be a friend of ZOU. Following ZOU's lead, CHEN devoted himself to identifying sources of various definitions and methods as presented in Chinese translations of Euclid's *Elements*. Among the alleged identifications CHEN made are: 1) that *duan* as defined in Canon I, 61, is equivalent to the *point* of Euclidean geometry; and 2) that the four Chinese characters used to describe *duan* in the same Canon: 體之無序, are equivalent in meaning to the Euclidean *line* (see page 32, note 55).[75]

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[73] See ZHANG Binglin 1985, 215.

[74] ZOU Boqi 1873, page 20b of the second volume of the *Xueji yide* 學計一得 (Some Reflections on Reading). ZOU's view that ancient China already had all Western methods obviously was an echo of the theory: "Chinese origins of Western learning," which was initiated by MEI Wending, one of the prominent Chinese mathematicians during the Kangxi reign (1660-1772 CE), a view sanctioned by the Emperor Kangxi. Nevertheless, ZOU was one of the earliest scholars who saw some "connection" between the *Mo Jing* and Euclid's *Elements*. For the "Chinese origins of Western learning" and its impact upon Chinese scholars during the eighteenth and nineteenth centuries, see WONG 1963; and HAN 1998.

[75] CHEN Li also claimed that the fragments in Canon I of the *Mo Jing*, like "even [means] the same height (*ping, tong gao ye* 平，同高也)," "middle or center [means] the same length (*zhong, tong chang ye* 中，同長也)," and "circle [means] one center and the same lengths (*yuan, yi zhong tong chang ye* 圓，一中同長也)," were similar in meaning to the definitions for parallel, mid or central point, and circle as those in Euclid's *Elements*. See CHEN Li 1936, 13; A. C. Graham 1978, 71.

CHEN's specific "evidence," together with ZOU's initial false assertion, in turn convinced SUN Yirang, whose book, *Mozi jiangu* 墨子閒詁, has traditionally been considered one of the most important contributions to textual study of the works attributed to MO Zi. SUN not only accepted CHEN's interpretation of *duan*, but also believed that the *Mo Jing* contains "theories on geometry, optics and mechanics."[76] Though SUN himself provides no new geometric interpretations for fragments of the *Mo Jing*, he did hope his junior, LIANG Qichao 梁啟超 (1873-1929), who was to become one of the most influential scholars in China in the early twentieth century, would continue with further studies along similar lines.[77]

Later LIANG did follow SUN's advice and used Western science, including geometry, to offer a very free interpretation of the *Mo Jing*. As for mathematics, besides repeating CHEN's views, LIANG also provided new interpretation of his own, including the Chinese term *jian* 兼 for *whole part* in the sense of Euclidean geometry, *ti* 體 for *partial part*, *xian* 線 or *line*, *qu* 區 for *plane*, *hou* 厚 for *solid*, *bei* 倍 for *addition*, *jian* 減 for *subtraction*, *ying* 盈 for *volume*, and *ci* 次 for *permutation* or *combination*, among others.[78] With these identifications between Chinese terms and those in Euclidean geometry, LIANG went on to interpret parts of the *Mo Jing* exactly as if it were a treatise on Euclidean geometry.

Due to LIANG's prestigious status in the Chinese academic world, his view of geometry in the *Mo Jing* only served to reinforce what had already been argued by ZOU

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[76] SUN Yirang 1975, 450; 445.

[77] Quoted from A. C. Graham 1978, 71-72.

[78] LIANG 1921, 142-146.

Boqi, CHEN Li, and SUN Yirang. Consequently, using Euclidean geometry to interpret the *Mo Jing* has become standard practice ever since. The scholars mentioned above: QIAN Baocong, LI Jimin, MEI Rongzhao, ZOU Dahai, and Angus C. Graham, have all been influenced by this approach.[79] Nevertheless, no scholars after CHEN Li have provided any solid historical evidence to justify such geometric interpretations of the *Mo Jing*.

Whether or not geometry is a major concern of the *Mo Jing*, however, should still be regarded as an open question. A thorough discussion of this matter would require a dissertation of its own, but discussion here will be confined to whether or not it is legitimate to use Euclidean geometry to interpret the *Mo Jing*. One of the key questions that must be addressed is whether traditional Chinese mathematics included the concept of geometrical magnitude.

Due to the fact that the extant texts of the *Mo Jing* have been heavily corrupted, and even more unfortunately, that very little is known of their intellectual contexts, it is futile to decide whether the original *Mo Jing* itself had something equivalent to the Euclidean concept of magnitude. But what can be learned about the question of "magnitude" from other ancient Chinese mathematical texts?

The earliest Chinese mathematical text yet known is on the bamboo strips unearthed between December of 1983 and January of 1984 from Han Tomb 247 at Zhangjiashan in Jiangling, Hubei province. On the *verso* of one bamboo strip, there are three Chinese characters: 筭數書. Based on this, the text was named the *Suan Shu Shu* (*A*

[79] Joseph Needham in *Science and Civilisation in China*, vol. 3, *Mathematics and the Sciences of the Heavens and the Earth*, also discussed mathematics in the *Mo Jing* at some length. His discussion seems to

Book on Numbers and Computation), although no such title is mentioned in any transmitted written historical records.[80] According to one of the leading archaeologists of the excavation, PENG Hao 彭浩, the text as a whole was written no later than 186 BCE, and parts of its contents can be traced back as far as the Warring-states period (475 – 221 BCE).[81]

The *Suan Shu Shu* is comprised of 190 bamboo strips. In all there are 69 sections or headings, each of which is devoted to a mathematical question or rule.[82] The main contents deal with basic operations of natural numbers and fractions, computations involving different ratios, the rule of three, and calculations of areas or volumes of physical objects of different shapes.[83] There are 16 sections in particular that deal with questions of plane and solid geometry, but none conveys the concept of geometrical magnitude.

Slightly later than the *Suan Shu Shu* is a largely astronomical text, the *Zhoubi Suanjing*. Although this has long been considered one of the ten Chinese mathematical

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be couched almost exclusively in the language of Euclidean geometry. For Needham's account, see Needham 1959, 91-95.

[80] The three Chinese characters, 算數書, appear on the *verso* of the sixth bamboo strip. For photographs of the bamboo strips and their transcriptions, see Zhangjiashan 2000; 2001, 83-96, 249-272; and PENG 2001. English versions of the *Suan Shu Shu* have been prepared, independently, by Joseph W. Dauben and by Christopher Cullen. Dauben's version will appear in the *Archive for History of Exact Sciences*. Cullen's translation is entitled "The *Suan shu shu* 算數書'*Writings on Reckoning*'," which was printed and circulated to a small group as Needham Research Institute Working Paper 1, 2004. The electronic version of Cullen's manuscript is available on the website of the Needham Research Institute: http://www.nri.org.uk. The English translation used in this thesis for the title of this work, "*A Book on Numbers and Computation*," is borrowed from Dauben, forthcoming.

[81] PENG 2000, 85-86; 2001, 4-6.

[82] ZJSHMZJ 2001, 249.

[83] For detailed descriptions of the contents of the *Suan Shu Shu*, see PENG 2000; 2001, 12-25; and Dauben in press.

classics, the *Zhoubi* actually contains very little mathematics, of which operations of natural numbers and fractions are a major component, along with material on triangles and the *gougu* 句股 relation (the Chinese equivalent to the Pythagorean theorem; the *Zhoubi* even includes a proof of the *gougu* relation that holds for right triangles in general). Nevertheless, as with the *Suan Shu Shu*, there is no evidence of any concern for a concept of magnitude in the *Zhoubi*.[84]

Among the ten mathematical classics, the one following almost immediately after the *Zhoubi* chronologically is the *Jiuzhang Suanshu* 九章算術(*Nine Chapters on Mathematical Procedures*). Like the *Suan Shu Shu*, the *Jiuzhang* is also a collection of questions and answers. As its title indicates, this work is composed of nine chapters: *Fangtian* 方田, *Sumi* 粟米, *Cuifen* 衰分, *Shaoguang* 少廣, *Shanggong* 商功, *Junshu* 均輸, *Ying buzu* 盈不足, *Fangcheng* 方程, and *Gougu* 句股. In all there are 246 problems, the general form of which begins with a question, followed by the answer(s), after which there is usually a description of method(s) or rule(s) by means of which the problem may be solved. Questions are loosely grouped together under the same method or rule. The methods presented or used in the later part of the text depend on, more or less, the previous ones, and they generally increase in complexity or difficulty. But this is by no means to suggest that the *Jiuzhang* has a deductive logical system similar to that in Euclid's *Elements*. The contents of the *Jiuzhang* that may be categorized as geometry are found primarily in four chapters: *Fangtian*, *Shaoguang*, *Shanggong*, and *Gougu*. The first three of these provide correct formulas for calculating areas of farm lands in different shapes, and volumes of certain constructions, including silos and warehouses.

[84] The *Zhoubi Suanjing* has recently been translated into English by Christopher Cullen. See Cullen 1996.

The last chapter, *Gougu*, is devoted to solving practical questions, all devoted to finding one side given the other two sides of a right-angled triangle.[85] Nevertheless, in none of these chapters is there any evidence of the concept of geometrical magnitude.

Since the original text of the *Jiuzhang* did not present proofs, or even explanations for the validity of the rules or procedures for finding the correct answers, scholars since the Eastern Han dynasty (22 - 220 CE) have tried to provide their own explanations through commentaries. Among the most important of these was LIU Hui 劉徽, about whom virtually nothing is known, except that he flourished in the mid-third century. From extant commentaries, historians unanimously agree that he was one of the most original, creative, and influential mathematicians in the history of Chinese mathematics.[86] LIU Hui has left some very important commentaries about methods for calculating areas and volumes, among other topics, in the *Jiuzhang*. For instance, in the first chapter, *Fangtian,* he comments at length on a formula for the area of a circle, explaining why the area is equal to the product of half the circle's circumference and half its diameter. Moreover, in the *Shaoguang* chapter, LIU Hui proves that the ratio of the volume of the *yangma* 陽馬(a pyramid with rectangular base and one lateral edge perpendicular to the base) to that of the *bienao* 鱉臑 (a similar kind of pyramid but with a right-triangular base), which together with the *yangma* forms a right prism with a right-triangular base (and in Chinese is termed *qiandu* 塹堵), is always 2 to 1. Whether LIU Hui employed a concept of infinity in these commentaries on the circle and the *yangma*/*bienao* ratio, as

[85] See LI and DU 1987, 75-80; Martzloff 1997, 136; and SHEN, Crossley and LUN 1999, 61-133; 196-250; 254-306; and 458-517.

[86] There is a substantial literature in many languages on Liu Hui's commentaries on the *Jiuzhang Suanshu*. Among the most recent, see GUO 1990; LI Jimin 1990, 1993; SHEN, Crossley, and LUN 1999, and Chemla and GUO 2004.

well as in other places, will be scrutinized in the next chapter. However, there can be no doubt that LIU Hui's proofs are valid in general, although the geometric figures he used to illustrate and facilitate his reasoning such as circles, polygons, cubes, and other solids, all have specific dimensions. Nonetheless, there is no evidence that he had any general concept of geometrical magnitude. His own favorite work, the *Haidao Suanjing* 海島算經, which he wrote as an appendix to the *Jiuzhang*, contains nine practical surveying problems. But here again, none betrays any evidence of a concept of magnitude.[87]

Following LIU Hui, other Chinese mathematicians have made further contributions to geometry by solving both theoretical questions and practical problems, sometimes with ingenious new methods or more accurate computations. For instance:

- ZU Chongzhi 祖沖之 calculated the value of the ratio of the circumference of a circle to its diameter determinate, i.e. the value of *pi,* to an accuracy between 3.1415926 and 3.1415927;
- The correct formula for the volume of a sphere was provided by ZU Chongzhi's son, ZU Geng 祖暅;
- The method of *yanduan* 演段術, which involves the reconfiguration of the parts of a rectangular figure in order to establish a sort of equation, was introduced and then widely applied by mathematicians during the Song dynasties;

[87] For a lengthy introduction and complete English translation of the *Haidao Suanjing*, see SHEN, Crossley, and LUN 1999, 518-559.

- LI Ye 李冶 (1192-1279) studied the complicated relationships among the different sides of a diagram involving a right-angled triangle with either an inscribed or circumscribed circle.

Still, despite these four significant advances in Chinese mathematics, there is no clear and solid evidence among them to support the view that there was any concept of magnitude in Chinese mathematical texts before the Chinese translation of the first six books of Euclid's *Elements* was published in 1607. In fact, the translators of *Euclid*, Matteo Ricci and XU Guangqi, when faced with the problem of finding a Chinese term for "magnitude," could find nothing equivalent in existing Chinese mathematical works and decided to render this concept sometime as "*du* 度," or more frequently, by "*jihe* 幾何."[88] The former was a standard for measuring lengths in Ancient China, including such units of length as *zhang* 丈, and *chi* 尺. But as a concept more or less equivalent to a ruler, "*du*" does not imply the dimensional character of "magnitude," either a one-dimensional magnitude or straight line, let alone the corresponding sense of magnitudes in two or three dimensions. Perhaps Ricci and XU realized the very limited meaning of "*du*," since they relied more heavily on "*jihe*" to express "magnitude" when translating the especially complicated Book V of Euclid's *Elements* for which the concept of "magnitude" was a key. The basic meaning of "*jihe*" is "how many," or "how much." It appears in the texts of the *Suanshu Shu*, *Zhoubi Suanjing*, *Jiuzhang Suanshu*, and subsequently in many other Chinese mathematical texts. When translating Aristotle's *Categories*, the Jesuit Françisco Furtado, one of Ricci's few colleagues in China, along with his co-translator LI

[88] See *TH*, vol. 5, 1164, 1234-1261.

Zhizao 李之藻, borrowed the term "*jihe*" for "magnitude."[89] It was also adopted later for the Chinese translation of the last nine books of the *Elements* by Alexander Wylie and LI Shanlan 李善蘭 in the early 1850s. By the twentieth century, when Chinese students began to go abroad to study mathematics, especially in Europe and North America, they soon realized that "*jihe*" is not a good translation for "magnitude" at all, and accordingly preferred "*liang* 量," the basic meaning of which is "to measure." The translation "*liang*" for "magnitude" later was adopted by the national Committee for Chinese Scientific Terminology in the 1950s, and has become the standard ever since.

In light of the above discussion it is difficult to maintain that traditional Chinese mathematics had any concept of geometrical magnitude. Therefore, using this concept to interpret certain fragments from the *Mo Jing* and the *tian xia* section in *Zhuang Zi*, is not only anachronistic but highly misleading.

In addition to analysis of those works already presented, there are several commentaries on pertinent fragments in the *Mo Jing* and *Zhuang Zi* written before the Chinese translation of the first six books of Euclid's *Elements* that are also revealing. SIMA Biao 司馬彪 (? – ca. 306), a prominent historian of his time, commented on the first thesis in the *tian xia* section of the *Zhuang Zi*, namely: "The greatest is nothing outside itself, which is called *da yi* 大一; the smallest is nothing inside itself, which is called *xiao yi* 小一 ," as follows:

> If there is no outside, then there is no such thing that can be called *yi* (one); If there is no inside, then there is nothing that can be divided. Therefore, name [a

[89] The title of the Chinese translation of the *Categories* is *Mingli Tan* 名理探. The translation is based on the section *Isagoge Porphyrii*, which is taken from the Coimbra commentary, *Universam Dialecticam Aristotelis*, of 1606. The Chinese translation consists of ten *juan* 卷 (sections), the first five of which were published in 1636, the rest between 1637 and 1641. See Albert Chan 2002, 283-284. For a discussion of the Chinese translation, see CAO 1982; and Wardy 2000, 69-87.

> situation like this as] *yi*. [A thing's] bigness and smallness in the world are not determined by their shapes. One or Two are not their true names. The ultimate shape is shapeless, and the ultimate name is nameless.
>
> 無外不可一，無内不可分，故謂之一也。天下所謂大、小皆非形，所謂一、二非至名也。至形無形，至名無名。[90]

Clearly, this commentary has nothing to do with mathematical magnitude, infinite or infinitesimal. The essence of this commentary is to say that the bigness or smallness of a thing, tangible or intangible, is not determined by its actual shape, and there is a limitation of language for describing things in the world.

LIN Xiyi 林希逸, a scholar of the Northern Song dynasty, also commented on the fragment in question. He says:

> The greatest has nothing outside itself and is the grand emptiness; The smallest has nothing inside itself and is something like a speck of dust in Fall. The eight Chinese characters (which refer to the original texts for "the greatest has nothing outside itself; and the smallest has nothing inside itself") are the same as those in *Zhuang Zi*. But naming them as *da yi* and *xiao yi*, respectively, ignites debate and different interpretations. If called *yi,* then there should be no concern for big or small. Differentiating big or small among *yi* is [exactly like finding] the difference among the same, or the same among the difference.
>
> 至大無外，太虛也；至小無內，秋毫之類也。此八字自与《莊子》所說同。但謂之「大一」、「小一」，便生辯說之端，謂之「一」則無大小矣。於「一」之中，又分大小，便是同中之異，異中之同也。[91]

LIN's interpretation quoted above places its emphasis upon change and oneness or unity, the "*yi*." Although his concern differs from that of SIMA Biao, it is clear that LIN was not interpreting the original text as devoted to discussion of the infinite, or as providing any sort of definitions for the infinite or infinitesimal.

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[90] The Chinese text is quoted from *SBBY* vol. 132, 10: 20b.

[91] LIN Xiyi 1997, 507.

As for the second thesis in the *tian xia* section of the *Zhuang Zi*: "Thicklessness (*wuhou*) cannot be accumulated, yet its bigness may reach a thousand miles," SIMA Biao also provides the following note:

> A thing is said to exist (*you* 有) because of its shape, and outside the shape is shapeless (*wu* 無). The shapeless and the shape coexist, depend on each other, and cannot be separated. Therefore, the thickness of a [tangible] thing is exhausted at thicklessnesses (*wuhou*), which is, together with thickness, in the same body. The thickness of the thing has size, so does its thicklessnesses. Height is based on length, and thickness is an accumulation of thicklessnesses. Therefore, accumulation of thickness depends on non-accumulation of thicklessnesses. If the thing is accumulatable, is its size limited to one thousand miles?
>
> 物言形為有，形之外為無，無形與有相為表裏。故形物之厚，盡於無厚。無厚與有同一體也。其有厚大者，其無厚亦大。高因廣立，有因無積，則其可積因不可積者。苟其可積，何但千里乎？[92]

Here SIMA Biao uses a pair of philosophical terms, "*you* 有" and "*wu* 無," upon which his interpretation rests. What the exact meanings of these two terms is for SIMA Biao, and how the two are related, is still an open question among historians of Chinese philosophy and ideas. But here one thing is certain: neither has anything to do with the concept of magnitude.

YANG Jing 楊倞, a scholar of the Tang dynasty, also left some thoughts on the fragment in question while commenting on the *Xun Zi*, a classic attributed to XUN Zi, who was, and still is, famous for his theory of that human's nature is evil inherently and yet it can be corrected by good education, right ritual and strict regulations. YANG's comments read:

> *Wuhou* is an extremity of *hou* when it reaches a point where it cannot have any thickness. Non-accumulation means that accumulation reaches a point where there is no more accumulation. Non-accumulation of *wuhou* is the result of accumulation of *youhou*. Therefore, its size could be as big as one thousand miles, which is just a metaphor for very large things.
>
> 無厚谓厚之极不可为厚薄也。不可積言其委積至多不可复積也。凡無厚不可積，因于有厚可積，故得其大千里。千里者，舉大之極也。[93]

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[92] See *Zhuang Zi, SBBY* vol. 132, 10: 20b.

Once again, these comments do not provide any helpful insights related to understanding the exact meanings of *wuhou* and *youhou*, and their relation, indeed, serves to disprove such current interpretations as: the accumulation of points cannot generate a line; or the accumulation of lines cannot generate an area.

There are numerous commentaries made before the seventeenth century on the often-quoted thesis: "A stick one foot long, if one takes away a half every day, will not be exhausted for ten-thousand generations." For instance, SIMA Biao, mentioned above, wrote:

> *Chui* 棰 is a cane. If it can be divided, then there are always two parts. If it cannot be divided, then the one part stands forever. Therefore [the thesis] says it "will not be exhausted for ten-thousand generations."
>
> 棰，杖也。若其可析，則常有兩；若其不可析，其一常存。故曰萬世不竭。[94]

HONG Mai 洪邁 (1123-1202), a scholar of the Southern Song dynasty, in his famous book, *Rongzhai Suibi* 容齋隨筆 (*Jottings from the Studio of Rongzhai*), provides a different interpretation:

> [The thesis], though, is a fable, but contains a certain truth. This is because [one] takes only half of [the stick], and despite breaking the half into dust, the other half exists still. [Therefore,] it can be said that [the stick] cannot be exhausted.
>
> 雖為寓言，此理固具，蓋但取其半，雖碎為塵埃，余半猶存。謂無窮可也。[95]

The above commentaries on fragments in the *Mo Jing* and *Zhuang Zi*, provided by Chinese scholars prior to the first introduction of Western mathematics, may not capture the true meaning of the original texts. The commentaries, nevertheless, provide

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[93] YANG Jing 1977, 45.

[94] *Zhuang Zi*, *SBBY* vol. 132, 10: 23.

[95] HONG Mai, *juan* 9, the section on "*Chichui quban.*"

meaningful historical evidence to show that using Euclidean geometry or the paradoxes of Zeno to interpret these fragments is ill founded. The evidence provided above, together with this brief sketch of Chinese mathematical texts before 1607, should suffice to reject those modern interpretations which hold that the *Mo Jing* and *Zhuang Zi* both include rich concepts of mathematical infinity. One question, however, that has yet to be addressed, is how these fragments from the two classic texts should be interpreted?

2.3 A New Interpretation of the Fragments in the *Zhuang Zi* and *Mo Jing*

LU Sheng 魯勝, a contemporary of SIMA Biao, once edited and commented on the *Mo Jing*, which he called the *Mo Bian* 墨辯 (*The Deputation among the Moists*). Unfortunately, neither the edition by LU nor his commentary survived as late as the mid-seventh century. But at best the preface he wrote was quoted in the short biography that appears for him in the *Jin Shu* 晉書 (*History of the Jin Dynasty*), and this survives today.[96] According to this preface, the main point of the *Mo Bian* is the subject of *ming* 名 (name). Part of LU Sheng's preface reads as follows:

> [Anything that has a] name must have [a tangible] shape. There is no better way of observing shapes than differentiating colors. Therefore, there was a debate/dispute over hardness and whiteness (堅白之辯). Things having the same name must be differentiated among themselves. There is no better way of separating them than [by discussing] *you* 有 and *wu* 無. Accordingly there was a debate over *wuhou* 無厚. [One thing can be both] yes and no, right and wrong, and this is named right in both cases (兩可). The same things have something different, and the different things have something in common, and this is called disputing similarities and differences. The ultimate communality is nothing but the same, and the ultimate difference is nothing but the different, and this is a disagreement/argument/dispute over what is the same and the different. Things that are the same and (things that) differ stir up [different views of] what is right and wrong, which in turn causes good fortune or disaster. Debate/disagreement

[96] See *Jin Shu*, vol. 94, biography 64, under the heading "Recluse."

> over one thing eventually leads to the origins of the generation or decay of everything in the world, which is ultimately a name (*ming*).
>
> 名必有形，察 [形] 莫如別色，故有堅白之辯。名必有分，明分（明）莫如有無，故有無（序）[厚]之辯。是有不是，可有不可，是名兩可。同而有異，異而有同，是之謂辯同異。至同無不同，至異無不異，是謂辯同、辯異。同異生是非，是非生吉凶。取辯於一物而原极天下之污隆，名之至也。[97]

According to the preface, the debates over hardness and whiteness and over *you* 有 and *wu* 無 were directly related to the fundamental meanings of *ming*. Though mentioning that hardness and whiteness have something to do with the shape and color of a physical object, and that *you* and *wu* relate to the actual size of the object, the preface does not provide details about the reasoning of the actual argumentations. Other classics such as the *Zhuang Zi*, *Xun Zi,* and the *Spring and Autumn of Mr. LÜ* also mention the two debates in passing, but again, provide no further explanations.[98] Some scholars hold that the debate over hardness and whiteness (堅白論) in the *Gongsun Long Zi* 公孫龍子 (*The Book of GONGSUN Long*) should, in essence, address the same issues as the debate over hardness and whiteness with which the author(s) of the *Mo Jing* as well as HUI Shi and GONGSUN Long were all familiar. Nevertheless, this is problematic because the *Gongsun Long Zi* may not be the work of GONGSUN Long himself, but a forgery of the early fourth century.[99] Accordingly, the reasoning applied in the debate over hardness

[97] Quoted from SKQS, vol. 256, 524, with modifications and punctuation borrowed from HU Daojing 1934, 82. The original text omitted the character "*xing* 形" immediately after "*cha* 察," and "*xu* 序," most likely, is a mistake for "*hou* 厚," because the other extant classics mention the debate on "*wuhou* 無厚," but never "*wuxu* 無序." See A. C. Graham 1978, 175, and note 53 of this chapter. For an alternative English translation of this passage, see again A. C. Graham 1978, 175.

[98] See GUO Zhanbo 1932, 26-31; and HU Daojing 1934, 75-84.

[99] For an overview of debate about the authenticity of the *Gongsun Long Zi*, see FENG Yaoming 2000, 8-24. For an English account of the life and works of GONGSUN Long, see Perleberg [1952] 1973.

and whiteness may be different from that related to the thesis of the hardness and whiteness.[100]

SHEN Youding 沈有鼎 (1908-1989), the late Harvard-educated logician and historian of Chinese logic, proposed that the debate over hardness and whiteness concerned the composition of physical objects. Specifically, he thought that the debate was about whether or not the two distinct characters of an object co-exist, or exist independently or separately, in the smallest unit of an object. For instance, given a hard, white stone, it must be comprised of the smallest substances which have the characteristic features of hardness and whiteness, respectively. Are the two smallest parts of the stone combined one over the other, like layers of meat and vegetable? Or do the smallest parts of the stone have both hardness and whiteness, like a magnetic iron bar which exhibits both a positive and a negative pole, regardless of how small a part of it may be? If divided repeatedly, even to a very small part, it will still have north-seeking and south-seeking poles. According to SHEN, the "*li jianbai* 離堅白" (the isolation of hardness and whiteness) argues for the latter, and he holds that the hardness of the stone felt by one's hand is only the result of an accumulation of those smallest parts which have hardness, and that the whiteness of the stone seen by one's eyes is produced by those smallest parts which have whiteness only, nothing else. Although the whiteness and the hardness look as if they are mingled together and occupy the same space, in fact they take up different room: what one sees is a white stone, and what one touches is a hard stone.[101]

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[100] See A. C. Graham 1978, 170-176.

[101] SHEN Youding 1992, 479-482.

What then is the debate over *wuhou* all about? The extant documents provide no explanation, but indicate that DENG Xi 鄧析, a contemporary of LAO Zi, once wrote something about this matter.[102] Among the surviving treatises attributed to him only two chapters: *wu hou* 無厚 (No benevolence) and *zhuan ci* 轉辭 (Turning words), are now extant. The heading of the former reads exactly the same as the debate at issue here. But DENG Xi is concerned with causes and consequences of "no benevolence," and what he has to say has nothing to do with any debate. Taking a clue from LU Sheng's preface quoted above, as well as inspiration from SHEN's explanation about the debate on hardness and whiteness, the debate on *wuhou* may well relate to philosophical considerations regarding the composition of physical objects.

It is quite natural to suppose that the debate on *wuhou* was inspired by philosophical speculations on the origin of the universe. As briefly discussed in the first section of this chapter, the *Lao Zi* holds that *wu* and *you* are two complementary aspects of the Dao, which is ultimately responsible for the origins of everything in the universe, and that the former represents its subtleties, and the latter its tangible results. But due to its vague description of the relation between *you* and *wu* in the *Lao Zi*, and moreover because of the later trend of emphasizing the more fundamental role of *wu*, the Daoists during the Warring States period (479-221 BCE) may have pondered such questions as: Are subtleties themselves tangible things? If they are, then *wu* is not that much different than *you*; If they

[102] For instance, the above mentioned preface to the *Mo Bian* by LU Sheng; and the *Hanfei Zi* 韓非子, wengbian 問辯 (Question and Debate) section, vol. 41.

are not, then how do they interact with tangible things? Does a tangible thing dissolve and eventually turn into nothing? Or does it just change into another form, a process that goes on forever? Or does it not change at all? Most likely, "*wuhou*" is a term which refers to debate on such questions among scholars of that time, one of whom was the above-mentioned DENG Xi.

The focal point of the debate over *wuhou* may thus have been on divergent views of *wuhou* and *youhou*, as well as their relations. The Moist Canon I, 55, provides a definition of *youhou* which reads: "厚，有所大也."[103] This sentence may be translated as: "Thickness is something which has a certain size." But the explanation given for this canon: "厚，惟無所大" is vague, and may be interpreted in different ways.[104] This could mean the size of *hou* 厚 is *only* in contrast with *wu*, or as TAN Jiefu, an authority on the *Mo Jing*, has suggested that the definition was meant to be explained in terms of the opposite point of view, i.e. only those objects without thickness can be called *wuhou*.[105] The extant version of the *Mo Jing* does not provide a definition for *wuhou*, nor do other Chinese classics before the Qin dynasty.

The term "*wuhou*" may have at least two different meanings, which are closely relate to different views of *wu*. If "*wu*" is taken to have the same meaning in the *Lao Zi*, i.e., it represents the subtleties of a physical object, then *wuhou* may refer to things without dimension, to the smallest parts of an object. If "*wu*" is

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[103] Quoted from TAN 1964, 137.

[104] *Ibid.,* 138.

[105] *Ibid.*

taken to mean absolutely nothing, or null, then *wuhou* may be taken to refer to the opposite status of a tangible object which has a certain size. Accordingly, it is impossible to form a tangible object from *wuhou* because absolute nothingness cannot generate a tangible object. By the same token, an object with thickness cannot be turned eventually into *wuhou*, no matter how it may be diminished because a tangible object cannot change into a non-tangible object.

If *wuhou* means something subtle, insubstantial, how can it form a tangible object? Since subtleties are formless, a new concept may have been needed in order to provide an explanation to this question. A possible candidate is *duan* 端, which is defined by the Moist Canon I, 61, as: The foremost *wuhou* which precedes all others (端，體之無厚而最前者也).[106] The *duan* cannot be interpreted as a geometrical point (for reasons why not, see the previous section), but it may be taken as the extremity of a tangible object. Although extremities do not have a definite shape, one contained in a specific object nevertheless does. The *duan*, therefore, is a *wuhou* which has a certain form. With this understanding of *duan* and *wuhou* in mind, it is now possible to reinterpret the quoted Moist Canon II, 60, and its explanation (p. 24), as well as thesis II and thesis twenty-one in the *Zhuang Zi*.

Thesis twenty-one says: "[Given] a stick one foot long, if one takes away a half every day, it will not be exhausted for ten-thousand generations." If this thesis is meant to argue that any tangible object is self-generated, and is *not* from absolute nothing, then the example of a one-foot-long stick is meant to illustrate

[106] See note 54.

the point that a tangible object will be tangible forever. With this in mind, it is interesting to note that after translating (with Matteo Ricci) a postulate added to Euclid's *Elements* by Christopher Clavius: "Magnitudes can be increased and they can also be decreased. As far as increasing is concerned: there can be added to without limit. As far as decreasing is concern[ed]: if you subtract from them, neither do you reach a limit,"[107] XU Guangqi provided the following commentary:

> I have read in Zhuangzi that he says about a stick of 1 foot that if every day half of it is taken away it will not be exhausted within 10,000 generations. That is the same principle. Why? Starting with something and dividing you will always be left with something. If you could exhaust it by diminishing it that would mean that something would have changed into nothing. Changing something into nothing can still be said. [But now] let what has been divided be put together again. Keep putting it together till it is again a one-foot stick. That would mean that the two "nothings" that were there before you started reassembling were capable of forming together one something. Two nothings together forming one something cannot be said [is unimaginalbe].[108]

The above passage makes clear that XU Guangqi's reasoning does not include the divisibility of magnitude as a geometrical postulate, and that it is logically insufficient to conclude that thesis twenty-one is "the same in principle" as the divisibility of magnitude. His arguments, however, do provide some support for the interpretation on the possible meaning of thesis twenty-one, namely: a physical object cannot be generated from nothing.

Thesis twenty-one may also be interpreted from the relation between *youhou* and *wuhou*. Suppose there are two physical objects of the same sort. Connect them at any

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[107] Quoted from Engelfriet 1998, 170. Christopher Clavius's concept of magnitude has nothing to do with any physical object; it can be added, subtracted or divided, *ad infinitum*. This is contrary to Aristotle, who rejects the very idea, based on the premise that nothing can be greater than the finite universe. See Aristotle's *Physics* III, 7. $207^{a}33$-$^{b}34$; and Heath 1949, 110.

two ends. Doing so, a larger object is formed. Repeating this process, a much larger object can be generated. Accordingly, it can be argued that any physical object with a certain thickness may be formed by *duan*, namely, *wuhou* of some sort.[109] But the question that naturally arises is: Can anything be added by connecting the two *duan* or ends? If there is, then where does it come from? If not, what is the consequence of reversing the process? Thesis twenty-one very likely is an argument for the latter question. If two pieces of a rod added together, can form a larger one without adding anything else, then it is correct to conclude that dividing one piece of the rod into two smaller pieces should result in no loss. Accordingly, dividing a one-foot-long rod into ten-thousand pieces, nothing has been lost.

As for thesis II, it reads as follows: "*wuhou* cannot be accumulated, yet its extent may reach a thousand miles." This thesis suggests that formless *wuhou*, although it cannot exist independently, depends for its size upon a specific object which has certain thickness. The commentary on this thesis by SIMA Biao (quoted earlier, page 48) may be very close to its original meaning.

Finally, how should the Moist Canon II, 60, be interpreted, along with its explanation? The Canon says: "If bisecting something [solid] again and again, the process will stop at some point. This is because of *duan*." The "Explanation" provides two different ways of bisecting the solid in question: "If bisecting something [solid]

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[108] This English translation is due to Peter Engelfriet. Quoted from Engelfriet 1998, 170-171. For the original text, see TH vol. 5, 1164.

[109] The *Moist Canon* includes the following text and explanation: "次, 無間而不相攖也;" and "次, 無厚而後可," which may be translated as "*Chi* has neither space [between] nor coincidence [over them]," and *Chi* is possible only after *wuhou* is involved," respectively. This canon and its explanation account for how small objects may be comprised from a bigger one. The Chinese texts are quoted from TAN Jiefu 1964, 151-152.

always lies ahead, then the remainder, which may be treated as a sort of *duan*, cannot be further halved after certain moves. If bisecting it ahead and then behind, then the *duan* would be in the center. If every cut must halve [the solid], then the cutting cannot go on [forever]," (see page 32). This canon and explanation must have something to do with thesis twenty-one.[110] In connection with the two different explanations provided above on thesis twenty-one, Canon II, 60 and its Explanation may also be interpreted in two different ways. The author of this Canon may have acknowledged that a physical object, say, a foot-rod, cannot be exhausted by halving, but that this only happens in thought or in theory; in actuality, the halving, no matter if it is "always ahead" or "ahead and then behind," must cease. This is the first interpretation. The second may go like this: Since a physical object is formed by connecting the ends of smaller ones, then dividing or cutting this object cannot go on *ad infinitum* because the shape of the object is getting smaller and smaller, and eventually the smallest parts which only have ends will be reached. Given that the Moists emphasized practical applications, these interpretations fit well into their philosophy.

2.4 Conclusion

In 1930, CHEN Yinke (1890-1969), a leading Chinese historian of the twentieth century, in evaluating volume one by FENG Youlan of the soon-to-be classic *History of Chinese*

[110] Interpretations of the relation between the Moist Canon II, 60, its explanation and thesis twenty-one depend on different views regarding such questions as: What is the relationship among HUI Shi, the dialectician GONGSUN Long, and the author(s) of the *Mo Jing*? Which came first: The Moist canons and their explanations or the ten theses and the twenty-one paradoxes in the *Tian Xia* section (Under the Heaven) of the *Zhuang Zi*? Modern scholars, especially those who study the history of Chinese logic, Chinese intellectual history, and Chinese classics, have discussed and debated these questions. Their discussions have in turn generated more complicated questions at length, with diverse conclusions, but with

Philosophy, wrote: "Current studies of the *Mo Jing* cannot find support [for their arguments or explanations] from any documents or texts surviving from ancient times. Adding or removing words [in the original texts] by one's sparkling wit is very much like an able magician who turns a thing into a donkey or a chicken with a magic spell."[111] Though hypercritical in tone, CHEN's words, nevertheless, rightly describe the severely corrupted nature of the extant texts of the *Mo Jing*, and the lack of related historical evidence in the pre-Qin classics. Much more literature on the *Mo Jing* has been published over the past seventy years, but the situation described by CHEN remains true.[112]

Some historical studies of Chinese science use Western science to explain and analyze Chinese texts, and try to discover their modern meanings as well as links to modern science and technology. But this approach makes the fatal mistake of using concepts, methods, and theories that are foreign to Chinese indigenous ones. More than one hundred years ago, the renowned British sinologist James Legge (1815-1897) noted the defectiveness of this approach, as he explains with wit and humor:

> Chinese scholars and gentlemen who have got some acquaintance with western science are fond of saying that all the truths of electricity, light, heat, and other branches of European physics are in the eight trigrams. When asked how then they and their countrymen have been and are ignorant of those truths, they say that they have to learn them first from western books, and then, looking into the *I* [The Book of Change], they see that they were all known to Confucius more than two thousand years ago. The vain assumption thus manifested is childish, and until the Chinese drop their hallucination about the *I* as containing all things that

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no conclusive end in sight. For an outline of the historiography related to these issues, see LI, ZHOU, and ZHOU 1989, 225-278.

[111] See note 2.

[112] D. C. Lau in the early 1950s also echoed CHEN Yinke's view, and said that the texts of the extant *Mo Jing* are beyond restoration. See Lau 1952/3, 189.

> have ever been dreamt of in all philosophies, it will prove a stumbling-block to them, and keep them from entering upon the true path of science.[113]

The fragments of the *Mo Jing* and *Zhuang Zi* discussed in this chapter concern the relation between *youhou* and *wuhou* in the context of *zheng ming* 正名 (rectifying names) in the Pre-Qin periods.[114] Those fragments are basically about what physical objects are made of, and whether a tangible object could be annihilated, or something created from an intangible object, among others. These are philosophical speculations, and not even scientific paradoxes, as Jean-Paul Reding has pointed out.[115] The fragments, though, do bear a similarity to Western texts, but it does not follow that they embody the same concepts of mathematical infinity. This is because the ancient Chinese texts, and indeed before Western mathematics was first introduced into China at the beginning of the seventeenth century, no Chinese mathematical texts contain a concept of "magnitude." Without a concept of magnitude, there is no way to conceptualize such concepts as the infinite or infinitesimal in a mathematically meaningful way.

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[113] Quoted from Needham 1956, 335-336.

[114] For a discussion of the *zheng ming* 正名 in the Pre-Qin periods, see Makeham 1994, 51-65.

[115] Jean-Paul Reding provides a detailed account, arguing that HUI Shi's dicta are not scientific paradoxes at all. For his arguments as well as alternative interpretations of the fragments in the *Zhuang Zi*, see Reding 1985, 274-385.

Chapter 3 Concepts of Infinity in the *Nine Chapters* and Liu Hui's Commentary

> Mathematicians do not need or make use of it [the infinite]; they only require that the finite straight line shall be as long as *they please*, and that, given a ratio in which the greatest magnitude is cut, another magnitude of any size whatever can be cut in the same ratio.
>
> -Aristotle[1]

> Many mathematical traditions confronted the computational anomaly posed by such quantities as $\sqrt{2}$; there was little problem in producing approximations which would satisfy any given context. But the Greeks advanced to the assertion that such quantities were irrational. This is a statement of an entirely different theoretical order. Its justification can be based only upon logical deduction, and a theory of such quantities can have but one criterion of validity: logical consistency.
>
> -Wilbur Richard Knorr[2]

The extant Chinese mathematical texts, either as transmitted from antiquity or recently unearthed, were all compiled after the Qin dynasty. The *Suan Shu Shu*, newly brought to light from a Han tomb and written on bamboo strips sometime before 186 BCE, is believed to be the oldest one. A careful check of all of its sixty-eight problems or sections reveals no relevant contents related to the concept of infinity. The same can also be said of the *Zhoubi Suanjing*, which traditionally has been held to be the earliest among the ten Chinese mathematical classics. Accordingly, this chapter focuses mainly on the next mathematical work in chronological order, namely the *Nine Chapters*, and especially its commentary by LIU Hiu. Specifically, we shall investigate whether or not the *Nine Chapters*, and/or LIU Hui, introduces the concept of infinity, or as some have claimed,

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[1] Aristotle, *Physics* III, 7: 207^{b} 30-^{b}34. Quoted from Heath 1949, 111.

[2] Knorr 1975, 4.

the allied concept of irrational number; we shall also discuss to what extent Liu Hui may have applied concepts of infinitesimals and limits, if any.

3.1 Does the *Nine Chapters* Include the Concept of Irrational Number?

In ancient Greek mathematics, the concept of irrational number is associated with incommensurable magnitude. Historians of mathematics agree that it was the Pythagoreans who first discovered incommensurability, but have divergent views on how they made this unexpected discovery. Whether the Pythagoreans conceived of incommensurability while studying anthyphairesis of lines in mean and extreme ratio, as argued by Kurt von Fritz and by S. Heller, or whether it was discovered as an accidental by-product of arithmetic study of the side and diameter of a square, as recorded by Aristotle and reiterated recently by Wilbur Knorr, it is clear that the concept of geometrical magnitude, the Pythagorean theorem, and the Pythagoreans' doctrine that natural numbers or their ratios suffice to express all things, physical and mathematical, are the three major foundations on which the Greeks' discovery of incommensurability depended.[3] Whether or not this discovery caused a crisis and eventually led to a *revolution* in Greek mathematics is debatable, but it is safe to say that the discovery was one of the major achievements of Greek mathematics.[4]

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[3] Kurt von Fritz argues that Hippasus of Metapontum was the first to encounter incommensurability in considering the construction of the regular pentagon, see von Fritz 1945. This view was supported by S. Heller (Heller 1956), who elsewhere also argued that incommensurability arose from an application of anthyphairesis to the side and diameter of a square (Heller 1958). For Aristotle's account of the incommensurability of the side and diagonal of a square, see *Prior Analytics* $41^{a}29$. For Wilbur Knorr's discussion, see Knorr 1975, 29-36, 298-303.

[4] See Knorr 1975, 36-49; 306-313; Knorr 1982, 112-145; and Dauben 1984, 84-89.

Traditionally, historians of Chinese mathematics have never claimed that ancient Chinese mathematics included any notion of irrational numbers. However, in 1986 a prominent Chinese scholar, LI Jimin, argued that in the *Nine Chapters* and its commentary by LIU Hui, there was indeed the notion of irrational numbers.[5] Subsequently, LI's view has found some support among historians, for instance, from Karine Chemla.[6] Both LI's and Chemla's arguments are mainly based on the *kaifang shu* 開方術 (method for finding the side of a square), *kailifang shu* 開立方術 (method for finding the side of a cube), and *kailiyuan shu* 開立圓術 (method for finding the diameter of a sphere), as well as their related commentaries by LIU Hui in the *Shaoguang* chapter of the *Nine Chapters*. In order to facilitate analysis of LI's arguments, as well as those of other historians, we first quote the original texts, including LIU Hui's commentary on *kaifang* 開方, as follows:

> **開方**求方冪之一面也
> **術曰：置積為實，借一算，步之超一(位)[等]。**言百之面十也，言萬之面百也。
> **議所得，以一乘所借一算為法，而以除。**先得黃甲之面，上下相命，是自乘而除也。
> **除已，倍法為定法，**倍之者，豫張兩面朱冪，定貿以待復除，故曰：定法。
> **其復除，折法而下。**欲除朱冪者，本當副置所得(成)[乘]方，倍之為定法。以折、議、乘，而以除。如是當復步之而止，乃得相命，故使就上折下。
> **復置借算，步之如初，以復議一乘之，**欲除朱冪之角黃乙之冪，其意如初之所得也。
> **所得副以加定法，以除。以所得副從定法，**再以黃乙之面加定法，是則張兩青冪之袤。
> **復除，折下如前。若開之不盡者，為不可開，當以面命之。**
> 術或有以借算加定法而命分者。雖粗相近，不可用也。凡開積為方，方自乘當還復(有)[其]積分。令不加借算而命分，則常微少；其加借算而命分，則又微多，其數不可得而定。故惟以面命之，為不失耳。譬猶以三除十，以其餘為三分之一，而復其數可以舉。不以面命之，加定法如前，求其微數。微數無名者以為分子，其一退

[5] LI Jimin 1989; [1990]1992, 105-110. The basic view that irrational numbers occur explicitly in the *Nine Chapters* and its commentary by LIU Hui, however, was expressed by LI Jimin as early as 1986. See *ZGSXJS* 1986, 153-154.

[6] Chemla 1992; Chemla and Keller 2002, 102-129.

以十為母，其再退以百為母。退之彌下，其分彌細，則朱冪雖有所(乘)[剩]之數，不足言之也。

若實有分者，通分內子為定實，乃開之。訖，開其母，報除。若母不可開者，又以母乘定實，乃開之。訖，令如母而一。[7]

Here the character "開" literarily means "open," and "方" means "square." Together they convey the meaning of "open a square." Perhaps this seemed unclear or imprecise to LIU Hui, who added a note explaining that "*kaifang*" meant: 求方冪之一面也 (Find the side of a square area). What then follows in the original text is the detailed description of the method for *kaifang* or square root extraction, to which a substantial literature has subsequently been devoted.[8] Above all, it should be pointed out that the method, at least according to LIU Hui's commentary, is based on a geometric process, namely cutting up a square into smaller squares and gnomons (see Figure 1).

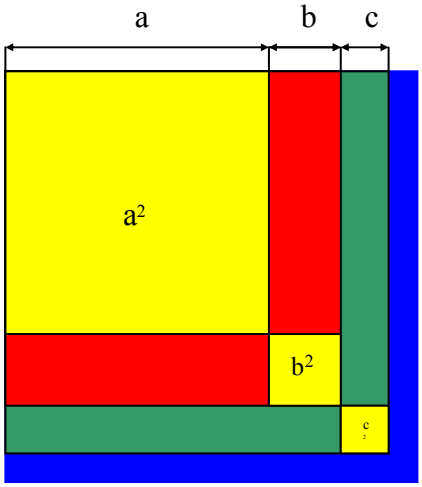

Figure 1. *The Procedures of Kaifan*

[7] Quoted from *TH* 1, 131a & b, with punctuation added. The *Nine Chapters* in *TH* is a photocopied reproduction from the edition of *Wuyingdian juzhen ban* 武英殿聚珍版 (1774 CE). The words in parenthesis are found in the original text, and those in square brackets are emendations, except for: [剩], suggested by QIAN Baocong; see QIAN [1963]1998: 4, 116. The Chinese characters in bold font represent the text of the *Nine Chapters*, those in smaller font are the commentary by LIU Hui.

[8] For explanations of the *kaifang* method as well as that of the commentary by LIU Hui, see MIKAMI [1913]1972, 12-14; WANG and Needham 1955, 350-365; QIAN [1964]1981, 46-49; LI and DU, 1987, 50-55; Martzloff 1997, 224-226; LI Di 1997, 118-121, and SHEN, Crossley and LUN 1999, 204-213.

The crucial phrase with respect to "incommensurability" in the above quoted texts are the words: 若開之不盡者, 為不可開, 當以面命之. Before discussing different interpretations of these words provided by LI Jimin and other historians, it may be helpful to see what "*mian* 面" and "*yi*…*ming*…以…命…" mean, respectively, in other contexts. *Mian* 面 is a technical term that appears once in the *Suan Shu Shu*, Problem *Yifang Caiyuan* 以方材圓 (From a square cut a circle), which reads:

> From a square [piece of] wood cut a circular one, say [one side of] the square is 7 3/5 *cun*, how large is the [circumference of the] circular [piece of] wood? [The answer] says: 42 14/25 *cun*. The method says: one side of (*mian*) the square [piece of] wood is the diameter of the circular [piece of] wood, multiplying it by 4 [and by 7] as the dividend, also dividing by 5 thus yields the result.
> 以方為圜材，曰：方七寸五分寸三，為圜材幾何？曰：四韋二寸二十五分十四。術曰：方材之一面即圜材之徑也，因而四之[七之]以為實，令五而成一。[9]

In this example *mian* clearly means the side of the square piece of wood. In the *gougu yuanfang tuzhu* 句股圓方圖注 (Notes on the Diagrams of the Base and Altitude, Circle and Square) by ZHAO Shuang 趙爽, a contemporary of LIU Hui, we also find the following commentary on the *Zhoubi Suanjing*:[10]

> [D]oubling the hypotenuse area fills the outer large square, [leaving as] excess the yellow area. Now the excess of this yellow area is just the base-altitude

[9] The Chinese text is quoted from ZJSHMZJ 2001, 268. The text is on slips nos. 154 and 155, see ZJSHMZJ 2001, 95. The original order of the two words, 曰材, is reversed here according to GUO Shirong 2001, 284. The insertion of two other words, 七之, is suggested by DUAN and ZOU, 2003, 171, but their placement is moved from before 因而四之. For an alternative English translation as well as detailed comments on current studies of this problem, see Dauben, forthcoming.

[10] About ZHAO Shuang's life, historians know virtually nothing, and are even unsure of when he lived. QIAN Baocong suggests that ZHAO may have lived in the State of Wu during the Three Kingdom Periods, see QIAN [1964]1981, 57. Recently, LI Di has put forward a different view, and suggests that ZHAO may have written his commentary on the *Zhoubi Suanjing* between 190 and 220 CE, see LI Di 1997, 161-162. Both QIAN and LI think that ZHAO lived before LIU Hui. Nevertheless, Christopher Cullen argues that it may have been the other way around, and ZHAO's commentary may have been influenced by LIU Hui's work. For Cullen's detailed arguments, see Cullen 1996, 160-161. Considering all these different views, it may be safe to say that ZHAO Shuang and LIU Hui were more or less contemporary.

> difference area. If you take away the difference area [from double the hypotenuse area], and 'open' the excess, you get the outer large square, [and then] the side of (*mian*) the outer large square, which is the base-altitude sum. Let this sum be multiplied by itself, [take] double the hypotenuse area then subtract [the result] from it, 'open' the excess, and you obtain the central yellow square. The side of (*mian*) this central yellow square is the base-altitude difference. Subtract this difference from the [base-altitude] sum and halve, to make the base. Add the difference to the sum and halve, to make the altitude.
>
> 倍弦實滿外大方而多黃實，黃實之多，即句股差實。以差實減之，開其餘，得外大方。大方之面，即句股并也。令并自乘，倍弦實乃減之，開其餘得中黃方[之面]。黃方之面，即句股差。以差減并而半之為句；加差于并而半之為股。[11]

Here the two *mian* refer to sides of the two squares, which have no specific length. This suggests that *mian* denotes the side of a square in a general sense. In his commentary on the *Nine Chapters*, LIU Hui uses the word, *mian*, many times. Besides referring to the side of a square, *mian* is also used by LIU Hui to represent:

1. Sides of regular polygons such as the hexagon, dodecagon, 24-gons, 1536-gons, and the like;[12]
2. The front or any other face of a regular pyramid;[13]
3. Two sides of a right-angled triangle. The shorter one 句 is named *duan mian* 短面, and the longer one 股 is termed *chang mian* 長面;[14] and
4. The side of a cube.[15]

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[11] The English translation is by Christopher Cullen, and quoted from Cullen 1996, 215-216. For an alternative English translation, see Gillon 1977, 267-269. The original texts are quoted from *TH* 1, 12a, with punctuation and emendations added.

[12] See LIU Hui's commentary on Problem 32 in the *Fangtian* Chapter of the *Nine Chapters*. *TH* 1, 05b-108b. Note that the problems are not numbered in the original texts. Here and throughout this thesis problem numbers are inserted for easy reference.

[13] See LIU Hui's commentary on Problem 34 in the *Fangtian* Chapter of the *Nine Chapters*. *TH* 1, 109b.

[14] See LIU Hui's commentary on Problem 3 in the *Gougu* Chapter of the *Nine Chapters*. *TH* 1, 189b.

[15] See LIU Hui's commentary on *kailifang*, which follows immediately after Problem 22 in the *Shaoguang* Chapter of the *Nine Chapters*. *TH* 1, 133b.

The basic meaning of *mian*, therefore, is *side* of a rectilinear figure. Now let us turn to the expression: "*yi*…*ming*…以…命…." This expression, like the word *mian*, also can be found in the *Suan Shu Shu*. In the Problem *Jingfen* 径分, the expression appears twice. First, in the sentence: 径分, **以**一人**命**其實, where the expression explains the heading and may be translated as "[*Jingfen* is] *taking* one person, *determine* his share." The expression appears for a second time in the phrase: **以命**其實, which in turn may be translated as: "*using* [the product of 5 by 6], *determine* all of its shares."[16]

In the Problems: *Nüzhi* 女織, *Fuzhi* 婦織, *Yushi* 羽矢, *Ruzhi* 挐脂, *Qicong* 啟從, *Shaoguang* 少廣, and *Daguang* 大廣, there are similar words: 不盈寸 [or 步] 者, 以法命分.[17] Here the meaning is: [For those numbers which are] less than one *cun* or one *bu*, *use* the divisor [as the denominator] *to determine/express/denote* a fraction.

The expression: *yi*…*ming*…以…命…, appears several times in the *Nine Chapters*. In addition to its occurrence in the *Shaoguang* Chapter, this terminology can also be found in the *Fangtian* Chapter with respect to the methods for *hefen* 合分 and *pingfen* 平分; in the *Cuifen* Chapter, with respect to the method for *cuifen*; and in the *Ying buzu* Chapter, Problem no. 19. For instance, in the method for *hefen*: 不滿法者, **以**法**命**之.[18] The meaning of this sentence is basically the same as in the *Suan Shu Shu*, namely, "use the divisor to denote a fraction." The expression in the *Ying buzu* Chapter is a little different from those discussed so far. This reads: 不盡者, **以**等數除之而**命**分, which basically means that if there are two integers, say m and n, and m is not an exact multiple of n, suppose $m = p \times n + r$ (r

[16] For a full English translation of the *Jingfen* problem as well as others in the *Suan Shu Shu*, see Dauben, forthcoming. The Chinese texts are quoted from ZJSHMZJ 2001, 252.

[17] See ZJSHMZJ 2001, 254, 256, 259, 269-271.

[18] Quoted from *TH* 1, 100a. The same phrase also appears in the method for *cuifen*, see *TH* 1, 121a.

≠ 0), for the reminder *r*, *use* the greatest common factor to simplify *r* [and *n*] , and then *express* [the result as] a fraction.[19]

Having discussed the meanings of *mian* and the expression, *yi*…*ming*…, we are now ready to examine the crucial sentence: 若開之不盡者, 為不可開, 當以面命之. Historians of Chinese mathematics have interpreted this in many divergent ways. As early as the beginning of the nineteenth century, LI Huang 李潢 (?-1812), a *literati* and once a deputy officer for compiling the *Complete Catalogue of the Library of the Four Divisions* 四庫全書總目協纂官, wrote in his *Jiuzhang Suanshu Xicao Tushuo* 九章算術細草圖說 (The Detailed Explanations on and Supplemented Diagrams to the *Nine Chapters*): [20]

> What the sentence (若開之不盡者，為不可開，當以面命之) means is to use the side [of the square area which has been cut] as the denominator, the remainder area as the numerator. For instance, there is an area in 10 [square] *bu*, cut a square, whose side is 3 *bu*, and take it away, with the remainder area 1 [square] *bu*. Denote the remainder as 1/3 *bu*. Multiply the denominator [by the side opened] and add the product into the numerator, which gives back its original number of area.
>
> "若開之不盡者，為不可開，當以面命之"者，以面為母，實餘為子。譬如，積一十步，開方除之，得三步為面，餘一步，命為三分步之一。通分內子，便還復其積分也。[21]

LI Huang interprets "*mian*" as the side of the square area which has been dissected by following the *kaifang* procedure, and he treats "*zhi* 之" as the remainder area. His explanation of "*yi mian ming zhi*" clearly fits the pattern of the above discussed expression: "*yi*...*ming*…以…命…." Perhaps it was because of all of this that MIKAMI

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[19] Quoted from *TH* 1, 175b. For a full English translation of this problem, see LAM 1993, 33; and SHEN, Crossley and LUN 1999, 378.

[20] The manuscript of the *Detailed Explanations on and Supplemented Diagrams to the Nine Chapters* was finished in 1812, and was published posthumously in 1820. For a brief description of the book, see a short introduction by GUO Shuchun in *TH* 4, 945-946.

[21] Quoted from *TH* 4, 1031, with punctuation added.

Yoshio and QIAN Baocong, both pioneering historians of Chinese mathematics, adopted LI Huang's view. In his influential book entitled *The Development of Mathematics in China and Japan*, MIKAMI translates the Chinese text, 若開之不盡者，為不可開，當以面命之, as follows:

> When evolving, if we don't come to a conclusion, the number cannot be evolved definitely. In such a case, divide the remainder with the evolved root and take the result to the fractional part of the root.[22]

QIAN Baocong also explicitly states:

> Suppose $A = a^2 + r, r > 0$, the meaning of "*yi mian ming zhi*" is to say that the tiny part of the square root A is represented by a fraction whose denominator is a and numerator is r, namely, $\sqrt{A} = a + \frac{r}{a}$.[23]

LI Huang's interpretation, although adopted by these eminent historians, is not above reproach, especially when one considers his further explanation: "Multiply the denominator [by the side opened] and add the product to the numerator which gives back its original *number of the area*." Although, it is possible to work back to the original number from the final result, the operation is not the inverse of *kaifang*, namely, squaring, as LIU Hui points out in his commentary. Moreover, using the remainder over the side of the opened area to express the side of the unopened area yields a result that is actually bigger than it should be. The most serious defect of this interpretation, however, is that it does not work for numbers whose remainders are larger than their "opened" sides. For instance, taking the *kaifang* of 75, according to LI Huang's interpretation, the side of the

[22] MIKAMI [1913]1972, 13.

[23] See QIAN [1957] 1998: 2, 547.

remainder area should be expressed as 11/8; when added to the "opened" side, 8, the result obviously is not even close to what the actual square root should be.

Perhaps the discrepancy between LI Huang's interpretation and LIU Hui's commentary, and the fact that the fractional representation of the remainder over the side of the opened area is not as accurate as historians would have expected, and indeed in certain cases it is totally wrong, prompted LI Yan, another pioneering historian of Chinese mathematics, to take a different view on the meaning of "*yi mian ming zhi*."

One of the sources LI Yan used to support his interpretation is the *Zhuobi Suanjing*, where he found the two words, *you ji* 有奇 (have something more), used to represent the concept of remainder.[24] In his commentary on the summer and winter solstices, ZHAO Shuang explains how the answers 59,598½ and 214,557½, to the original questions in the *Zhoubi Suanjing*, were calculated.[25] For the former, ZHAO's calculations go roughly like this: Take 238,000 as *xian* (hypotenuse), and double 103,000, which yields 206,000. Then take 206,000 as *gu* (altitude). Multiply the altitude by itself, and then subtract the result from the square of the hypotenuse, which results in 14,208,000,000. Taking the *kaifang* of this number yields 119,197 *gou* (base). Halve 119,197, and get the final answer: 59,598½. ZHAO is aware that 119,197 is not the exact result of taking the *kaifang* of 14,208,000,000, but it is the closest whole number one gets by following the *kaifang* procedure. In fact, ZHAO writes: "*de gou 119,197 you ji*

[24] LI Yan [1958]1998: 3, 103.

[25] The Chinese texts can be found in *TH* 1, 30. For an English translation of the original description of the two solstices, see Cullen 1996, 180-181.

得句一十一萬九千一百九十七里有奇."[26] In his commentary on the winter solstice, the same phraseology: "*de 42,9115 you ji* 得四十二萬九千一百一十五里有奇," also appears.[27]

In the *Xiahou Yang Suanjing* 夏侯陽算經, another Chinese mathematical classic, the extant version of which is believed to have been compiled during the Tang dynasty (618 - 907 CE), LI Yan considers the following:

> Now there is a field of 21 *qin* 78 *mu* 180 *bu*. How long is the side of its square? The answer says: 723 *bu* with a remainder of 171 *bu*.
>
> 今有田二十一頃七十八畝一百八十步，問為方幾何？答曰：七百二十三步奇百七十一步。[28]

Here the result of taking the *kaifang* of 52,290 is clearly written as 723 with a remainder of 171.[29] This, as well as ZHAO's commentary discussed above, both involve numbers or areas which cannot be extracted or opened exactly in whole numbers, and indicate whatever is left over either by the words: "*you ji* 有奇" or "*ji* 奇." In view of these examples, LI Yan suggests that "*yi mian ming zhi*" has the same meaning as "*you ji*" or "*ji*."[30] This interpretation avoids using a fraction, and accordingly does not face the problems we have just discussed with LI Huang's understanding. But like LI Huang, MIKAMI, and QIAN, LI Yan holds that "*zhi* 之" in "*yi mian ming zhi*" means "remainder". LI Yan's

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[26] Quoted from *TH* 1, 30a.

[27] *Ibid.,* 30b.

[28] The Chinese text is quoted from *TH* 1, 385a, with punctuation added.

[29] *Qin*, *mu*, and *bu* are Chinese measures for fields. The rates among them are: 1 *qin* = 100 *mu*; 1 *mu* = 240 *bu*. Accordingly, (21 × 100 + 78) × 240 + 180 = 52,290.

[30] LI Yan [1958] 1998, 103.

interpretation still finds support among current experts in the field, for instance, LI Di 李迪.[31]

However some historians, notably, LI Jimin and Karine Chemla, do *not* think that "*zhi* 之" refers to "remainder," but to the original number or square to be extracted or "opened".[32] Moreover, LI Jimin argues that "*mian*" should be interpreted as meaning an irrational number (Karine Chemla, however, does not go this far, but only claims that "*mian*" refers to a quadratic irrational number), and thus, "*yi mian ming zhi*" is taken to refer to irrational number (or for Chemla, a quadratic irrational number) representing an un-extractable root.[33] Both LI's and Chemla's arguments are mainly based on LIU Hui's commentary on the *kaifang shu* 開方術 quoted above, as well as his remarks on the *kai lifang shu* 開立方術 and *kai liyuan shu* 開立圓術.

Since there is at least a two-hundred-year gap between the time when the extant version of the *Nine Chapters* was compiled and when LIU Hui made his commentary, it must be asked whether LIU Hui truly captured the real meaning of "*mian*" in the *Nine Chapters*.[34] Considering the fact that LIU Hui in his commentary often makes original contributions to or ingenious explanations of the methods and terms that occur in the *Nine Chapters*, there is good reason to

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[31] See LI Di 1997,189.

[32] SHEN Kangsheng, and GUO Shuochun also think that "*zhi*" refers to the original number set to be extracted. See SHEN 1997, 228; SHEN, Crossley, and LUN 1999, 204; and GUO 1998, 270.

[33] *ZGSXJS* 1986, 153-154; LI Jimin 1989; [1990]1992, 105-110; Chemla and Keller 2002.

[34] There is no consensus on the dating of the *Nine Chapters*. Some argue it was completed in the first part of the first century before the Common Era, some hold it was complied at the end of the first century of the Common Era, and others believe it was composed at some time in between. For a survey of this issue, see LI Di 1982, 28-34; Chemla and GUO 2004, 41-70.

suspect that LIU Hui also provided his original thoughts on the procedure for determining square roots. This is what LIU Hui actually has to say about this in his commentary:

> Among the methods there is one which uses *jiesuan* plus *dingfa* to determine a fraction. [This method], although it provides a rough approximation, cannot be used. Open any number of an area to find the number of its side; the number of the side multiplied by itself should give back the original number of the area. Suppose *jiesuan* is not used in a denominator which is to denote a fraction, the fraction is always a little smaller; If *jiesuan* is employed, the fraction is then a bit bigger. [In either case] they are not the actual value. Accordingly, only use "*mian*" to represent the number of the area to be cut; it is no more or no less than its actual value. It is like using 3 to divide 10, with the remainder expressed as 1/3. And a return to the original number can be obtained.[35]

LIU Hui here explicitly mentions the two approximation methods for opening a square area. For the sake of clarity, we may use symbols to express the two methods as follows: $\sqrt{N} \approx a + \frac{N - a^2}{2a}$ (*jiesuan* is not used), and $\sqrt{N} \approx a + \frac{N - a^2}{2a + 1}$ (*jiesuan* is employed), where N is the number of the area to be dissected and a is the side of the square area which has been cut, represent the closest approximations to the square root of N. LIU Hui also recognized that the actual value of the square root of N was bounded above and below as follows:

$$a + \frac{N - a^2}{2a + 1} \le \sqrt{N} \le a + \frac{N - a^2}{2a}.$$

The first three words of the above quoted commentary (術或有) indicate that the two approximation methods had already been developed before LIU Hui. In the text of

[35] The *jiesuan* and *dingfa* are two technical terms that occur in the method of *kaifang*. The number 1 is always used for *jiesuan*. But *dingfa* is not a fixed number, and it changes with the number to be extracted, and is equal to twice the number extracted. For detailed explanations of the two terms, see WANG and Needham 1955, 349-365; and SHEN, Crossley, and LUN 1999, 204-210. For an alternative English translation of this passage, see SHEN, Crossley, and LUN 1999, 205. For the original Chinese text and its source, see note 6 of this chapter.

the *Nine Chapters*, we find no words, explicit or implicit, about the two methods, but there is evidence of their use in later classics such as the *Sunzi Suanjing* 孫子算經 (*A Mathematical Classic by Master SUN*), the *Zhang Qiujian Suanjing* 張丘建算經 (*A Mathematical Classic by ZHANG Qiujian*), and the *Wujing Suanshu* 五經算術 (*A Mathematical Classic on the Five Classics*).[36] It is very likely, however, that the two approximation methods had already been developed by the time the earliest extant version of the *Nine Chapters* was compiled. It is interesting to note that in the *Suan Shu Shu* there is a problem which requires extraction of the square root of 240. But in this case the method used to solve the problem is the *ying buzu shu* 盈不足術 (rule of false position for excess and deficit). The result, $15\frac{15}{31}$, is actually the same as that obtained by the *kaifang* method using the *jiesuan*.[37] In fact, the result of taking the square root of any whole number by the rule of false position for excess and deficit is the same as that obtained by the *kaifang* method using the *jiesuan*. It is possible that the latter was derived from the former.

The approximations given by the two methods mentioned in LIU Hui's commentary should have been good enough for solving practical questions in ancient times. In fact, in his commentary on problem 17 of Chapter 4, *fangtian*, LIU Hui gave an alternative answer which is actually less accurate than those obtained from the two approximations. But why did LIU Hui regard the two approximation methods as useless? From his commentary it is clear that what LIU Hui emphasized was the inverse relation between *kaifang* and squaring:

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[36] LI Yan [1958] 1998, 103.

[37] For the original text of the *Fangtian* problem in the *Suan Shu Shu*, see *ZJSHMZJ* 2001, 98, 272. For an English translation of this problem, see Dauben forthcoming.

Taking the *kaifang* of the number of a square area, one obtains the side; multiplying the side by itself should restore the original number of the square area. The relation between *kaifang* and squaring is similar to that between division and multiplication. But are all these matters really concerns of the *Nine Chapters*? From the flow of the description of the procedures for *kaifang* in the *Nine Chapters*, it is clear that the procedure stops when the unit position of a root is fixed, regardless of whether the remainder is zero or not. If the remainder is zero, then the number extracted is a whole number; but if the remainder is not zero, then the square root cannot be extracted *exactly* as a whole number. Thus it seems apparent that the inverse relation between *kaifang* and squaring was *not* a concern of the author(s) of the *Nine Chapters*.

Moreover, assuming that the original meaning of "*yi mian ming zhi*" was to use the side of a square area which could not be extracted as a whole number to express the result of taking the *kaifang* of the number of the square area, then there arise a number of difficulties in interpreting the four sentences immediately following:

> 若實有分者，通分內子為定實，乃開之。訖，開其母，報除。若母不可開者，又以母乘定實，乃開之。訖，令如母而一。
>
> If a *shi* [which is a number to be extracted] has a fractional part, multiplying the whole number part by the denominator, add the product to the numerator, and then let the sum be *dingshi*. Extract this *dingshi*. After doing that, extract the denominator, and then divide. If the denominator cannot be extracted exactly as a whole number, multiply it by the *dingshi*, and then extract the product. After finishing that, let [the extraction of the product] be divided by the denominator. [38]

[38] For the source of the original text, see note 6 of this chapter. For an alternative English translation of this passage, see SHEN, Crossley, and LUN 1999, 204.

For the sake of clarity, we again use symbols to illustrate the meanings of the above passage. Suppose a *shi* (or square area) *N* is in the form: $a\frac{b}{c}$, then

$$\sqrt{a\frac{b}{c}}=\sqrt{\frac{ac+b}{c}}=\frac{\sqrt{ac+b}}{\sqrt{c}}, \text{ if } c \text{ is a square number;}$$

$$\sqrt{a\frac{b}{c}}=\sqrt{\frac{ac+b}{c}}=\sqrt{\frac{(ac+b)c}{c\times c}}=\frac{\sqrt{(ac+b)c}}{c}, \text{ if } c \text{ is not.}$$

Where $ac+b$ and $(ac+b)c$ could be either a square number or a non-square number. If both are non-square numbers, taking the *kaifang* requires naming them by "*mian*" if one accepts LIU Hui's interpretation. But the original text just says, 乃開之 (then open/extract it), instead of 以面命之 (use the side to express it).

Based on the above considerations, it is reasonable to suspect that LIU Hui's commentary may not capture the original meaning of the text in question in the *Nine Chapters*. The sentence: 若開之不盡者, 為不可開, 當以面命之, may be better understood as follows: If extracting a number, and there is a remainder left when the *kaifang* operation ends, then the number cannot be extracted exactly in a whole number, and a fraction must be used, the numerator of which is the remainder and the denominator of which is twice the extracted number or the opened side, representing whatever number or area is left. Note that the words: 為不可開, should not be interpreted as "un-extractable" as some historians have done. The phrase may really mean: 為不可開盡 (cannot be extracted or cut *exactly*). The supporting evidence for this interpretation is whether or not numbers in the forms $ac+b$, and $(ac+b)c$, are square, the *kaifang* procedure can still be carried out. Similar evidence can also be found in the method for *kai lifang shu* 開立方術 (the

method for finding the side of a cube).[39] Moreover, consider LIU Hui's commentary on Problem 11 of Chapter 9, *gougu*, where he writes:

> If *gou* and *gu* are both 5, the square of *xian* should be 50. Extracting it gives 7 with the remainder 1. [Accordingly it] cannot be extracted exactly. If *xian* is 10, its square is 100. Halving it gives the squares of *gou* and *gu*, 50 each. [These] also cannot be extracted [exactly].
>
> 假令句、股各五，弦冪五十，開方除之，得七尺，有餘一，不盡。假令弦十，其冪有百，半之為句、股二冪，各得五十，當亦不可開。[40]

Here extracting the square root of 50, which gives 7 with a reminder of 1, is called "不盡" and also "不可開."[41] This also supports the above interpretation of "為不可開."

Following the lead of LI Huang, MIKAMI Yoshio, QIAN Baocong, LI Yan, and LI Di, we believe that *zhi* 之 in the context of "*yi mian ming zhi* 以面命之" refers to the remainder. As for the meaning of this sentence, it is meant to cover the cases in which a fraction is needed to represent the remainder, an interpretation which follows closely the pattern of "*yi*…*ming*…以…命…" in the earliest Chinese mathematical texts, including the *Suan Shu Shu* and the *Nine Chapters*. More specifically, the instructions are to take the remainder as the numerator and twice of the extracted number or side as the denominator. The difference between this interpretation and that offered by LI Huang is the word "twice." Doubling the extracted number as the denominator not only serves to

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[39] See *TH* 1, 134a-134b.

[40] Quoted from *TH* 1, 194a, with punctuation added. For alternative translations of this passage, see SHEN, Crossley, and LUN 1999, 477; and Chemla and Keller 2002, 115.

[41] LIH 1994, 50.

make the fraction very close to the true value of the square root of the remainder, but this fits well with the original text which can indeed be interpreted in this way.

Having discussed the meaning of "*yi mian ming zhi* 以面命之" in the *Nine Chapters*, we now turn to examine LIU Hui's commentary on this phrase. As already explained briefly, LIU Hui's commentary emphasizes the inverse relation between squaring and extracting the square root. But there he provides no example for the term "*mian*" to illustrate his point. Several specific numbers associated with the term "*mian*," however, can be found in his commentary on the method for finding the diameter of a sphere (*kai liyuan shu* 開立圓術). It is mainly based on this commentary that LI Jimin and Karine Chemla argue, respectively, that "*mian*" equates to the concept of irrational number, or at least to that of a quadratic irrational number. The relevant texts should be quoted as follows:[42]

1. 假令丸中立方五尺，五尺為句。句自乘，冪二十五尺。倍之得五十尺，以爲股冪，謂平面方五尺之弦也。以此弦冪為股，亦以五尺為句，并句股冪得七十五尺，是為大弦冪。開方除之，則大弦可知也。大弦即中立方之長邪，邪即丸徑。故中立方自乘之冪于丸徑自乘之冪三分之一也。令大弦還乘其冪，冪即丸外立方之積也。大弦冪開之不盡，令（困）[其] 冪七十五再自乘之為面，命得外立方積四十二萬一千八百七十五尺之面。又令中立方五尺自乘，又以方乘之，得積一百二十五尺。一百二十五尺自乘為面，（句）[命] 得積一萬五千六百二十五尺之面。[43] 皆以六百二十五約之，外立方積六百七十五尺之面，中立方積二十五尺之面也。

[42] Quoted from *TH* 1: 136b – 137a. The commentary added by DAI Zhen is omitted here. The paragraph numbers and punctuation are added for easy reference and reading. Paragraphs 2 through 8 comprise a single paragraph in the original source.

[43] The two emendations are suggested by DAI Zhen, see *TH* 1: 136a.

2. 張衡算又謂：立方為質，立圓為渾。衡言：質之與中外之渾，六百七十五尺之面，開方除之不足一，謂外（質）[渾] 積二十六也。[44] 内渾二十五尺之面，謂積五尺也。

3. 今徽令質言中渾、渾又言質，則二質相與之率猶衡二渾相與之率也。衡蓋亦先二質之率推以言渾之率也。

4. 衡又言：質六十四尺之面、渾二十五尺之面。質復言渾，謂居質八分之五也。

5. 又云：方八之面、圓 [五之面]。[45]

6. 圓、渾相推知其復以圓困為方率，渾為圓率也，失之遠矣。衡說之自然慾協其陰陽、奇偶之說而不顧疏密矣。雖有文辭，斯亂道破義，病也。

7. 置外積二十六，以九乘之、十六而一，得積十四尺八分之五，即質中之渾也。以分母乘全内子，得一百一十七。又置内質積五，以分母乘之，得四十。是謂質居渾一百一十七分之四十。而渾率猶為傷多也。

8. 假令方二尺，方四面并得八尺也，謂之方周。其中令圓徑與方等，亦二尺也。(丸)[圓]半徑以乘圓周之半，即圓冪也。[46]半方以乘方周之半，即方冪也。然則方周知方冪之率也，圓周知圓冪之率也。按：如衡術方周率八之面、圓周率五之面也，令方周六十四尺之面，則圓周四十尺之面也。又令徑二尺自乘，得徑四尺之面。是為圓周率十（二）之面，而徑率一之面也。[47] 衡亦以周三徑一之率為非，是故更著此法。然增周太多，過其實矣。

The quoted texts are parts of the lengthy commentary in which LIU Hui first points out that the formula for calculating the volume of a sphere: $V_{sphere} = \frac{9}{16} d^3$ (d is the diameter), is not accurate (the correct formula should be

[44] This emendation is proposed by WANG Lai, see GUO Shuchun 1990, 274; 2004: 1, 166.

[45] This emendation is made by LI Huang, see GUO Shuchun 1990, 274; 2004: 1, 166.

[46] Here the word "丸" should be a mistake of the character "圓." This emendation was proposed by LI Huang, see GUO Shuchun 2004: 1, 168.

$V_{sphere} = \frac{\pi}{6} d^3$). He then suggests a way of fixing it, which eventually led ZU Gengzhi to succeed some three hundred years later. What follows are the quoted texts in which LIU Hui supplies his explanation and criticism of the related work done by ZHANG Heng. Examining the text, it seems apparent that ZHANG Heng's mathematical book, *Suan Wang Lun* 算罔論 (*On Difficult Mathematical Problems)*, was not available *in toto* to LIU Hui, and may already have been lost by the mid-third century. Paragraphs 2, 4, 5, and parts of 8 of the quoted original texts record some results of ZHANG Heng's mathematical work which includes:

a) The ratio of the volume of a sphere to that of the sphere inscribed in the cube which in turn is inscribed in the first sphere is *mian* of 675 to *mian* of 25. Taking root extraction of 675, with 1 short, gives 26. Meanwhile *mian* of 25 is equal to 5. Therefore, the ratio is very close to 26 to 5 (Paragraph 2);

b) The ratio of the volume of a cube to that of the inscribed sphere is *mian* of 64 to *mian* of 25, namely, 8 to 5 (Paragraph 4);

c) The ratio of the area of a square to that of the inscribed circle is *mian* of 8 to *mian* of 5 (Paragraph 5); and

d) The ratio of the circumference of a square to that of the inscribed circle is also *mian* of 8 to *mian* of 5. Accordingly, the ratio of the circumference of a circle to its diameter is *mian* of 10 to *mian* of 1. (Paragraph 8).

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[47] This correction was suggested also by LI Huang, see GUO Shuchun 1990, 275; 2004: 1, 168.

The details of how ZHANG Heng got these results were obviously not available to LIU Hui. Accordingly, he is compelled to supply his own explanations. For the ratio of the volumes of the two spheres inscribed within and circumscribed about a cube (Figure 2), LIU Hui points out that it is the same as the ratio of the volumes of the two cubes inscribed within and circumscribed about a sphere (Figure 3). The latter case is explained in Paragraph 1, which may be translated as follows:

> If the side of the cube inscribed in a sphere is 5 *chi*, take 5 *chi* as the *gou*. Multiplying the *gou* by itself [results] in an area of 25 *chi*. Doubling the area gives 50 *chi*. Take [50 *chi*] as the square of *gu*, which also can be considered as the diagonal of the square on the base with its side as 5 *chi*. Let the square of the *xian* be [the square of the] *gu*, and the *gou* be 5 *chi* again, then the sum of the squares of the *gou* and the *gu* is 75 *chi*, that is the square of the larger *xian*. Taking *kaifang* of 75 gives [the result for] the larger *xian*, which is the diagonal of the inscribed cube. The larger *xian* is the diameter of the sphere. Therefore, the square of the side of the inscribed cube is 1/3 of the square of the diameter of the sphere. Multiplying the larger *xian* by its square, which gives the volume of the outer cube. [Since] the square of the larger *xian* cannot be extracted exactly [as a whole number], let its square, 75 *chi*, be multiplied by itself three times, and name the product as a "*mian*." The operation gives the volume of the outer cube, the *mian* of 421,875 *chi*. Also let the side of the inscribed cube, 5 *chi*, multiplied by itself, and again by the side, [which] yields the volume, 125 *chi*. Multiply 125 *chi* by itself, and name the product as a "*mian*." This operation gives the volume [of the inscribed cube], the *mian* of 15,625 *chi*. Dividing both by 625 gives the volume of the outer cube, the *mian* of 675, and the volume of the inscribed cube, the *mian* of 25, respectively.

Let AB be the side of the inscribed cube (see Figure 3), LIU Hui's explanation may be illustrated in symbols as follows:

Since $AB = 5$,

then $AB^2 = 25$;

accordingly, $2AB^2 = 50$;

thus $AC^2 = 50$.

Also $AC^2 + CD^2 = AD^2$,

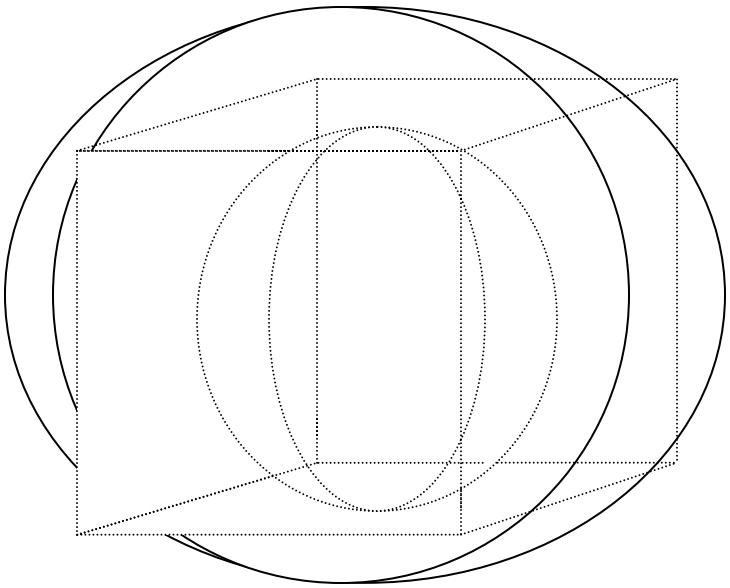

Figure 2. Two spheres inscribed within and circumscribed about a cube.

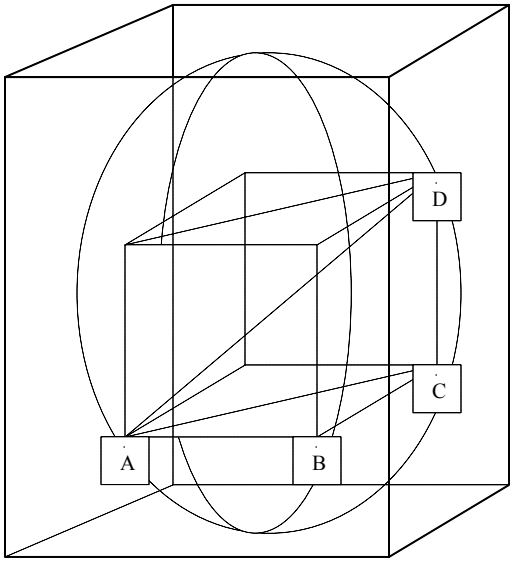

Figure 3. Two cubes inscribed within and circumscribed about a sphere.

then $AD^2 = 75$ (AD is the diameter of the sphere),

therefore $AB^2 = \frac{1}{3} AD^2$.

Also $V_{outer\ cube} = AD \times AD^2$

Since 75 cannot be extracted exactly, then let

$$AD^2 \times AD^2 \times AD^2 = 75 \times 75 \times 75 .$$

The volume of the outer cube is the *mian* of the product: $75 \times 75 \times 75$, namely, 421875.

Since $AB = 5$,

the volume of the inscribed cube is: $AB \times AB \times AB = 5 \times 5 \times 5 = 125$;

Name the product of 125×125 as a *mian,* namely, the *mian* of 15625.

$$\frac{V_{outer\ cube}}{V_{inscribed\ cube}} = \frac{AD^3}{AB^3} = \frac{mian\ \ of\ \ 421875}{mian\ \ of\ \ 15625}$$

$$= \frac{mian\ \ of\ \ \frac{421875}{625}}{mian\ \ of\ \ \frac{15625}{625}} = \frac{mian\ \ of\ \ 675}{mian\ \ of\ \ 25} .$$

In addition to providing an explanation for the ratio of the volume of the circumscribed cube to the volume of the inscribed cube above, LIU Hui also criticizes ZHANG Heng's ratio of the circumference of a square to that of the inscribed circle as *mian* of 8 to *mian* of 5 (in Paragraph 8 of the quoted original text). There he also provides several more examples of numbers named with the term, "*mian*." The text may be translated as follows:

> Suppose the side of a square is 2 *chi;* the four sides of the square added together gives 8 *chi*, which is the perimeter of the square. Let the diameter of the circle be equal to the side of the square, namely 2 *chi.* Half of the diameter multiplied by half the circumference gives the area of the circle. Half of the side of the square multiplied by half of its perimeter yields the area of the square. Therefore, the perimeter of the square is in proportion (ratio) to its area (as $4s : s^2$, or $4 : s$); the circumference of the circle is in proportion (ratio) to its area (as $2 : r$ [i.e. $2\pi r : \pi r^2$]). Note: According to [ZHANG] Heng's method: the ratio of the circumference of a square to that of the [inscribed] circle is *mian* of 8 to *mian* of 5. Let the circumference of a square be *mian* of 64 *chi*, this gives the circumference of the circle, *mian* of 40 *chi*. Also let the diameter 2 *chi* multiplied by itself, which yields a circumference of *mian* of 4 *chi*. Therefore, the ratio of the circumference of the circle to its diameter is *mian* of 10 to *mian* of 1.

There are several numbers designated as "*mian*" in this lengthy text as quoted above. Apparently, they do not fit into any interpretation of the expression "*yi mian ming zhi*" in the *Nine Chapters* as put forward either by LI Huang or by LI Yan. Moreover, it is LI Jimin who first links the term "*mian*" here and in Chapter 4 (*shaoguang* or "Short Width") of the *Nine Chapters*, and argues that "*mian*" has the same meaning as "surd root," and as such represents the introduction of irrational numbers in ancient Chinese mathematics.[48]

LI Jimin's view has been elaborated recently by Karine Chemla, who carefully puts her discussion in context, and confines her interpretation of "*mian*" to quadratic irrationals.[49] Chemla's analysis of the original text overall is quite accurate, and several observations she makes are insightful. For instance, after presenting LIU Hui's procedures on the ratio of the volumes of the two cubes which are inscribed within and circumscribed around a sphere, she comments:

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[48] LI Jimin 1989; [1990]1992, 105-110.

[49] Chemla and Keller 2002, 102-119; Chemla and GUO 2004, 41-70.

> It (*mian* or side of *N*) expresses a quantity the square of which is *N*. And in case the interpretation of the result of a sequence of operations shows that *N* corresponds to the square of the value sought, then this value can be found by applying a root extraction algorithm to *N* and may therefore be expressed as "side of *N*," irrespective of the actual object it refers to. It is not the number N as such that may or may not be qualified as a square; it is the object, the value of which it represents, which may be shown to be the square of some other magnitude. The qualification of a magnitude as the square of another expresses in this case, with full generality, an operational relation between the two. Such quantities as "side of *N*" do not pertain to a given geometrical meaning. In other words, the operation of taking the square root appears to be conceived of not as determining the length of the side of a given square, but as the inverse of the operation of squaring, and this "operational meaning" is conveyed to such quantities as "side of *N*"; they are qualified as the quantities the square of which is *N*. [50]

Chemla's arguments as a whole, however, are not sufficient enough to warrant her conclusion. Firstly, the term "*mian*" is *not* limited to these numbers which are non-squares. The above quoted text provides such examples as "*mian* of 15625," "*mian* of 25," "*mian* of 64," and "*mian* of 16." This point has already been raised by LIH Kowei 李國偉 in his criticism of LI Jimin.[51] But Chemla does not address this issue.

Secondly, no single "*mian* of *N*" is found without regard to a ratio. In other words, all the numbers termed as "*mian*" are presented in the context of discussing a ratio. This fact may suggest that "*mian* of *N*" was used only for operational purposes to bridge steps. (Chemla does observe the "operational meaning" of "*mian* of *N,*" but fails to recognize its significance.) Above all, it is not treated as an independent number. This is one of the major reasons why we think that "*mian*" in the original text of the *Nine Chapters* and in LIU Hui's

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[50] Chemla and Keller 2002, 106-107.

[51] See LIH 1994.

commentary has a different meaning. Providing the answer to a practical problem as the "*mian* of *N*" seems very odd! It is more reasonable to suspect that the answer was meant to be an approximation.

Thirdly, Chemla's argument that LIU Hui had realized the different nature of a quadratic irrational number, say $\sqrt{2}$, and a fraction, say $\frac{10}{3}$, is not convincing.[52] LIU Hui indeed says that when extracting a number, if the result is not a whole number, then "*mian*" should be used to represent the number, because in doing so the actual value is preserved. But this is like using 3 to divide 10, the remainder of which should be expressed as $\frac{1}{3}$.[53] But LIU Hui's focus is to emphasize the inverse relation between squaring and root extraction, which is similar to the inverse relation between division and multiplication. LIU Hui may well have realized that the procedure of extracting the root of a non-square number goes on indefinitely, exactly just like that of dividing 10 by 3. But he does not recognize that the result of the former operation is non-repeating, whereas the latter one *is*. Without distinguishing this feature, how is it possible to say that their natures are totally different?

Chemla quotes one sentence from LIU Hui's commentary on Problem 11 of Chapter 9 in the *Nine Chapters*, namely, "雖不正得盡理，亦可言相近爾," which she translates as: "even though one cannot exactly (*zheng* 正) manage to exhaust their inner constitution (*li* 理), one can still express an approximation to it, but that

[52] See Chemla and Keller 2002, 115-119.

[53] See note 35 of this chapter.

is all."[54] The sentence appears in the context of discussing the ratio of the circumference of a circle to its diameter, very imperfectly, as 3 to 1; and the ratio of the side of a square to its diagonal, 5 to 7. These ratios are practically very useful, even though they are only fair approximations and by no means express the true ratios. Chemla, however, argues that the above-quoted sentence reflects LIU Hui's knowledge of the irrational nature of these two ratios. The word "*li* 理" is a philosophical term, which Chemla appreciates, and has diverse meanings, of which "inner constitution" is one. But the meaning of this sentence is so vague that this alone is of little help; the sentence could also be interpreted as something like this: "the two ratios, though not precise, are practically useful." In any case, the sentence in question is not clear enough to warrant the conclusion Chemla draws.

Chemla also quotes another part of the text to support her conclusion. This text is from LIU Hui's commentary on the famous "Method of Cutting a Circle" from Chapter 1: "此以周、徑，謂至然之數，非周三徑一之率也," which she translates as follows: "Here, by circumference and diameter we designate (*wei* 謂) the values (*shu* 數) that attain what is so (*zhi ran* 至然), what the *lǚ* of 3 for circumference and 1 for diameter are not."[55] This sentence comes immediately after LIU Hui has demonstrated and concluded that the area of a circle is equal to the product of half of its circumference and half of its diameter. What he tries to say here is that the circumference and diameter in the method or formula are

[54] Chemla and Keller 2002, 115.

[55] Chemla and Keller 2002, 116.

general and precise, not like the specific yet imprecise ratio: 3 to 1. Accordingly, this sentence does not add more weight to Chemla's position.

From all of the evidence that we have examined (and in fact there is very little related to the term "*mian*" in the *Nine Chapters*), we can only conclude that the expression "*mian* of *N*" represents a quantity, which may either be a specific number, or the side of a square area, the squaring of which yields *N* itself. It is named for the purpose of convenience of calculation. It is not a number that exists independently, let alone an irrational number.

At the beginning of this chapter, we pointed out that the concept of irrational number in ancient Greek mathematics is based on the concept of geometrical magnitude, the Pythagorean theorem, and the Pythagoreans' doctrine that natural numbers or their ratios can determine or express all things, whether physical or mathematical. There is no doubt that ancient Chinese mathematics has its own version of the so-called Pythagorean theorem, namely the "*gougu*" 句股 theorem. What is lacking here is the Pythagoreans' doctrine that ratios of whole numbers sufficed to measure everything in nature, along with the concept of geometrical magnitude, which we have discussed in the previous chapter. For Chinese mathematicians, all entities such as lines, areas, and volumes can be measured out according to various units, and therefore can be represented by a combination of whole numbers. This fundamental belief in no way disposed them to discover incommensurability or irrational numbers. This would have required the realization that there were some entities that could not be described as a ratio of whole numbers, but no such statements are to be found in any of the currently-

known treatises or commentaries on ancient Chinese mathematics. On the other hand, Chinese mathematicians had little difficulty in providing fine approximations for the ratio of the circumference of a circle to its diameter or for the square root of a non-square number. It was the lack of the concept of geometrical magnitude which prevented them from confronting the logical dilemma that first confounded the Pythagoreans, and in turn led to the discovery of incommensurable magnitudes and their arithmetic equivalent, irrational numbers. But ancient Chinese mathematicians never asked the question which plagued the Greeks about commensurability, and were never forced to confront the fact that there was no unit common to both the side and diagonal of the square.

Part II: Western Concepts of Infinity in Transition

Chapter 4 Introduction of the Concepts of Infinity in Euclid's *Elements*

> Book X is perhaps the most remarkable, as it is certainly the most finished, of all the books in the *Elements*. It deals with irrationals, by which must be understood, in general, *straight lines* which are irrational in relation to any particular straight line assumed as rational; and it investigates every possible variety of straight line corresponding to what we should express in algebra by $\sqrt{(\sqrt{a}\pm\sqrt{b})}$, where $\sqrt{a}$ and $\sqrt{b}$ are surds and incommensurable with one another.
>
> -Sir Thomas L. Heath[1]

4.1 Introduction: Euclid's *Elements* in China

It is widely known among historians that Euclid's *Elements* may first have come to China as early as the Yuan dynasty, sometime between 1250 and 1270. This has at least been the case ever since the historian of Chinese mathematics YAN Dunjie pointed out in 1943 that a book mentioned in the "Catalogue of the Muslim Books (*Huihui shuji* 回回書籍)" might be interpreted as a version in 15 books of Euclid's *Elements* (namely a work listed in chapter seven of the *History of the Imperial Archive and Library,* the *Mishujian zhi* 秘書監志, which was compiled in the mid-14th century).[2] Moreover, in the same article YAN argued that one sentence in the *Jām` al-tawārīkh* (*Compendium of Chronicles*) written by Rashīd al-Dīn (ca. 1247–1318), a high functionary at the Mongolian court of

[1] Quoted from Heath 1963, 241.

[2] See YAN Dunjie 1943, 35; and WANG and SHANG 1992, 129. The term "*Mishujian*" is rendered here as "imperial archive and library," but it functioned much more broadly than an archive or a library does, and undertook astronomical observations, oversaw calendar-making, and even organized grand book projects, among others. For a relatively detailed discussion of the history of this institution and its main functions in the 12th and 13th centuries, see Hucker 1985, 376b-377b, as well as the introduction to the latest Chinese edition of the *Mishujian zhi* by GAO Rongsheng in WANG and SHANG 1992, 1-2.

Iran,[3] praises the intelligence of Möngke Khan (Mengge Han 蒙哥汗, reigned 1251–1259) for "solving some propositions of Euclid," which clearly suggests that Euclid's *Elements* was known in the Mongolian court by the early 1250s.[4] One year before YAN's publication, the Japanese scholar TASAKA Kōdō, in his study of the introduction of Islamic culture into China during the Yuan dynasty, also discussed the possible meanings of the recorded book title in the *Catalogue of the Muslim Books*. But TASAKA held that it might refer to a work of Abu Ja`far Muhammad ibn Musa al-Khwarizmi.[5] Fifteen years later, in his English article, "An Aspect of Islam Culture Introduced into China," TASAKA changed his view, and also concluded that the title refers to an Arabic version of Euclid's *Elements*, based upon a sounder linguistic analysis than YAN had used.[6]

Although there is little doubt about the book in question being Euclid's *Elements*, it is unclear whether any Chinese scholar of that time was familiar with any of the contents of the book. It is also unknown whether there may have been a Chinese translation of all or only part of the copy of Euclid's *Elements* which was in the library of the Northern, or Islamic, Observatory (*Bei sitiantai* 北司天台) built in 1271 by the Mongolians in the upper capital (Shangdu 上都), situated near Dolon Nor (Zhenglanqi 正藍旗) in Inner Mongolia. Moreover, Rashīd's brief and ambiguous description is really

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[3] For a detailed biography of Rashīd al-Dīn, see Morgan 1995, 443a-444b.

[4] YAN Dunjie 1943, 36. The original words in Persian may be found in Rashīd al-Dīn 1836, 324. The French translation of the sentence by Etienne Marc Quatremère is: "Mangou-kaân se distinguait entre tous les monarques Mongols par ses lumières, son esprit, sa prudence, son tact, sa sagacité, sa finesse; il était si instruit, qu'il avait expliqué plusieurs des figures d'Euclide." See Rashīd al-Dīn, 1836, 325. An alternative English translation of the sentence by Wheeler McIntosh Thackston reads: "Mänggü Qa'an was distinguished among Mongol rulers for his intelligence, perspicacity, sharp wit and mind, and he even solved some problems left by Euclid." See Rashīd al-Dīn, 1998-1999, 502a.

[5] TASAKA 1942, 444-445.

[6] TASAKA 1957, 103-104.

not sufficient to prove that Möngke Khan had studied Euclid's *Elements* as Yan suggested. Nevertheless, the *Elements*, in either an Arabic or a Persian version, certainly existed in China no later than 1273, the year in which the Islamic Observatory reported on its collection of books to the Mongol authorities.[7]

Euclid's *Elements* was again brought to China some 300 years later. This time it was in the form of a Latin version by Christopher Clavius, professor of mathematical sciences at the *Collegio Romano* (the Jesuit seminary founded in Rome by Ignatius Loyola in 1551). It was the Jesuit Matteo Ricci, a student of Clavius, or one of his Jesuit brothers who first brought a copy of Clavius's *Euclid* with them to China. Unlike the Muslims before him, Ricci was determined to translate the book, along with other Western works, in hopes of gaining the support that he and his fellow Jesuits sought from the Chinese *literati*. In doing so the Jesuits expected to secure a footing in pagan China, and eventually to convert the Chinese to Christianity. Not long after his arrival in China, Ricci sought help from one of his early converts and together they began to translate the *Elements*. But at the time he was only able to finish the first book, the manuscript of which was later lost.[8]

Ricci, however, never lost interest in translating the *Elements*. In 1604, when he eventually settled in Beijing after experiencing many years of hardship in the south of China, he approached XU Guangqi 徐光啓, who not only showed a religious inclination toward Christianity, but was a well-educated member of the *literati* who was on the verge

[7] For recent studies of scientific transmission between Chinese and Arabs under the Mongols, see LI Di 1999, 135-151; and van Dalen 2002.

[8] Engelfriet 1998, 59-66.

of getting the highest degree the Ming dynasty could offer.[9] Ricci proposed they collaborate in translating the *Elements*. Fascinated by its style and the entirely different approach to geometry it represented in contrast to traditional Chinese geometry, XU Guangqi devoted himself wholeheartedly to assisting Ricci in translating Euclid's *Elements*.[10] After three years of hard work, they had finished the first six books. Although XU Guangqi wanted to continue to translate the rest of the book, Ricci thought it would be better to publish their partial translation first. Depending upon its reception, they could then decide whether to continue the project or not. Accordingly, they published their Chinese translation of the first six books in 1607.[11]

Reactions to the publication of the Chinese Euclid were somewhat mixed. On the one hand, its exotic style and formality (not its axiomatic–deductive system) was warmly accepted, and the newly invented term *Jihe* for geometry was widely adopted in titles of Chinese mathematical books published soon thereafter. Moreover, a number of books were written to elucidate parts of the Ricci/XU translation. On the other hand, some mathematicians, including MEI Wending 梅文鼎, who was one of the most prominent mathematicians in the early Qing period, held that traditional Chinese mathematics, represented in particular by the *Nine Chapters*, included all of the methods to be found in the translation, with but one possible, if notable, exception—namely, Euclid's

[9] For a biography of XU Guangqi, see HASHIMOTO 1988.

[10] There are three editions of Clavius's *Elements*, namely, those of 1574, 1591, and 1603. Which of these was actually used by Matteo Ricci and XU Guangqi in preparing their translation is not clear. Only the last two editions have ever been found in China. See Verhaeren 1969, 376.

[11] Engelfriet 1998, 460.

introduction of extreme and mean ratios.[12] Moreover, Chinese mathematicians regretted that the translation was only partial, and some, notably MEI, even suspected that Ricci was trying to hide something important from the Chinese in the later books of the *Elements*.[13] As we shall see below, MEI's false accusation was to become a major factor in the decision, made early in the 1850s by a later missionary, Alexander Wylie, to complete the work left undone by Ricci.

During MEI's time, the Emperor Kangxi 康熙 (reigned 1662–1722) was especially interested in Western science in general and mathematics in particular. In order to learn Euclidean geometry, sometime around 1690 he requested two French Jesuits, Joachim Bouvet and Jean-François Gerbillon, who had recently arrived in China, to lecture at his court. Bouvet and Gerbillon, both of whom held the title "King's Mathematician" granted by the Sun King, Louis XIV, compiled their lessons for the Emperor in Manchu from a short textbook by their countryman Ignace-Gaston Pardies: *Eléments de Géométrie* (first published in Paris in 1671).[14] The resulting lecture notes for Emperor Kangxi were subsequently translated into Chinese and later incorporated into a mathematical encyclopedia, the *Essence of Mathematics* (*Shuli jingyun* 數理精蘊), which was compiled under the auspices of Kangxi, but not published until 1723, the year immediately after his death.[15]

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[12] For reactions to the Chinese Euclid in the 17th century, see Martzloff 1981; 1993a; and 1993b; MEI *et al*. 1986/1990; and Engelfriet 1998, 289-448.

[13] LIU Dun 1989, 57.

[14] See Martzloff 1997, 26-27; LIU Dun 1991, 90; and Jami 1994, 233. For the King's mathematicians, see Landry-Deron 2001.

[15] A Manchu manuscript of Euclid's *Elements*, three volumes (*san ce*) in bulk, which undoubtedly came from the lecture notes by Joachim Bouvet and Jean-François Gerbillon, has survived and is preserved in the Palace Museum in Beijing. The manuscript had been subsequently translated into Chinese. There are three different copies of the Chinese manuscripts, one preserved in the National Library in Taipei and the other

The Chinese translation (with some additions and deletions) of Pardies' *Eléments*, when it appeared in the *Essence of Mathematics*, bore the same title as the one used by Ricci and Xu, but its contents and style were very different. The Pardies translation was divided into 12 chapters (*juan*), the first five of which covered, respectively, lines and angles, triangles, quadrilateral figures and polygons, circles, and solids. Pardies' Book VI, on proportion, constituted the contents of chapters six to nine in the translation. However, Books VII and VIII of Pardies' *Eléments*, devoted to incommensurables, progressions, and logarithms, were left out, presumably because of their difficulty. The last book of the original text, Book IX, which included 48 practical geometrical problems, provided the contents for chapters 11 and 12 of the translation. The contents of chapter 10 were devised by Bouvet and Gerbillon themselves or by other missionaries.[16]

The *Essence of Mathematics* version of the *Elements* was not, in reality, even a close translation of Euclid's *Elements*, nor was it complete. But the drastic negative change in Qing dynasty policy toward the missionaries certainly did nothing to encourage either the Chinese or the Jesuits—or indeed any other missionaries—to complete Ricci's unfinished project for another 130 years. The Emperor Yongzheng 雍正, Kangxi's immediate successor, adopted harsh measures against the missionaries in China as a direct result of the rites controversy that had started during the Kangxi reign. The controversy was mainly about whether ancestor worship should be an allowable practice for Chinese converts to Catholicism or whether the Jesuit missionaries should be allowed

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two in the Palace Museum in Beijing. Each of these three Chinese manuscripts represents a different stage of a working version which was eventually published in the *Essence of Mathematics*. For the relations among Pardies' *Eléments de Géométrie*, the Manchu and Chinese manuscripts, and the published Chinese version in the *Essence of Mathematics*, see CHEN 1931; LI Zhaohua 1984; LIU Dun 1991; and Martzloff 1993a.

[16] See HAN Qi 1993, 3-4.

to follow the practice of ancestral rites and thus "accommodate" themselves to Chinese rites.[17] The ruling of Pope Benedetto XIV against this in 1742 eventually led the Qing court to close China's doors to all foreigners, but especially to missionaries, and this situation was only reversed by British gunboats in the 1840s as a result of the so-called Opium Wars.

With China's doors reopened to foreigners in 1842, missionaries from the West—especially from Great Britain—flooded in. Among them was Alexander Wylie from Scotland, who was sent to Shanghai by the London Missionary Society in 1847.[18] Wylie soon found that the work left unfinished by Ricci and XU Guangqi was worth pursuing. He described his thoughts about this as follows:

> After arriving in China, I read the translation of the first six books of the *Elements* by the European [Matteo] Ricci in the Ming dynasty. The translation was highly regarded by most [Chinese] mathematicians. They knew it was incomplete, and therefore, expressed their dissatisfaction. MEI [Wending] of Xuancheng once said that [Ricci] had hidden something from the translation. [But that incompleteness] was due to the sophistication of the theories and to the difficulty of translating. The nature of knowledge is to provide means to all people. Why hide it and not reveal it?. . . My translation has just been completed. It has not only fulfilled Ricci's wish, but also dispelled MEI's doubts.[19]

Wylie went on to explain that whereas Ricci believed the teachings of Jesus Christ were fundamental, he also emphasized that calendrical science, mathematics, and the other sciences, though far less important, were nevertheless useful.[20]

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[17] For details and historical meanings of the Chinese rites controversy, see Mungello 1994.

[18] For Alexander Wylie's life and his contributions to Chinese studies, see Edkins 1897; Thomas 1897; Cordier 1897; HAN Qi 1998; and WANG Xiaoqin 1998.

[19] Translated from Wylie and LI, 1857/1865, Wylie's preface, 3a, and 4a.

[20] See Wylie and LI, 1857/1865, Wylie's preface, 4a.

Like Ricci, Wylie carried out his translation in collaboration with a Chinese scholar, LI Shanlan, who later proved to be the ablest and most prominent Chinese mathematician in the second half of the 19th century.[21] Wylie and LI started their translation of Books VII through XV of the *Elements* in 1852, and took about four years to complete their work. The translation was published under the patronage of HAN Yingbi in Shanghai in 1857, and later was reprinted along with the first six books by Ricci and XU Guangqi in 1865.

Over the next half a century, Chinese interest in the last nine books of the *Elements* was exclusively technical. As time passed by, historical interest in the translation gradually increased. Historians quite naturally wanted to know about the original source, or sources, that Wylie and LI had used to prepare their translation in order to evaluate the Chinese translation itself, assess the transmission of Western mathematics into China in the second half of the 19th century, and study the influence of Euclid's *Elements* upon Chinese mathematicians, especially the translator LI Shanlan, among other historical interests. However, neither Wylie nor LI had said anything, specifically, in their prefaces to the translation about its original source. The only information Wylie offered was the following:

> This book commonly used in Europe is of six or eight books, none of which is complete. I had searched for a copy of the completed book in Shanghai since my arrival here, and yet found none. Then I turned to my own country and finally got a copy. The book is a translation from Greek into my native tongue. It has not been reprinted recently. The copy I have is an old version, but not well proofread and has mistakes. Such mistakes, even minor, would cause great difficulties in understanding for readers.[22]

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[21] For LI Shanlan's life and his contributions to mathematics, introduction of Western mathematics and mathematical education in China, see FANG 1943; and HORNG 1991.

[22] Translated from Wylie and LI, 1857/1865, Wylie's preface, 3a.

Though this provides an important, but not entirely correct, clue (namely, the original text was an old English translation of a complete Greek version of the *Elements*), the just-quoted passage does not mention the actual source of the translation done by Wylie and LI. Accordingly, early historians of Chinese mathematics, including LI Yan 李儼, ZHANG Yong 章用, and YAN Dunjie, all searched for the original source in vain.[23] However, another historian, QIAN Baocong 錢寶琮, ventured the following hypothesis: "The Englishman Isaac Barrow (wrongly spelled as "Issac" in the original text) first translated a Latin version from the Greek in 1655, and then rendered it into English in 1660. Barrow's English translation seems to be the original source of the Chinese translation."[24] QIAN's hypothesis has been widely cited ever since, but we shall see that QIAN's hypothesis is wrong. After describing in the next section the essential features of Barrow's English version of the *Elements*, and evidence that contradicts QIAN's claim, we will argue, based on more solid evidence, that the true source of the translation done by Wylie and LI was the first English translation of Euclid by Henry Billingsley of 1570.

4.2 Isaac Barrow and His English Translation of Euclid's *Elements*

Isaac Barrow was educated at Trinity College, Cambridge, where he was made a fellow in 1649. Inspired by the *Elementa geometriae planae et solidae* (*The Elements of Plane and Solid Geometry*), a popular Latin edition of the *Elements* (comprising only Books I–VI and XI–XII) by the French geometer, André Tacquet, Barrow decided to produce his

[23] LI Yan 1940, 801; and YAN Dunjie 1943, 35.

[24] QIAN Baocong 1981, 324.

own Latin version of the *Elements*. This resulted in a translation in 15 books, which was printed in London in 1655. Meanwhile, having been forced to leave the University because of his family's Royalist connections, with the Restoration of the monarchy Barrow was appointed to the professorship of Greek at Trinity College four years later. It was then that he prepared the second edition of his Latin version of Euclid, *Euclidis Elementorum libri xv* (1659), and then translated it, with corrections, into English. This first appeared in a small octavo edition in 1660.

In the English version of Barrow's *Elements*, like the previous two Latin editions, in order to achieve his goal of "[conjoining] the greatest compendiousness of demonstration with as much perspicuity as the quality of the subject would admit," Barrow used many symbols borrowed from the *Clavis Mathematicae* (*Key of Mathematics*) by William Oughtred, who devised new symbols, in particular, for his account of Book X of the *Elements*.[25] For instance, Barrow used "×" for multiplication; "+" for addition; "−" for subtraction; "√‾" for a root of any degree; ". :" for the difference of the two quantities or magnitudes on the two sides of the symbol; "**Q&q"** for a square; "**C&c"** for a cube; and "**Q**.**Q**." for the ratio of two square numbers.[26] While employing symbols for the sake of brevity, Barrow preserved the total number and order of the propositions and also tried to provide the same demonstrations of the propositions as Euclid himself did. Barrow wrote:

> I have altered nothing in the number and order of the propositions, nor taken the liberty to leave out any one of *Euclid's* as less necessary, or to reduce certain of the easiest into the Classis of Axiomes. . .

[25] Barrow 1660/1983, preface; and Murdoch 1971, 451-452.

[26] Barrow 1660/1983, preface. For a list of the major symbols used by Isaac Barrow in his English *Elements*, see "The Explication of the Signs or Characters," which immediately precedes Book I.

> I had a different purpose from the beginning; not to compose *Elements of Geometry* any wise at my discretion, but to demonstrate Euclide himself, and all of him, and that with all possible brevity. . .
>
> I purposed to use generally no other than Euclide's own demonstrations, contracted into a more succinct form, saving perchance in the second and thirteenth, & sparingly in the seventh, eighth, and ninth Books, where it seem'd convenient to vary something from him.[27]

The basic character of Barrow's treatment of the *Element*s was well summarized by John Keill, Savillian professor of astronomy at Oxford and a supporter of Isaac Newton in the calculus priority controversy. In the preface to his own treatment of the *Elements* (containing Books I–VI, XI, and XII only), which first appeared in 1702, Keill wrote:

> Barrow's demonstrations are so very short, and are involved in so many notes and symbols, that they are rendered obscure and difficult to one [who is] not versed in geometry. There, many propositions which appear conspicuous in reading Euclid himself, are made knotty, and scarcely intelligible to learners, by his algebraical way of demonstration; as is, for example, prop. 13 Book I. And the demonstrations which he lays down in Book II, are still more difficult: Euclid himself has done much better, in shewing their evidence by the contemplations of figures, as in geometry should always be done. The *Elements* of all science ought to be handled after the most simple method, and not to be involved in symbols, notes, or obscure principles, taken elsewhere.[28]

Despite Keill's criticisms, Barrow's English version of the *Elements* was well received in Britain. Over the course of the century since it first appeared in 1660, the book was reproduced (reprinted or reset) at least six times, first in 1686, and then in 1696, 1705, 1714, 1722, and 1732. The last edition appeared in 1751. The editions of 1705 (and after) also included as an appendix Archimedes' *Theorems on the Sphere and Cylinder*. The 1714 edition added Euclid's *Data*, as well as François Foix-Candale's *Brief Treatise*

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[27] Quoted from Barrow 1660/1983, preface.

[28] Quoted from Keill 1708/1782, A4.

of Regular Solids.[29] The next edition of 1722 also included additional supplements, namely some corollaries Barrow had deduced himself from the propositions of Euclid's *Elements*. The 1732 edition was issued after Barrow's death, and thus added his portrait. The last edition was virtually the same as that of 1732, except for a new supplement on logarithms.[30] Despite slight differences in the layouts of these editions, the content of the Euclidean geometry in all editions remained virtually the same. Consequently, in what follows, Barrow's first English edition is used for comparison with Wylie and LI's Chinese translation.

4.3 Barrow's Version of the *Elements* Contrasted with the Translation by Wylie and LI

Before comparing Isaac Barrow's English Euclid and Wylie/LI's Chinese translation of Euclid (hereafter referred to as *IBE* and *WLC*, respectively), to determine whether the former was really the source of the latter, it is necessary to describe the format and major characteristics of the latter. As already mentioned above, Alexander Wylie considered it his mission to fulfill the unfinished project left behind by Matteo Ricci. Thus it was natural for him to adopt the terminology and format created by Ricci and XU Guangqi for their translation of the first six books.

Two basic formats, as shown in a full English translation of two typical examples taken from Ricci and XU's translation by Peter M. Engelfriet, were followed in the

[29] François Foix-Candale's full French name is François de Foix, Comte de Candale, known in Latin as Franciscus Flussatus and in English as Flussas. For a brief biography of François Foix- Candale, see Morembert 1979.

[30] Simpkins 1966, 242-244.

arrangement of propositions in *WLC*.[31] One of these used frequently begins with the description of a proposition, then "explanations of the proposition" (*jie yue* 解曰), which is immediately followed by a "proof" (*lun yue* 論曰). Since some propositions contain more than one argument, this format was sometimes extended to include "another explanation" (*you jie yue* 又解曰) and "another proof" (*you lun yue* 又論曰). The other basic format starts with the description of a proposition followed immediately by "the [construction] method says" (*fa yue* 法曰). Corollaries, lemmas, and porisms were all translated under the heading *xi* (literally, "connected" or "subordinate"), which usually came at the end of a proposition. Some propositions also include a "note" (*an* 案), an "example" (*li* 例), or a "comment" (*zhu yue* 注曰), and sometimes all of these. In *WLC*, everything was articulated in words without use of mathematical symbols or formulas, and most of the propositions were illustrated with diagrams. The Roman and Greek letters used in the diagrams were replaced by Chinese characters borrowed from the 10 heavenly stems and 12 earthly branches.[32]

With the above brief description of the basic format and characteristics of *WLC* in mind, it is now possible to compare it with *IBE*. The most obvious contrasts between the two books are that (1) *IBE* makes liberal use of symbols, whereas *WLC* is totally free of symbols, and (2) *IBE* provides no comments, examples, or notes after the proof of a proposition, whereas *WLC* supplies a considerable amount of such additional explanatory commentary. For example, consider Book X, Proposition 42, which plainly illustrates these differences (see Figures 4 and 5). In order to facilitate this comparison, and make

[31] Engelfriet 1998, 142-145.

[32] For a list of the correspondence between English and Greek letters and their associated Chinese characters, see Martzloff 1997, 121.

what follows below clearer, the following is a literal translation, back into English, of Proposition 42 as it was rendered in *WLC*:

> Book X, Proposition XLII:
>
> Two straight lines which are incommensurable in square, on which the sum of the squares is a medial. A rectangle made of the lines is also medial. The rectangle is incommensurable with the sum of the two squares. Thus, the sum of the two lines is irrational, naming it the line of the two medials.
>
> Explanation: The two straight lines AB, BC are incommensurable in square. As the Problem stated, adding them gives AC. The Problem says AC is irrational.
>
> Proof: Let DE be a commensurable line, on which construct a rectangle DF, making it equal to the sum of the squares AB and BC (*Book I*, 45). Construct another rectangle GH, making it equal to twice the rectangle AB, BC. Then the rectangle DH is equal to the square AC (*Book II*, 4). The sum of the squares AB and BC is a medial, and is equal to the rectangle DF, therefore DF must be a medial. But it is applied to the rational line DE, so DG is rational and incommensurable with DE (*this Book*, 23). GI is also rational and is incommensurable with FG, that is, DE. Same reason as above. Since the sum of the squares AB, BC, is incommensurable with twice the rectangle AB, BC, the two rectangles DF, GH are incommensurable (*Book VI*, 1; *this Book*, 10). The two lines [DF and GH] are rational. Therefore, DF and GI are rational lines commensurable only in square. So, DI is irrational and called binomial (*this Book*, 37). While DE is rational, so the rectangle DH is irrational (*this Book, a note in Proposition* 39), therefore the side of a square which has the same area is irrational. The rectangle DH is equal to the square AC, therefore, AC is irrational, and is called the line of the two medials.
>
> Note: A line, whose square is equal to the sums of the squares on AB, BC, and of twice the rectangle AB, BC, is equal to the two medials. Therefore, it is called a line of the two medials.
>
> Another Note: An irrational line can be divided into two lines at one point only. The following problems will illustrate this matter. One example is given here first.
>
> Example: Suppose the point D divides AB evenly, and AC is greater than EB. Subtract the common part CE, the remainder AE is greater than CB. Since AD is equal to DB, therefore ED is less than DC. The distances between the midpoint D and the points C and E are not equal. The sum of the rectangle AC, CB, and the square CD is equal to the square DB (*Book II*, 5). The sum of the rectangle AC, EB and the square ED is also equal to the square DB. Therefore, the sum of the rectangle AC, CB and the square CD is equal to that of the rectangle AE, EB, and the square ED. Since the square ED is less than the square CD, the remaining rectangle AC, CB is less than AE, EB. Therefore, twice the rectangle AC, CB is less than twice AE, EB. Their difference is greater than the difference between the sum of the squares on AC, CB, and that of the squares AE, EB.[33]

[33] Translated from Wylie and LI 1857/1865, Book X, part I, 47b-49a.

Comparing this literal translation with the corresponding proposition in *IBE*, note the following discrepancies:

(1) *IBE*'s description is concise and replete with symbols such as = and +, whereas *WLC*'s description is lengthy and only verbal;

(2) The lettering used in the two diagrams does not correspond to each other;

(3) References to previous propositions upon which X, 42 depends are provided in the margin of *IBE*, but are incorporated into the text of *WLC*; and

(4) The most dramatic difference is that *WLC* has an explanation (*jie yue* 解曰) of the proposition, along with two short notes (*an* 案and *you an*又案), as well as a lengthy example (*li* 例) after the proof, none of which can be found in *IBE*.

If *IBE* were the original source of *WLC*, Wylie and LI would have had not only to translate the symbols of *IBE* into verbal language, but then to rearrange the text, providing their own notes and examples, which in Book X are also given for Propositions 10, 17, 19, 22, 31, 32, 38–41, 54, 60, 73, 88, and 117; in Book XII for Propositions 2 and 4; and in Book XIII for Propositions 1, 2, 13, and 18, and indeed, for many other propositions throughout *WLC*. Doing all of this would have required tremendous effort and considerable ingenuity.

Further comparisons of the two books reveal other major discrepancies. For instance,

(1) Book VII of *IBE* includes 23 definitions, the last of which states: "One number is said to measure another, by that number, which, when it multiplies, or is multiplied by it,

it produces."[34] This definition cannot be found in *WLC*, and the three postulates and twelve axioms immediately following the definition in *IBE* are also missing from *WLC*.[35]

(2) Again, in Book VII of *IBE*, Proposition 3 has only one corollary, but the same proposition in *WLC* has two;[36] moreover, Propositions 18, 19, and 36 all have a scholium or corollary, but there are none for the corresponding propositions in *WLC*.[37]

(3) The definitions for a parallelepiped and inscribed and circumscribed solid figures (Definitions XXX, XXXI, and XXXII) in Book XI of *IBE* are not to be found in *WLC*.[38]

(4) At the beginning of Book XIV of *WLC*, the translators state clearly that neither Book XIV nor XV was written by Euclid, but both were added to the *Elements* by Hypsicles of Alexandria. Wylie and Li also rendered the preface by Hypsicles into Chinese but *IBE* does not include the Hypsicles preface.[39]

Many more discrepancies, major and minor, between *IBE* and *WLC* could easily be identified. But the comparisons made above suffice to cast considerable doubt on QIAN Baocong's hypothesis that Isaac Barrow's English version of Euclid's *Elements* served as the basis for the Wylie/Li translation of Books VII–XV into Chinese. The question that now arises concerns the book Alexander Wylie and Li Shanlan actually used in preparing their Chinese translation of the last nine books of Euclid's *Elements*,

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[34] Barrow 1660/1983, 143.

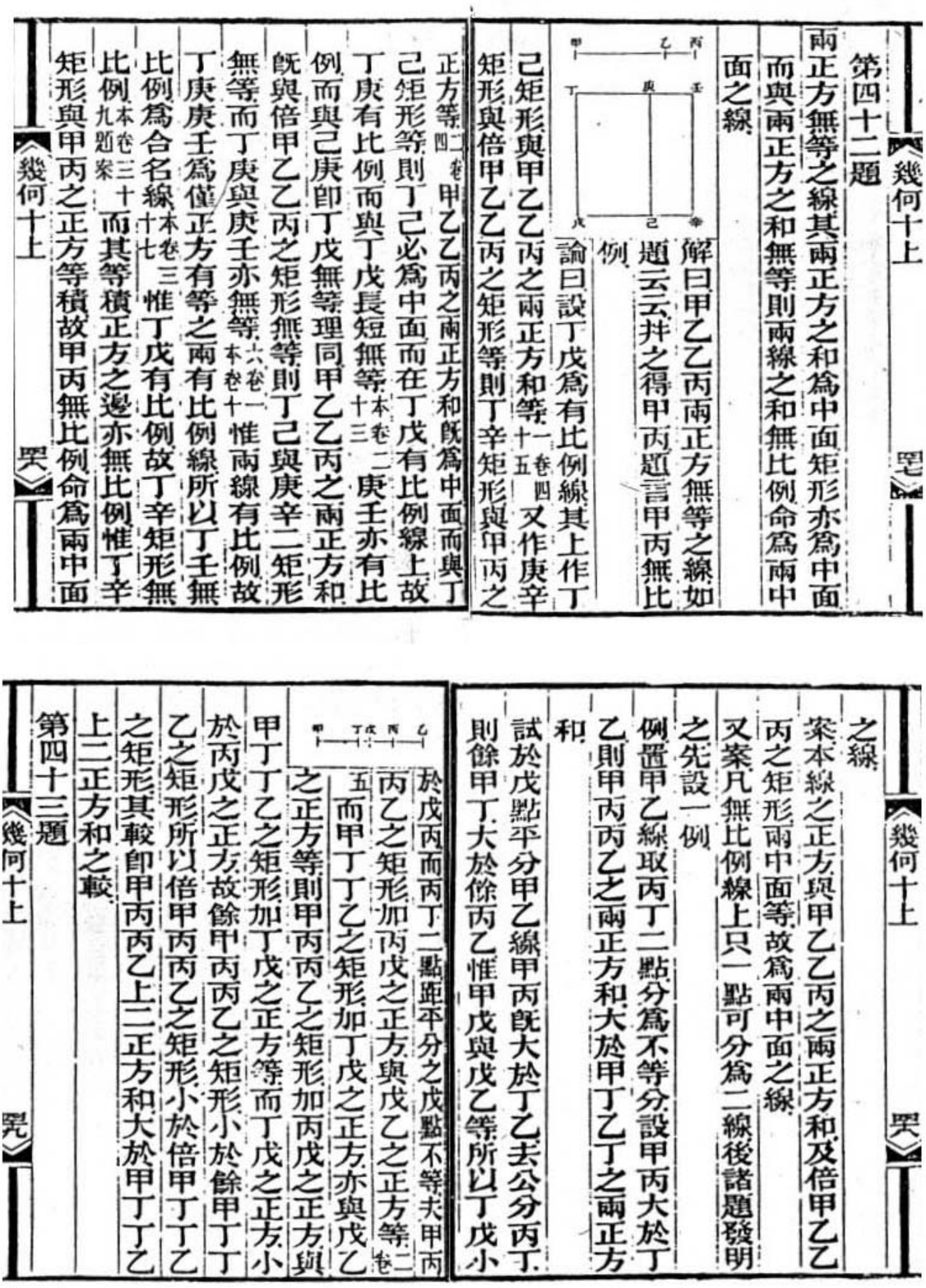
[35] Barrow 1660/1983, 143-144; Wylie and LI 1857/1865, Book VII, 2b-3a.

[36] Barrow 1660/1983, 146. Wylie and LI 1857/1865, Book VII, 4a.

[37] Barrow 1660/1983, 154, 159, and 161; Wylie and LI 1857/1865, Book VII, 13b-15a, and 25a-26a.

[38] Barrow 1660/1983, 270; Wylie and LI 1857/1865, Book XI, 4.

[39] Wylie and LI 1857/1865, Book XIV, 1; Barrow 1660/1983, 345.



第四十二題

兩正方無等之線其兩正方之和爲中面矩形亦爲中面而與兩正方之和無等則兩線之和無比例命爲兩中面之線

解曰甲乙乙丙兩正方無等之線如題云云幷之得甲丙題言甲丙無比例

論曰設丁戊爲有比例線其上作丁己矩形與甲乙乙丙之兩正方和等（一卷四十五）又作庚辛矩形與倍甲乙乙丙之矩形等則丁辛矩形與甲丙之正方等（二卷四）甲乙乙丙之兩正方和既爲中面而與丁己矩形等則丁己必爲中面而在丁戊有比例線上故丁庚有比例而與丁戊長短無等（本卷十三）庚壬亦有比例而與己庚即丁戊無等理同甲乙乙丙之兩正方和既與倍甲乙乙丙之矩形無等則丁己與庚辛二矩形無等而丁庚與庚壬亦無等（六卷一本卷十）惟兩線有比例故丁庚庚壬爲僅正方有等之兩有比例線所以丁壬無比例爲合名線（本卷三十七）惟丁戊有比例故丁辛矩形無比例（本卷三十九題案）而其等積正方之邊亦無比例惟丁辛矩形與甲丙之正方等積故甲丙無比例命爲兩中面之線

案本線之正方與甲乙乙丙之兩正方和及倍甲乙乙丙之矩形兩中面等故爲兩中面之線

又案凡無比例線上只一點可分爲二線後諸題發明之先設一例

例置甲乙線取丙丁二點分爲不等分設甲丙大於丁乙則甲丙丙乙之兩正方和大於甲丁乙丁之兩正方和

試於戊點平分甲乙線甲丙既大於丁乙去公分丙丁則餘甲丁大於餘丙乙惟甲戊與戊乙等所以丁戊小於戊丙而丙丁二點距平分之戊點不等夫甲丙丙乙之矩形加丙戊之正方與戊乙之正方等（二卷五）而甲丁丁乙之矩形加丁戊之正方亦與戊乙之正方等則甲丙丙乙之矩形加丙戊之正方與甲丁丁乙之矩形加丁戊之正方等而丁戊之正方小於丙戊之正方故餘甲丙丙乙之矩形小於餘甲丁丁乙之矩形所以倍甲丙丙乙之矩形小於倍甲丁丁乙之矩形其較即甲丙丙乙上二正方和大於甲丁丁乙上二正方和之較

第四十三題

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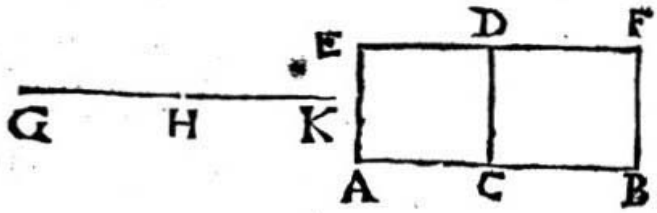
PROP XLI.

If two right lines AC, CB, *incommensurable in power, be added together, having that which is made of their ſquares added together mediall, and the rectangle conteined under them rationall, the whole right line* AB *ſhall be irrationall, and is called A line containing in power a rationall and a mediall rectangle.*

For 2 rectangles ACB ^{a} ρν, ^{b} □ ACq + CBq ^{c} μν. ^{d} therefore 2 ACB ^{d} □ ABq. wherefore ^{e} AB is ρ. *W. W. to be Dem.*

a hyp. and ſch 12. 10.
b ſch. 12. 10
c hyp.
d 17. 10.
e 11. def. 10.

PROP. XLII.



If two right lines GH, HK, *incommenſurable in power be added together, having both that which is compoſed of their ſquares mediall, and the rectangle contained under them mediall, and incommenſurable to that which is compoſed of their ſquares, the whole right line* GK *is irrationall, & is called A line containing in power two mediall figures.*

Upon the propounded line FB ρ make the rectangles AF = GKq, & CF = GHq + HKq. Being GHq + HKq (CF) ^{a} is μν, the breadth CB ^{b} ſhall be ρ. Alſo becauſe 2 rectangles GHK (^{c} AD) ^{a} is μν, therefore AC ^{b} ſhall be ρ. Moreover becauſe the rectangle AD ^{a} □ CF, ^{d} and [illegible]D. CF :: AC. CB. ^{e} thence ſhall ^{e} AG be □ CB. ^{f} wherefore A is ^{g} ρ. therefore the rectangle AF. *i.e.* GKq is ρν; ^{h} and conſequently GK is ρ. *W. W. to be Dem.*

a hyp.
b 23. 10.
c 4. 2.
d 2. 6.
e 10. 10.
f 37. 10.
g lem. 38. 10.
h 21 def. 10.

PROP.

Figure 5. Propositions 41 and 42 from Isaac Barrow's *English Elements*, Book X (1660 edition). From *Early English Books*, 1641–1700, 1383: 9, p. 224 (Ann Arbor, Michigan: University Microfilm International, 1981–1982).

and whether the exact edition of Euclid's *Elements* they used can be identified. The next section establishes beyond doubt that the source of the Wylie/LI translation must have been the first English edition of the *Elements*, published in 1570 by Billingsley.

4.4 Billingsley's Version of the *Elements* Compared with the Translation by Wylie and LI

By 1850 many English editions of Euclid's *Elements* were available when Alexander Wylie decided to translate the last nine books into Chinese. Nevertheless, most included only the first six books, although some added XI and XII as well.[40] Among English versions with 15 or even 16 books of the *Elements* are the first English translation in 15 books by Henry Billingsley of 1570 (hereafter referred to as *HBE*) and a translation by John Leeke and George Serle of 1661 (hereafter referred to as *LSE*). Of these, the New York Public Library's Rare Book Division has an original copy of *HBE*, and Columbia University's Butler Library has a microfilm copy of *LSE*. Comparing both of these with the Chinese translation leads to some interesting conclusions, but first it is necessary to introduce briefly these two English editions.

The first English translation of Euclid's *Elements*, by Billingsley, was made from the 1558 Latin edition: *Euclidis Megarensis mathematici clarissimi Elementorum geometricorum libri XV* (Figure 6). This in turn was the first Latin translation based upon a Greek text, and was made by Bartolomeo Zamberti who published it in Venice in 1505.[41] A brief description of *HBE* can be found in Charles Thomas-Stanford's *Early*

[40] Heath 1956, 109-112.

[41] Archibald 1950, 445-448.

Editions of Euclid's Elements, which provides a facsimile of the title page.[42] More detailed information may also be found in Walter F. Shenton's "The First English Euclid."[43] In addition to including a facsimile of the same title-page as shown in Thomas-Stanford's book, the article provides seven more illustrations, including a portrait of John Daye (also Day), the printer. Three-dimensional geometric models made out of folded pieces of paper serve to illustrate the text. Shenton also reprints the preface to the book in its entirety.[44]

Following "The Translator to the Reader," *HBE* has a lengthy introduction, 24 folios in all, by the Renaissance polymath John Dee, who was misattributed as the "true" translator of the book by August De Morgan.[45] The translation of the 13 books by Euclid covers 414 folios and are followed by the 14th and 15th books by Hypsicles (29½ folios). The book ends with an appendix as Book XVI (13 folios) and a "Brief Treatise of Mixed and Composed Regular Solids" (5 folios) by François Foix-Candale.[46] At the beginning of each book there is an outline, or as Billingsley puts it, "the argument" of the main contents of the book in question. For Books VII, X, and XI, explanations are also

[42] Thomas-Stanford 1977, 13-16, 43-44, Plate X.

[43] Shenton 1928.

[44] Shenton 1928, 505-512. The copy of Euclid that Walter F. Shenton consulted, now in the collection of the American University in Washington, DC, is a folio bound in two volumes, the first of which contains the first nine books of the *Elements*. The copy in the New York Public Library is of the same size, but bound in a single volume.

[45] De Morgan 1837, 38. A facsimile of the preface by John Dee, together with an introduction by Allen G. Debus, was published by Science History Publications in New York. See Dee 1975.

[46] The original pagination is not always correct. Certain numbers were skipped or repeated. For instance, there is a jump from folio 203 to folio 205, and in some cases the pages are wrongly numbered, for example, the three folios after 319 were numbered as 340, 341, and 317, respectively. But following these errors, the correct pagination is restored.

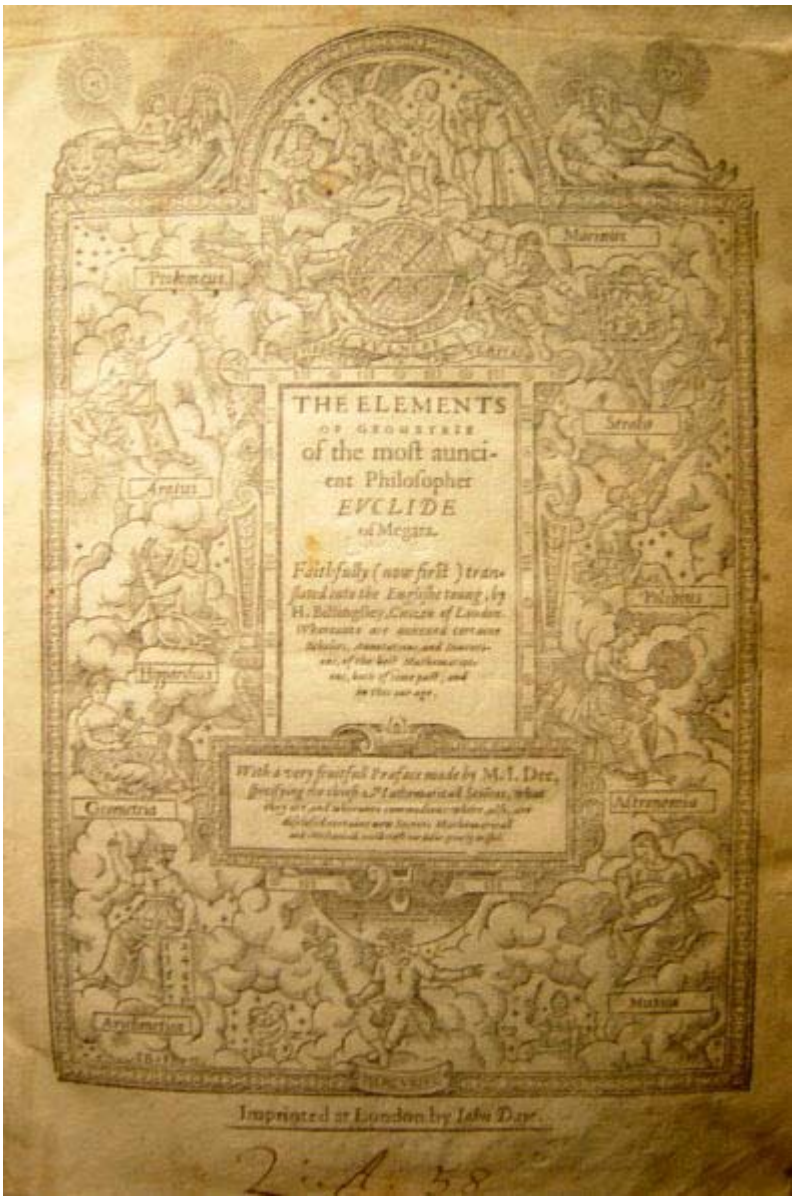


Figure 6. Title page of the Henry Billingsley's *Elements*. Stuart Collection, Rare Books Division, The New York Public Library, Astor, Lenox, and Tilden Foundations.

provided after every definition. A double numbering system, for both theorems and propositions, is used in the book. The translation is not a mere word-for-word translation, but contains "all the earlier and current commentary of importance."[47] Many corollaries, assumptions, and notes by Apollonius, Archimedes, Campanus, John Dee, Eudoxus, François Foix-Candale, Hypsicles, Montaureus, Pappus, Plato, Proclus, Pythagoras, Regiomontanus, and Theon of Alexandria, among others, are also incorporated.[48]

The *LSE* resembles the *HBE* in many ways. For instance, each book begins with "The Argument." Ornamental initial letters at the beginning of "The Argument" decorate the book throughout. Explanations, though shorter, are provided for each definition. A dual number system is adopted and descriptions of the propositions and their proofs are entirely verbal. Many corollaries and comments are also provided. Because of these and many other similarities, the *LSE* was mistaken for the second edition of *HBE* by Robert Potts, a 19th-century expert on Euclid.[49] Potts' view was widely accepted by many influential scholars, including August De Morgan and Thomas L. Heath.[50] His misattribution was only refuted later, in 1950, by Raymond C. Archibald. To quote Archibald again:

> It [the 1661 English edition] is simply an independent, and inferior, edition of the *Elements* in the preparation of which L[eeke] & S[erle]—the obscure 'students in mathematicks'—perhaps consulted B[illingsley's edition] constantly.[51]

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[47] Archibald 1950, 449.

[48] *Ibid*.

[49] Heath 1956, 110: 22, 233.

[50] De Morgan 1837, 38; Heath 1956, 110; Archibald 1950, 451.

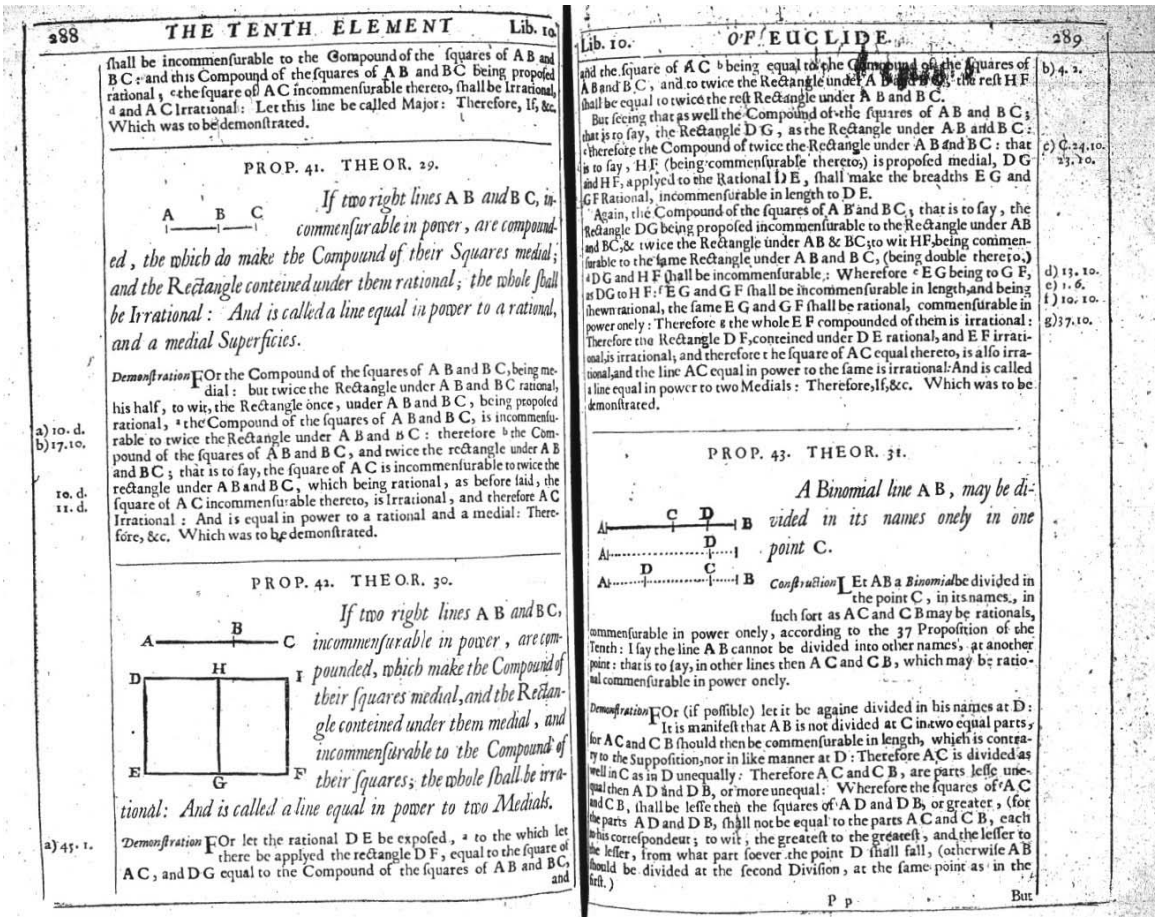
[51] Archibald 1950, 451.

Archibald provides some compelling evidence to make clear the differences between *HBE* and *LSE*, but these do not help much in deciding whether *HBE* or *LSE* was the model for *WLC*.[52] Nevertheless, there is direct evidence that shows that *LSE* could not have been the basis for *WLC*, and at the same time establishes the link between *HBE* and *WLC*. Book X, Proposition 42, which was used above to compare *IBE* and *WLC*, again offers a constructive basis for comparison. Proposition 42 in Book X is numbered as "PROP. 42. TEOR. 30." in *LSE*, and as "The 29. Theoreme. The 41. Proposition." in *HBE* (the discrepancy of these numberings is explained below). Comparing the proposition in *LSE* (Figure 7) with that in *WLC* as translated above (but see Figure 4 as well), the following differences can be observed: (1) The letterings *G*, *F*, and *H* in the diagrams do not correspond to one another; (2) *LSE* does not use words similar to those found in the section *jieyue* in *WLC*; and (3) like *IBE*, *LSE* does not include anything corresponding to the sections under the headings *an* and *li*.[53] But the *jieyue* appears in *WLC* throughout. Not one proposition in *LSE* provides such explanations accompanying the statement of each proposition. Moreover, the contents of the *an* and *li* in other places of *WLC*, some of which have been pointed out above, would have been added by Alexander Wylie and Li Shanlan themselves if *LSE* were the book they used for preparing *WLC*. But when we turn to compare Book X, Proposition 42 in *WLC* with its counterpart in *HBE*, we shall see there is no such possibility.

As the illustration (Figure 8) shows, the contents of "The 29. Theoreme. The 41. Proposition." Correspond very well to the literal translation of *WLC* provided in the

[52] Archibald 1950, 450-451.

[53] Leeke and serle 1661/1982, 288-289; Wylie and LI 1857/1865, Book X, part I, 47b-49a.



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ſhall be incommenſurable to the Compound of the ſquares of A B and B C: and this Compound of the ſquares of A B and B C being propoſed rational, c the ſquare of A C incommenſurable thereto, ſhall be Irrational, d and A C Irrational: Let this line be called Major: Therefore, If, &c. Which was to be demonſtrated.

PROP. 41. THEOR. 29.

A B C

If two right lines A B *and* B C, *incommenſurable in power, are compounded, the which do make the Compound of their Squares medial; and the Rectangle conteined under them rational; the whole ſhall be Irrational: And is called a line equal in power to a rational, and a medial Superficies.*

Demonſtration FOr the Compound of the ſquares of A B and B C, being medial: but twice the Rectangle under A B and B C rational, his half, to wit, the Rectangle once, under A B and B C, being propoſed rational, a the Compound of the ſquares of A B and B C, is incommenſurable to twice the Rectangle under A B and B C: therefore b the Compound of the ſquares of A B and B C, and twice the rectangle under A B and B C; that is to ſay, the ſquare of A C is incommenſurable to twice the rectangle under A B and B C, which being rational, as before ſaid, the ſquare of A C incommenſurable thereto, is Irrational, and therefore A C Irrational: And is equal in power to a rational and a medial: Therefore, &c. Which was to be demonſtrated.

a) 10. d.
b) 17. 10.
10. d.
11. d.

PROP. 42. THEOR. 30.

A B C
D H I
E G F

If two right lines A B *and* B C, *incommenſurable in power, are compounded, which make the Compound of their ſquares medial, and the Rectangle conteined under them medial, and incommenſurable to the Compound of their ſquares; the whole ſhall be irrational: And is called a line equal in power to two Medials.*

Demonſtration FOr let the rational D E be expoſed, a to the which let there be applyed the rectangle D F, equal to the ſquare of A C, and D G equal to the Compound of the ſquares of A B and B C, and

a) 45. 1.

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and the ſquare of A C b being equal to the Compound of the ſquares of A B and B C, and to twice the Rectangle under A B and B C, the reſt H F ſhall be equal to twice the reſt Rectangle under A B and B C.

But ſeeing that as well the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G, as the Rectangle under A B and B C: c therefore the Compound of twice the Rectangle under A B and B C: that is to ſay, H F (being commenſurable thereto,) is propoſed medial, D G and H F, applyed to the Rational D E, ſhall make the breadths E G and G F Rational, incommenſurable in length to D E.

Again, the Compound of the ſquares of A B and B C, that is to ſay, the Rectangle D G being propoſed incommenſurable to the Rectangle under A B and B C, & twice the Rectangle under A B & B C, to wit H F, being commenſurable to the ſame Rectangle under A B and B C, (being double thereto,) d D G and H F ſhall be incommenſurable: Wherefore e E G being to G F, as D G to H F: f E G and G F ſhall be incommenſurable in length, and being ſhewn rational, the ſame E G and G F ſhall be rational, commenſurable in power onely: Therefore g the whole E F compounded of them is irrational: Therefore the Rectangle D F, conteined under D E rational, and E F irrational, is irrational, and therefore the ſquare of A C equal thereto, is alſo irrational, and the line A C equal in power to the ſame is irrational: And is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated.

b) 4. 2.
c) C. 24. 10. 23. 10.
d) 13. 10.
e) 1. 6.
f) 10. 10.
g) 37. 10.

PROP. 43. THEOR. 31.

A C D B
A D
A D C B

A Binomial line A B, *may be divided in its names onely in one point* C.

Conſtruction LEt A B a *Binomial* be divided in the point C, in its names, in ſuch ſort as A C and C B may be rationals, commenſurable in power onely, according to the 37 Propoſition of the Tenth: I ſay the line A B cannot be divided into other names, at another point: that is to ſay, in other lines then A C and C B, which may be rational commenſurable in power onely.

Demonſtration FOr (if poſſible) let it be againe divided in his names at D: It is manifeſt that A B is not divided at C in two equal parts, for A C and C B ſhould then be commenſurable in length, which is contrary to the Suppoſition, nor in like manner at D: Therefore A C is divided as well in C as in D unequally: Therefore A C and C B, are parts leſſe unequal then A D and D B, or more unequal: Wherefore the ſquares of A C and C B, ſhall be leſſe then the ſquares of A D and D B, or greater, (for the parts A D and D B, ſhall not be equal to the parts A C and C B, each to his correſpondent; to wit, the greateſt to the greateſt, and the leſſer to the leſſer, from what part ſoever the point D ſhall fall, (otherwiſe A B ſhould be divided at the ſecond Diviſion, at the ſame point as in the firſt.)

P p But

Figure 7. Book X, 41 of the *Elements* translated by John Leeke and George Serle and published in 1661. From *Early English Books*, 1641–1700, 1283: 17, pp. 288–289 (Ann Arbor, Michigan: University Microfilm International, 1981–1982).

previous section. In particular, note the sentences “Let these two right lines AB and BC. . . Then I say that the whole line AC is irrational,” beginning at the ornately designed initial letter L, the words in the *jieyue* of *WLC* convey the same meaning. Just as “*jieyue* ” is part of the standard format of *WLC*, “Suppose that. . . Then I say,” or less frequently “Let there be. . . Then I say,” can be found throughout *HBE*. Furthermore, compare the first note *an* with the following words:

> A line whose power is two medials, is an irrationall line which is composed of two right lines incommensurable in power, the squares of which added together, make a medial superficies, and that which is contained under them is also mediall, and moreover it is incommensurable to that which is assumptes of the two quares added together.[54]

Similarly, compare the second note “*you an*” with this paragraph from *HBE*:

> And that the said irrationall lines are divided one way onely, that is, in one point onely, into the right lines of which they are composed, and which make every one of the kinds of those irrationall lines, shall straight way be demonstrated, but first will we demonstrate two assumptes here following.[55]

These comparisons clearly show that the two notes in *WLC* were doubtlessly translated from these two passages. Moreover, the last sentence of the previous quotation indicates that two “assumptions” are provided immediately after. Having compared the first “assumption” with the *li*, we must conclude that the contents accord directly with each other.

The above example is not an isolated case. Compare the last proposition of Book X, which is numbered as the 117th problem in *WLC*, and “The 92. Theoreme. The 116. Proposition.” in *HBE*.[56] (This proposition cannot be found in *IBE*—another solid piece of

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[54] Billingsley 1570, folio 261 *recto*.

[55] *Ibid.*

[56] Billingsley 1570, folio 310 *recto*.

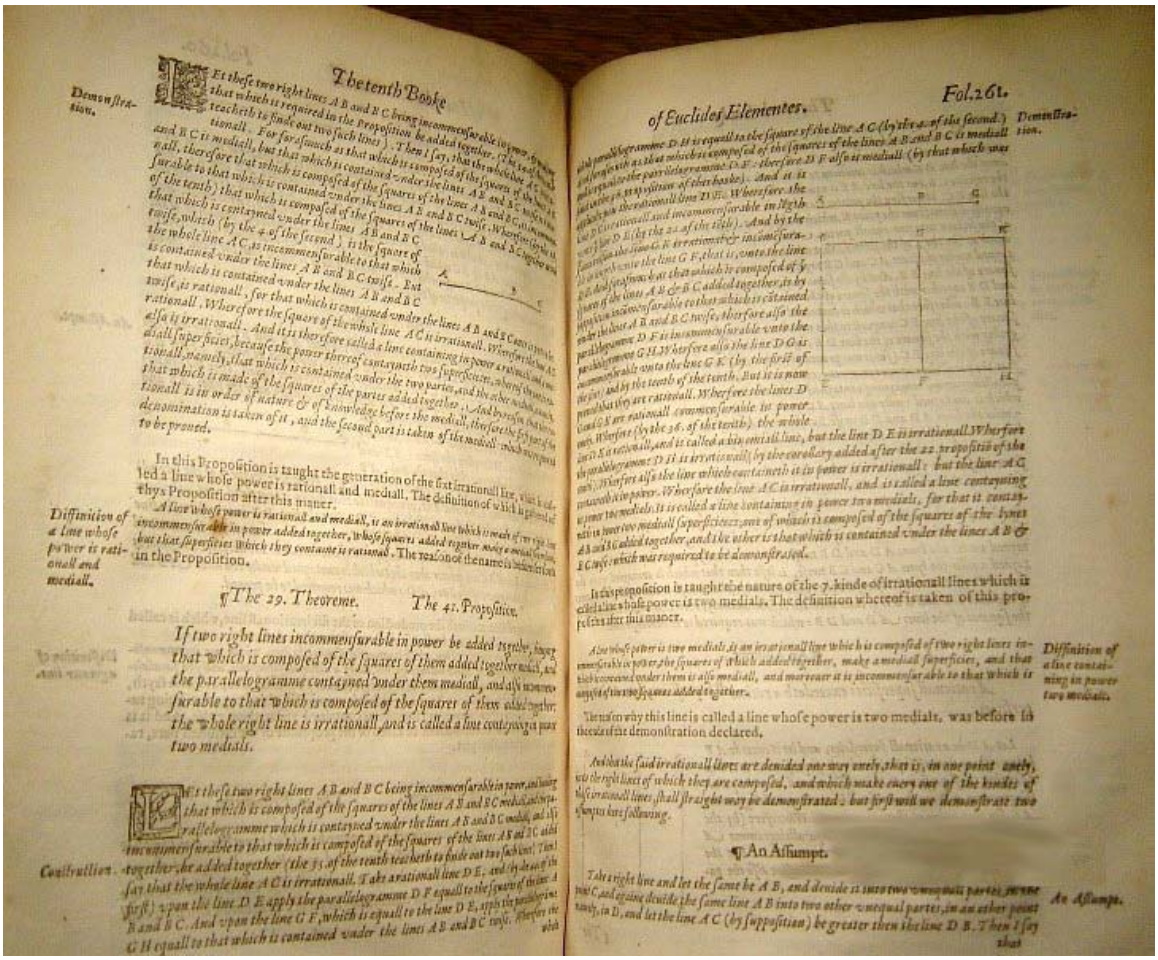

Figure 8. Book X, 41 of the Henry Billingsley's *Elements*. From Henry Billingsley's English translation (printed by John Daye in 1570), folios 260 *verso*, 261 *recto*. Stuart Collection, Rare Books Division, The New York Public Library, Astor, Lenox, and Tilden Foundations.

evidence that Barrow's *Elements* was not the basis for *WLC*.) At the end of the problem, *WLC* includes three notes (one "*an*" and two "*you an*"), which may be translated, once again *literally*, as follows (Figure 9):

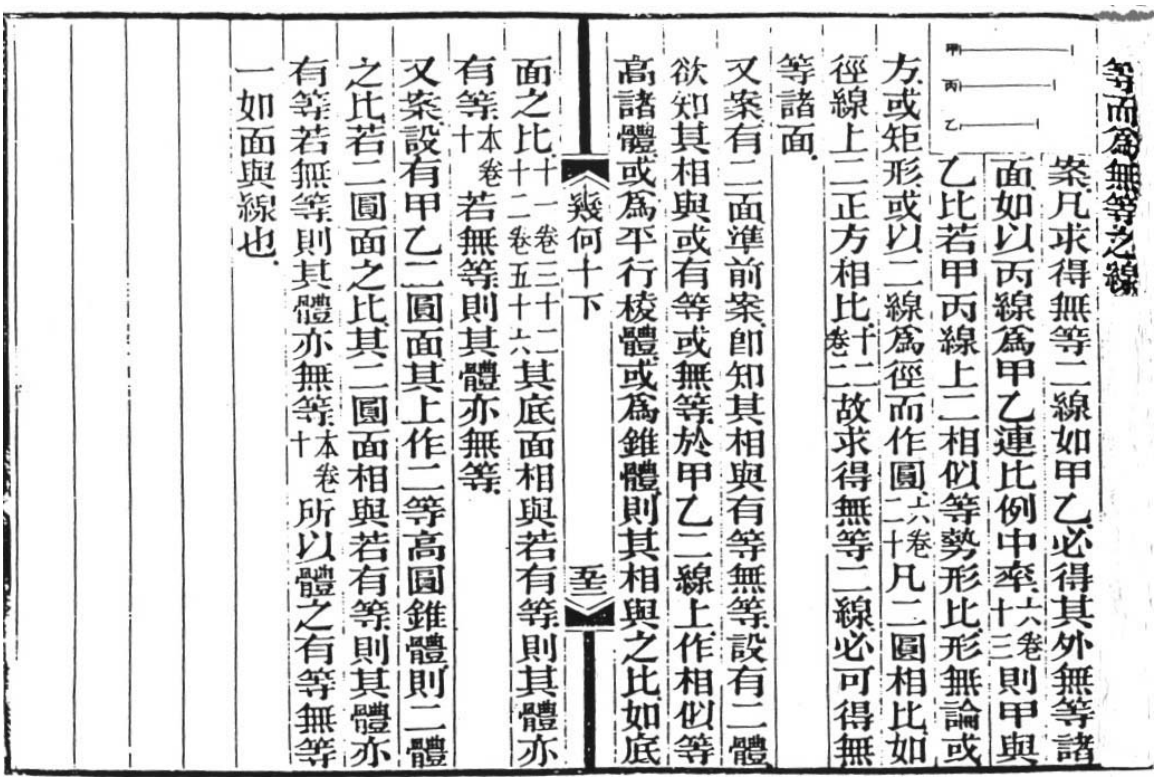

Figure 9. Alexander Wylie and LI Shanlan's Chinese translation of *Euclid's Elements*, Book X, 117. From the 1865 edition of *Jiheyuanben* , Book X, part III, p. 52.

> Note: For any two incommensurable lines, A, B, the squares on which must be incommensurable. If C is the mean proportional of A and B (*Book VI*, 13), the ratio of A to B is equal to that of any similar figures: squares, rectangles, or circles, on A and C (*Book VI*, 20). The ratio of two circles is equal to that of the two squares on their diameters (*Book XII*, 2). Therefore, from two incommensurable lines, many incommensurable areas can be deduced.
>
> Another Note: Any two areas, whether they are commensurable or incommensurable, can be judged according to the previous note.
> For any two solids, whether they are commensurable or incommensurable can be deduced from the similar solids, cylinders or cones, which have the same heights, constructed on lines A and B. The ratio of the two solids is equal to that of the two bases (*Book XI*, 32; *Book XII*, 56). If their bases are commensurable, so are

> the solids (*this Book*, 10). If their bases are incommensurable, so are the two solids.
>
> Another Note: On two circles, A and B, construct two cones with the same height. The ratio of the cones is equal to that of the circles. If the two circles are commensurable, so are the two solids. If incommensurable, so are their solids (*this Book*, 10). Therefore, whether solids are commensurable or not, is just like the cases of areas and lines.[57]

In *HBE*, there are the following related passages (Figure 10):

> Here Follows an instruction by some studious and skilful Grecian (perchance Theon) which teaches us of farther use and fruit of these irrational lines
>
> Seing that there are founde out right lines incommensurable in length the one to the other, as the lines A and B, there may also be founde out many other magnitudes having length and breath (such as are playne superficieces) which shall be incommensurable the one to the other. For if (by the 13. of the sixth) between the lines A and B there be taken the mean proportionall line, namely C, then (by the second corollary of the 20. of the sixth) as the line A is to the line B, so is the figure described upon the line A to the figure described upon the line C, being both like and in like sort described, that is, whether they be squares (which are alwayes like the one to the other), or whether they be any other like rectiline figures, or whether they be circles aboute the diameters A and C. For circles have that proportion the one to the other, that the squares of their diameters have (by the 2. of the twelfth). Wherfore (by the second part of the 10. of the tenth) the figures described upon the lines A and C being like and in like sort described are incommensurable the one to the other, wherfore by this means there are founde out superficieces incommensurable the one to the other. In like sort there may be founde out figures commensurable the one to the one, if ye put the lines A and B to be commensurable in length the one to the other. And seing that it is so, now let us also prove that even in solids also or bodyes there are some commensurable the one to the other, and other some incommensurable the one to the other. For if from each of the squares of the lines A and B, or from any other rectiling figures equal to these square be erected solids of equall altitude, whether those solids be composed of equidistance superficieces, or whether they be pyramids, or prismes, those solids so erected shal be in that proportion the one to the other that theyr bases are (by the 32. of the eleventh and 5, and 6, of the twelfth) Howbeith there is no such proposition concerning prismes. And so if the bases of the solids be commensurable the one to the other, the solids be commensurable the one to the other, the solids also shall be commensurable the one to the other, and if the bases be incommensurable the one to the other, the solids also shall be incommensurable the one to the other (by the 10. of the tenth) and if there be two circles A and B and upon ech of the circles be erected Cones or Cylinders of equal altitude, those Cones & Cilinders shall be in that proportion the one to the other that the circles are, which are their bases (by the 11. of the twelfth): and so if the circles be commensurable the one to the other, the Cones and Cilinders also shall be commensurable the one to the other. But if the circles be in

[57] Wylie and LI 1857/1865, Book X, part III, 52.

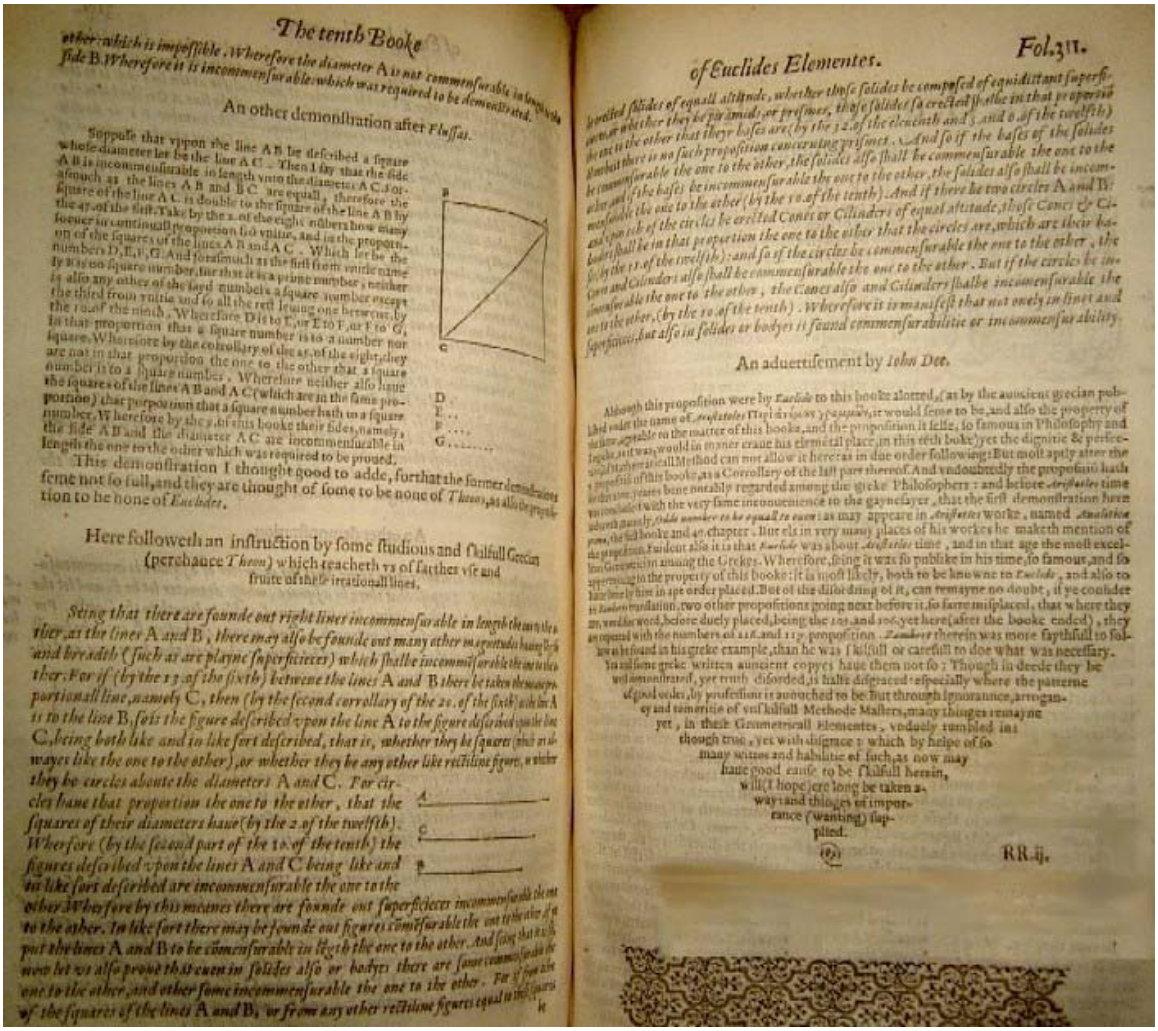

Figure 10. The Commentary on Book X, 116 of the Henry Billingsley's *Elements* by Theon of Alexandria. From Henry Billingsley's English translation (printed by John Daye in 1570), folios 310 *verso*, 311 *recto*. Stuart Collection, Rare Books Division, The New York Public Library, Astor, Lenox, and Tilden Foundations.

> commensurable the one to another, the Cones also and Cilinders shall be incommensurable the one to the other (by the 10. of the tenth). Wherfore it is manifest that not only in lines and superficieces, but also in solids or bodies is found commensurable or incommensurability.[58]

The three notes in *WLC* are all apparently translations from the "instruction" added by the prominent Greek mathematician Theon of Alexandria.[59] The above lengthy quotation not only provides further evidence to establish that *HBE* is indeed the source of *WLC*, but it also makes clear that *an* and *you an* are translations of the notes, comments, assumptions, and other supplements that Billingsley inserted in his book. They are not comments originally written by Alexander Wylie or LI Shanlan themselves.

Some historians have misattributed the three notes, as well as many others, to LI Shanlan, and wrongly praised him for having extended the theory of irrationals from straight lines to planes and solids.[60] Moreover, the contents under the commentary *zhu yue* are also translations. The commentary *zhu yue* in *WLC* Book X, Proposition 9, for instance, is a translation of "An Assumpt."of the same proposition in *HBE*.[61]

In comparing *WLC* and *IBE* above, it was pointed out that there is a preface by Hypsicles, the author of Books XIV and XV, before Book XIV in *WLC*. Although missing from *IBE*, it is to be found in *HBE*.[62] Nevertheless, *WLC* is not a cover-to-cover translation of *HBE*. In order to follow in general the format created by Ricci and XU

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[58] Billingsley 1570, folio 310 *verso* and folio 311 *recto*.

[59] For a biography of Theon of Alexandria, see Toomer 1976.

[60] WANG Yusheng 1990, 405-406; 1993, 1149.

[61] Billingsley 1570, folio 240 *verso*.

[62] Billingsley 1570, folio 416 *recto-verso*.

Guangqi, Wylie and LI Shanlan did not translate the outlines of each book provided in *HBE*, nor the lengthy explanation for every definition. Even the two definitions of Book XV, and many propositions, theorems, corollaries, assumptions, comments, and notes by scholars other than Euclid were omitted. Twenty propositions after Hypsicles' work, the entire Book XVI, and the appendix by François Foix-Candale in *HBE* are also missing from *WLC*.[63] In addition, the total number of propositions in Book X, and of the definitions in Book XI, is not identical. Book XI of *WLC* has 29 definitions, but *HBE* only 25. This difference was caused by breaking down certain original definitions into separate parts. For instance, the first definition in *HBE* was translated as the first two definitions in *WLC*.[64] As noted above, Book X, Proposition 42 in *WLC* actually corresponds to Proposition 41 in *HBE*. This difference was caused by "An Assumption" added after Proposition 12 in *HBE*: "If there be two magnitudes compared to one and the self same magnitude, and if the one of them be commensurable unto it, and the other incommensurable; those magnitudes are incommensurable the one to the other," which was translated, but numbered as Proposition 13 in *WLC*.[65] All of this evidence strongly suggests that *HBE* was the original source of *WLC*.

The specific English version of Euclid's *Elements* that Wylie used to prepare the first Chinese translation of Books VII to XV of the *Elements* was not the one by Isaac Barrow as some historians have speculated, but the one published in 1570 by Henry Billingsley, as argued above. This conclusion also coincides with Wylie's own brief description of the source he used. As he noted in his preface, "The book is a translation

[63] Billingsley 1570, folio 437 *recto* - 463 *recto*.

[64] Billingsley 1570, folio 312 *verso*; Wylie and LI 1857/1865, Book XI, part I, 1a.

from Greek into my native tongue. It has not been reprinted recently. The copy is an old version, but it was not well proofread and has mistakes."[66] Billingsley's translation of the *Elements* was published in 1570, and has never been reprinted since. Thanks to Raymond C. Archibald, we now know that Billingsley translated his edition from a 1558 Latin edition by Bartolomeo Zamberti. But Wylie's contemporary, the prominent mathematician August De Morgan, mistakenly wrote: "This translation [Billingsley's] of Euclid was either made from the Greek, or corrected by the Greek."[67] This widely held view may have led Wylie to believe that the source of the English *Euclid* he was using was based directly upon a Greek version. Wylie did not mention the name of the translator of the book he used, which may be explained by the fact that most writers of the 19th century did not bother to identify the sources they used, and also by the fact that De Morgan doubted that Billingsley was the true translator, and instead took John Dee to have been the translator in question.[68] Probably in order to avoid a possible mistake, Wylie may have thought it best to say nothing about the authorship of the English *Euclid* he was using. Billingsley's translation was beautifully printed by John Daye, but like any book, it was not free of printing errors. By saying that "it was not well proofread and has mistakes," Wylie may have intended to exaggerate the difficulties he and LI Shanlan had faced in translating Books VII through XV of Billingsley's *Euclid* for the first time into Chinese.

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[65] Billingsley 1570, folio 241 *verso*, 242 *recto*; Wylie and LI, 1857/1865, Book X, part I, 14a.

[66] Wylie and LI 1857/1865, Wylie's preface, 3a.

[67] De Morgan 1837, 39.

[68] *Ibid*.

4.5 Aspects of the Infinite as Treated by Wylie and LI in their Chinese Translation of Euclid's *Elements*

As early as the paradoxes of motion attributed to Zeno, the hare that can never catch the tortoise has prompted western mathematicians to view the infinite with suspicion. The Pythagoreans even told the story of Hippasus, who according to legend was killed in a shipwreck for having divulged the secret of incommensurable magnitudes, whose discovery presumably caused a crisis in Pythagorean philosophy, if not mathematics. Plato, however, delighted in how the human mind was able to prove the existence of incommensurable magnitudes, those seemingly irrational quantities that were certainly unnamable by simple arithmetic means of numbers or their ratios.[69] The Greek mathematician Eudoxus, realizing the importance of finding a means of dealing with incommensurable magnitudes, found a way that is basically the substance of Book V of the *Elements* that provides a rigorous theory of equal ratios for magnitudes, whether commensurable or incommensurable. But the proof that incommensurable magnitudes exist, along with a means of classifying them, is deferred until Book X of Euclid's *Elements*, where the theory due largely to Theatetus and his solution to the problem of incommensurable magnitudes are presented at length.[70] Historically, there is no doubt that the infinite, incommensurability, and related topics entail difficult and perplexing concepts, and when it came to translating the theory of proportion for magnitudes and such ideas as the incommensurability of the side and diagonal of the square into Chinese, the problem may well have seemed insurmountable.

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[69] The literature on the subject of the Greeks and discovery of incommensurable magnitudes is vast; the best overviews of the subject are presented in Lasserre 1964; and Fowler [1987] 1999. For specific analysis of the "crisis" in Pythagorean mathematics and a general assessment of the significance of the discovery of incommensurable magnitudes for Greek Mathematics generally, see Dauben 1992.

Indeed, how difficult that task was may be judged from the problems Wylie and LI faced in their treatment of Book X of the *Elements*, where concerns related to the infinite are confronted in the form of incommensurable magnitudes, and proved to be especially challenging. Book X is lengthy, and actually occupies more than a quarter of the entire *Elements*. In Sir Thomas L. Heath's definitive and popular English version, it requires 259 pages! Besides it physical size, Book X has always been considered by mathematicians, scholars and historians alike, as the most difficult part of the *Elements*. For instance, as Henry Billingsley wrote:

> This tenth booke hath ever hitherto of all men, and is yet thought & accompted, to be the hardest booke to understand of all the bookes of *Euclide*. Which common received opinion, hath caused many to shrinke, and hath (as it were) deterred them from the handeling and treatie thereof. [71]

Billingsley's contemporary, the Dutch mathematician and engineer Simon Stevin (1548 -1620), who advocated the use of decimals and once was very proud of himself for finding a key—namely a symbolic algebraic notation—to unlock the secrets of Book X, echoed Billingsley's view:

> La difficulté du dixiesme Livre d'Euclide est à plusieurs devenue en horreur, voire jusque à l'appeler la crois des mathématicians, matière trop dure à digérer, et en la quelle n'aperçoivent aucune utilité.
>
> The difficulty of the Tenth Book of Euclid has become for many a horror, even to calling it the cross of mathematicians, a subject matter too hard to digest and in which they perceive no utility.[72]

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[70] The details are discussed at length in Knorr 1975.

[71] Billingsley 1570, folio, 228 *recto*.

[72] The original French is quoted from Heath 1956: 3, 9. The English translation is by Wilbur R. Knorr, and quoted from Knorr 1983, 61.

These views of Billingsley and Stevin on the complexity of Book X have been shared by such prominent historians of mathematics as J. L. Heiberg, Sir Thomas Heath, B. L. van der Waerden, and Wilbur Knorr, among others. For instance, as Knorr puts it so succinctly: "Simply put, Book X is a pedagogical disaster."[73]

While understanding Book X alone requires no small effort, one must face even more considerable difficulties when it comes to translating it into a language where previously no foundation has been laid conceptually for such concepts as incommensurability. Moreover, if the translation is to a non-Western language such as Chinese, whose traditional mathematics lacked the basic concept of geometrical magnitude, the difficulty is even greater and far from trivial. In what follows, we limit our consideration to some aspects of Book X which take up the problem of the infinite in especially interesting ways. First we consider definitions, and then proceed to the opening propositions which deal with incommensurable magnitudes and anthyphairesis. We examine in particular Proposition X, 116, which proves the incommensurability of the diagonal and side of a square. Proposition 2 of Book XII, which treats of the "exhaustion" of the area of a circle by inscribing polygons of increasingly many sides, will also be considered.

Billingsley's English version of Book X of Euclid's *Elements* divides the definitions into three groups. The first group, which includes eleven definitions, immediately follows a summary of the Book. The second group is placed between Propositions 47 and 48. It includes six definitions. The third and last group is inserted between Propositions 84 and 85. Like the second group, the third also provides six

[73] Knorr 1983, 59.

definitions. Under each definition, Billingsley provides explanations, some lengthy and some brief, illuminating the essence of each definition.

The most basic and difficult definitions to translate are those in the first group, at the start of Book X, which begins as follows:

> Definition 1. Magnitudes commensurable are such, which one and the selfe same measure doth measure.[74]
>
> 第一界. 凡幾何有他幾何可度，為有等之幾何。[75]
>
> Heath 1: Those magnitudes are said to be commensurable which are measured by the same measure.[76]

"Magnitude" here is translated as 幾何 *ji he*, which Wylie and LI simply borrowed from Matteo Ricci and XU Guangqi's translation of the first six Books of the *Elements*. Traditional Chinese mathematics lacked this concept, as we have already discussed at length in Chapter 2. Before the seventeenth century, the words "幾何" were used in Chinese to describe numbers or to address questions like "how many?" or "how much?" When borrowing the term 幾何 *ji he* to translate "magnitude," Ricci and XU did not provide any explanation, or even a note, to differentiate the two different concepts of "magnitude" and "number."[77] Nor is such an explanation to be found in Wylie and LI's translation.

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[74] Quoted from Billingsley 1570, folio, 228 *recto*.

[75] Quoted from *TH* 1993: 5, 1340b.

[76] For comparison's sake, admitting that the florid English prose of Billingsley's translations is an added impediment to understanding the meaning of the Euclidean originals, we provide here as well the more familiar English translations of Euclid's definitions in Book X due to Heath. Quotation here is from Heath 1956: 3, 10.

[77] Peter M. Engelfriet in his study of the Chinese translation of the first six Books of Euclid's *Elements* argues that the term "幾何" in the translators' minds "actually mean[s] 'magnitude','' (Engelfriet 1998, 184). This certainly seems to be true. The problem is that Matteo Ricci and XU Guangqi did not provide

Similarly, the translation of the word "commensurable" as 有等 *you deng*, which literally means "something equal/are equal/have equality," is also ambiguous. In the original text, following the strict principle of homogeneity in Greek mathematics, magnitudes are always compared to other magnitudes "*of the same kind*." The Chinese translation does not explain the meaning of "*the same kind*," and as a whole conveys a meaning something like this: "Magnitudes or numbers which can be measured by another one have something equal."

But if the translation of "commensurability" leaves something to be desired, consider the following translation of Definition 2, which is actually more intelligible:

Definition 2:	Incommensurable magnitudes are such, which no one common measure doth measure.
第2界.	凡幾何無他幾何可度，為無等之幾何.
Heath 1:	and those incommensurable which cannot have any common measure.[78]

The Chinese translation may be rendered back into English as: "Magnitudes or numbers which cannot be measured by another one do not have something equal." Next comes an even trickier definition:

Definition 3	Right lines commensurable in power are such, whose squares one and the selfe same superficies, area, or plat doth measure.
第3界.	凡正方有面可度，其邊為正方有等之線.
Heath 2:	Straight lines are commensurable in square when the squares on them are measured by the same area.[79]

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any explanation to make Chinese readers aware of the new meaning this common term had been assigned, rather than its established meaning in previous mathematical practice.

[78] The original is quoted from Billingsley 1570, folio, 229, *recto*; the Chinese translation is from *TH* 1993: 5, 1340b; and Heath's version is from Heath 1956:3, 10.

[79] The original is quoted from Billingsley 1570, folio, 229, *recto*; the Chinese text is from *TH* 1993: 5, 1340b; and Heath's version if from Heath 1956:3, 10.

Definition 3 from the Chinese may be translated back into English as follows: "If squares can be measured by an area, then the sides of the squares are lines having something equal in square." Comparing this with the original text: "Right lines commensurable in power are such, whose squares one and the selfe same area doth measure,"[80] one may say it is not bad, but what would this have conveyed to a first-time reader with no previous familiarity with Euclidean geometry? In any case, as for the translation itself, the correctness of the concept it is meant to convey is apparent upon comparison with respect to the Heath translation of this same text. The same is true of the conversion from the Billingsley to the Chinese translation of Definition 4:

Definition 4:	Lines incommensurable are such, whose squares no one plat or superficies doth measure.
第4界.	凡正方無面可度，其邊為正方無等之線.
Heath 2:	and incommensurable in square when the squares on them cannot possibly have any area as a common measure.[81]

The original text of Definition 5 is more problematic because it is too concise. It reads: "And that right line so set forth is called a rationall line."[82] At this point Billingsley felt compelled to provide a lengthy explanation running to one-and-a-half folio pages, explaining "of that right line," which "may be composed infinite lines, some commensurable both in length & power, and some commensurable in power onely, and

[80] The original is quoted from Billingsley 1570, folio, 229, *recto*; and the Chinese text is from *TH* 1993: 5, 1340b.

[81] *Ibid*.

[82] Quoted from Billingsley 1570, folio, 229, *verso*.

incommensurable in length, and some incommensurable both in power & in length."[83]

The Chinese explanation runs as follows:

> 準上四說，凡線有無窮相與有等及無等，線或長短及正方俱有等，或僅正方有等，或俱無等。原線謂之有比例線。[84]

This may be translated back into English as follows:

> With the above four definitions, lines have infinite commensurable or incommensurable lines. Lines may be commensurable in length or in square, or in square only, or none is incommensurable. The set line is called a line of having a ratio.

Heath 3: With these hypotheses, it is proved that there exist straight lines infinite in multitude which are commensurable and incommensurable respectively, some in length only, and others in square also, with an assigned straight line. Let then the assigned straight line be called rational

In the above translation, "commensurable" is used to translate 有等 *you deng*, "having something equal;" just as "incommensurable" is used to translate 無等 *wu deng*, "not having something equal." Moreover, in the Chinese translation, "rational line" is rendered as "有比例線," which literally means "line which has a proportion." (This term was also borrowed from Ricci and XU's translation.) This is arguably correct, but using the same criterion of whether lines can have a proportion to define "irrational line," as in Definitions 7 and 11, is mistaken. This is because whereas any lines can have proportion, this is not necessarily something that can be expressed in terms of numbers (i.e. as a rational number). Although the side and diagonal of a square are incommensurable, the diagonal nevertheless expresses geometrically this incommensurable magnitude as a length; there was no corresponding number in Greek mathematics by which this could be

[83] Quoted from Billingsley 1570, folio, 230 *recto*.

[84] Quoted from *TH* 1993: 5, 1340b.

expressed numerically. Thus the Greeks regarded arithmetic as incomplete, lacking the concept of irrational number, and when desiring complete generality, confined their mathematics to geometry, which did not suffer this same incompleteness, and relied upon the Eudoxean theory of proportion to deal with such matters as in Book V, or in the case of incommensurable magnitudes, as in Book X of the *Elements*.

Having considered the evidence of Billingsley's translation and the difficulties encountered in trying to express such concepts as commensurable, incommensurable, commensurable in square, incommensurable in square, etc., in Chinese, what do we find in moving from Definitions to the actual Propositions of Book X? How are these handled in the Chinese translation?

Thus far, we have seen some worrisome evidence about the imprecise translation of basic definitions that would not have left a very clear sense in the minds of Chinese readers about exactly what the point of Euclid's Book X might have been. Most of the trouble here can be attributed to the lack of a concept for geometrical magnitude in traditional Chinese mathematics. Since the crux of the original Greek problems with incommensurable magnitudes all stemmed from the method of anthyphairesis, by which incommensurable magnitudes may well have been discovered in the first place, how was the anthyphairesis method introduced through the translation of Book X of Euclid's *Elements*?

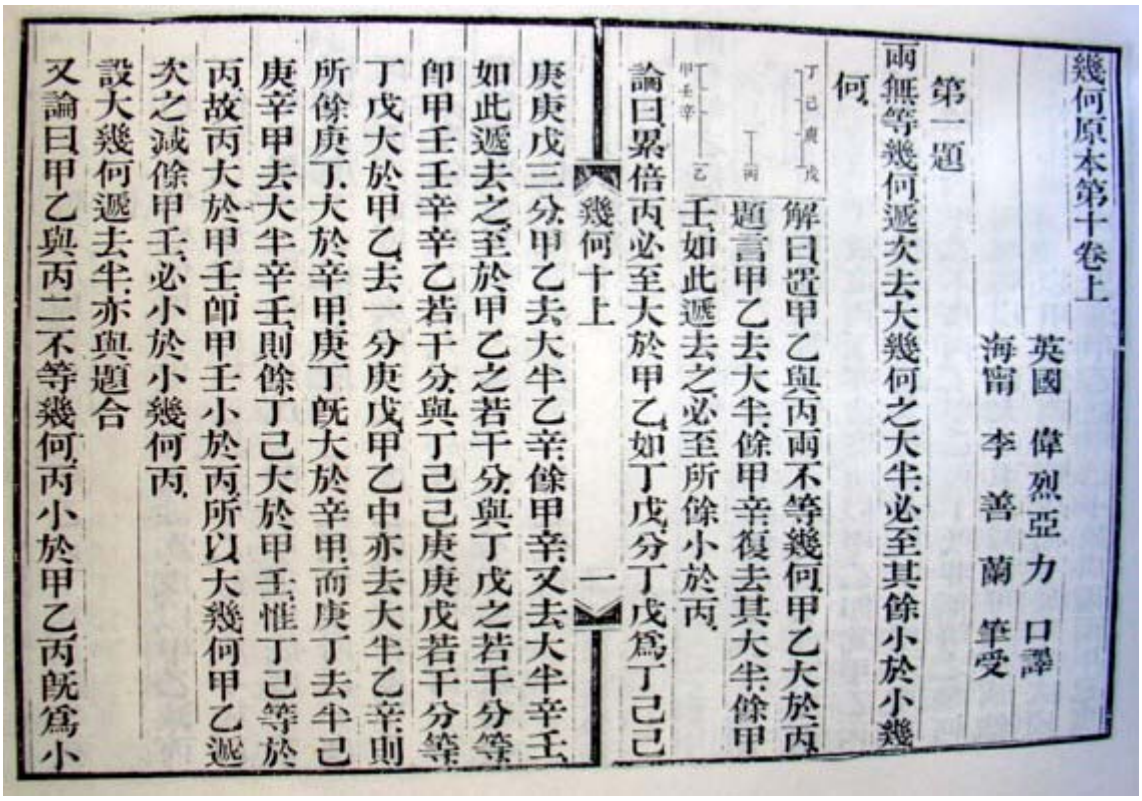
Anthyphairesis, or the "Euclidean algorithm," is used in Book X, Propositions 1-3. Proposition 1 reads as follows:

> Two unequall magnitudes being given, if from the greater be taken away more than the halfe, and from the residue be againe taken away more than the halfe,

and so be done still continually, there shall at length be left a certaine magnitude lesser than the lesse of the magnitudes first given.[85]

This is rendered into Chinese by Wylie and LI as follows (see Figure 11):

兩無等幾何遞次去大幾何之大半，必至其餘小於小幾何。[86]


幾何原本第十卷上

英國 偉烈亞力 口譯
海甯 李善蘭 筆受

第一題

兩無等幾何遞次去大幾何之大半必至其餘小於小幾何

解曰設甲乙與丙兩不等幾何甲乙大於丙
題言甲乙去大半餘甲辛復去其大半餘甲
壬如此遞去之必至所餘小於丙

論曰累倍丙必至大於甲乙如丁戊分丁戊爲丁己己

庚庚戊三分甲乙去大半乙辛餘甲辛又去大半辛壬
如此遞去之至於甲乙之若干分與丁戊之若干分等
即甲壬壬辛辛乙若干分與丁己己庚庚戊若干分等
丁戊大於甲乙去一分庚戊甲乙中亦去大半乙辛則
所餘庚丁大於辛甲庚丁既大於辛甲而庚丁去半己
庚辛甲去大半辛壬則餘丁己大於甲壬惟丁己等於
丙故丙大於甲壬即甲壬小於丙所以大幾何甲乙遞
次之減餘甲壬必小於小幾何丙

設大幾何遞去半亦與題合
又論曰甲乙與丙二不等幾何丙小於甲乙丙既爲小

Figure 11. Alexander Wylie and LI Shanlan's Chinese translation of Euclid's *Elements*, Book X, 1. From the 1865 edition of *Jihe yuanben*, Book X, part I, p. 1341b.

[85] Billingsley 1570, folio, 232 *verso*.

[86] *TH* 1993: 5, 1341b. The two characters 無等 *wu deng* in this sentence must be a printing mistake. In the above discussion of the translation of Definition 1, we point out that Wylie and LI use this term for "incommensurable." But here the original text says "unequall magnitudes," not "incommensurable magnitudes." Though the literal meaning of "無等" is "not equal," it should not be used with this meaning in mind once it has been used as a technical term. In fact, in the two lines left of "無等" here are another two characters, 不等 *bu deng*, used to mean "unequal" (see Fig. 8). Similar evidence can also be found at the beginning of Book XII, Proposition XII, 2 of the Chinese translation; see *TH* 1993: 5, 1447b, and 1449b.

This translation, as well as the following proofs, are on the whole quite accurate, except for the proper understanding of those terms explained less than clearly in the definitions discussed above. Two observations need to be made here. First, the proofs in the original text use the so-called Archimedean Axiom without mentioning it explicitly by name.[87] Even so, the Chinese translation rightly captures its essence. But the translators provide no explanatory comment, nor do they link this Axiom to a sentence in the *Moist Cannon*, as they might have done, to make this all more transparent to Chinese readers, namely, "窮，或有前不容尺也" (The *qiong* (exhaustion or limit) is wherever at the next advance there is no room for a measured length) [see Chapter 2, page 32]. Some scholars have argued that this statement is equivalent to the Axiom of Archimedes [see Chapter 2, page 36]. It seems, however, that LI Shanlan was not among those who thought so; at least no attempt is made here in the Chinese translation to draw this connection, or to exploit its significance. As for the method of anthyphairesis, it is correctly introduced in the context of Euclid's proof. This is perhaps more evident from the translation of the next proposition, Book X, 2:

> Two unequall magnitudes being given, if the less be continually taken from the greater, & that which remayneth measureth at no time the magnitude going before: then are the magnitudes given incommensurable.
>
> 大小兩幾何，輾轉相減而所餘幾何，俱不度原幾何，則為兩無等幾何。[88]

The words, "be continually taken from" are translated as "輾轉相減," which conveys the meaning that the process may go on and on. In Problem 6 of Chapter 1, *Fangtian*, of the *Nine Chapters*, there is a method named "*geng xiang jian sun* 更相減損,"

---

[87] Billingsley 1570, folio 232 *verso*, 233 *recto*.

[88] The English text is quoted from Billingsley 1570, folio, 233 *recto*, and the Chinese translation is from *TH* 1993, 5: 1342a.

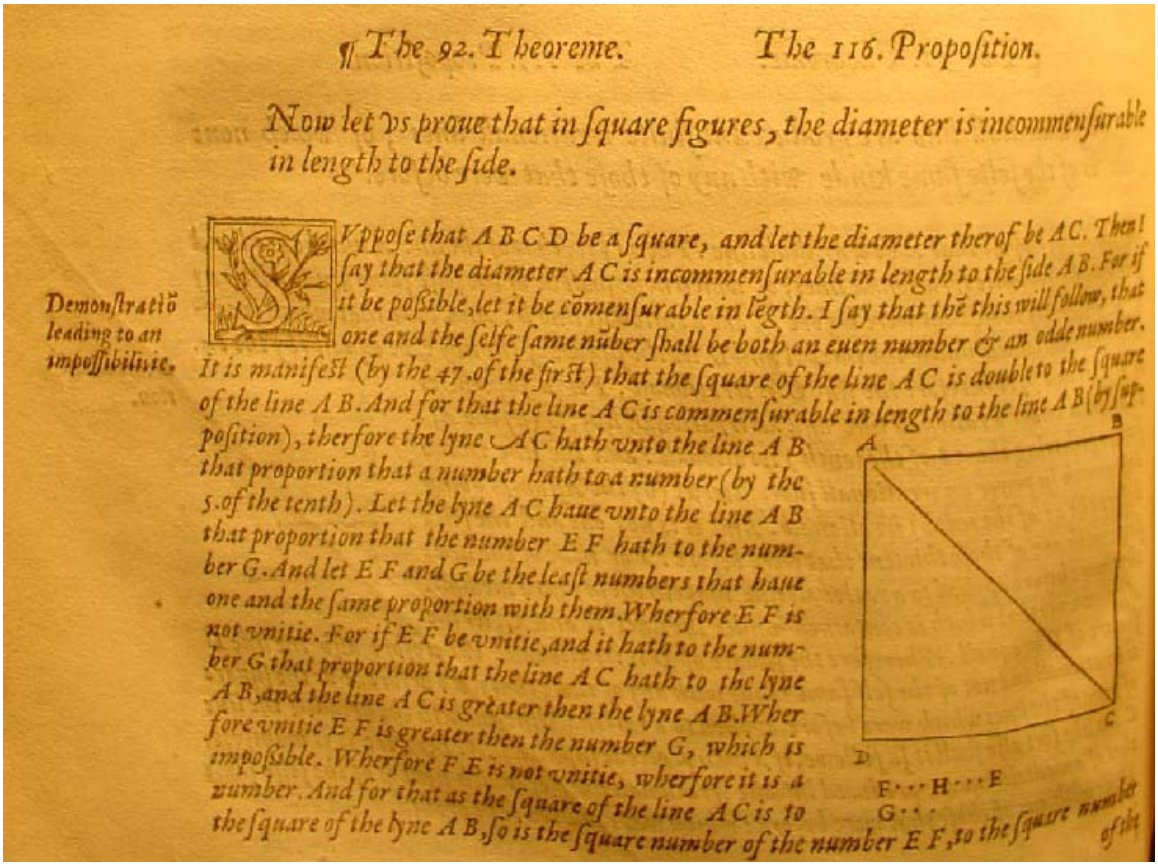
which continually subtracts the numerator and the denominator of a fraction in order to find a common factor to simplify the fraction. However, this method in the *Nine Chapters* (and its applications in other Chinese texts) is confined to whole numbers, and the process is apparently taken to be finite. Perhaps LI Shanlan here tried to connect, and at the same time differentiate, the method of anthyphairesis and the *geng xiang jian sun* method of the *Nine Chapters*, by using "輾轉相減" to describe a continuous process with regard to magnitudes.

Proposition X, 1 is important in a number of ways. It provides not only a logical foundation for the theory of incommensurability, but it also provides a rationale for the method of exhaustion that plays a crucial role in the proof of Book XII, Proposition 2, to which we shall turn after discussing Wylie and LI's translation of the argument which establishes the existence of incommensurable magnitudes.

The Proposition establishing the incommensurability of the diagonal of a square with its side is the last in Book X of Billingsley's English translation of the *Elements* (see Figure 12). Historians including Heiberg and Heath believe it is an interpolation.[89] The proof in essence is the same as that recorded by Aristotle in his *Prior Analytics*.[90] It uses a *reduction ad absurdum* argument, and basically goes like this:

[89] See Heath 1956, 3: 2.

[90] See Heath 1949, 23-23; Heath 1956, 3: 2.



¶ The 92. Theoreme. The 116. Proposition.

Now let vs proue that in ſquare figures, the diameter is incommenſurable in length to the ſide.

Demonſtratiõ leading to an impoſſibilitie.

Vppoſe that A B C D be a ſquare, and let the diameter therof be A C. Then I ſay that the diameter A C is incommenſurable in length to the ſide A B. For if it be poſſible, let it be cõmenſurable in lẽgth. I ſay that thẽ this will follow, that one and the ſelfe ſame nũber ſhall be both an euen number & an odde number. It is manifeſt (by the 47. of the firſt) that the ſquare of the line A C is double to the ſquare of the line A B. And for that the line A C is commenſurable in length to the line A B (by ſuppoſition), therfore the lyne A C hath vnto the line A B that proportion that a number hath to a number (by the 5. of the tenth). Let the lyne A C haue vnto the line A B that proportion that the number E F hath to the number G. And let E F and G be the leaſt numbers that haue one and the ſame proportion with them. Wherfore E F is not vnitie. For if E F be vnitie, and it hath to the number G that proportion that the line A C hath to the lyne A B, and the line A C is greater then the lyne A B. Wherfore vnitie E F is greater then the number G, which is impoſsible. Wherfore E F is not vnitie, wherfore it is a number. And for that as the ſquare of the line A C is to the ſquare of the lyne A B, ſo is the ſquare number of the number E F, to the ſquare number of the

Figure 12. Book X, 116 of Henry Billingsley's *Elements*, folio 309 *verso*. From the Stuart Collection, Rare Books Division, The New York Public Library, Astor, Lenox and Tilden Foundations.

Assuming the diagonal *AB* and *AC* are commensurable, then, the ratio of *AB* to *AC* can be set in the proportion of a ratio of one number, *m*, to another number, *n* (*m* and *n* are assumed to be reduced to lowest terms at the outset). According to the Pythagorean theorem, the square of *AC* is twice of that of *AB*. Therefore, the square of *n* is twice the square of *m* (i.e. $n^2 = 2m^2$). Accordingly, *n* must be an even number. Let it be *2k*, and then substitute this for *n*, resulting in quadruple the square of *k* (i.e. $4k^2$), which is equal to twice the square of *m* (i.e. *2m*). But if $4k^2 = 2m$, then $2k^2 = m$, and as a result, *m* must also be an even number. Since *m* and *n* were supposed to have already been reduced to lowest terms at the outset, this contradiction establishes the result. Since *m* and *n* cannot

both be even, the ratio of the side and diagonal of the square *AB* : *AC* cannot be expressed as a ratio of numbers *m* : *n*. Therefore, the diagonal *AC* and the side *AB* are incommensurable.

The details of the proof in the original text are faithfully translated into Chinese. Even the term "unitie" (unity) is rightly rendered as "*yi* 一" (one).[91] But as one might have expected, the Chinese translation makes no comment to link this very proposition to the term of "*yi mian ming zhi* 以面命之." This is related to another matter, namely the claim some historians of Chinese mathematics have made that this phrase (to be found in LIU Hui's commentary on the *Nine Chapters*) is evidence that ancient Chinese mathematicians knew of the incommensurability of the side and diagonal of the square at least since the time of LIU Hui. But were this so, surely Wylie and LI would have been expected to make mention of this fact here. But clearly no such link was evident to LI Shanlan.

One final example of how the infinite was treated by Wylie and LI in their translation of the *Elements* brings us to the method of "exhaustion" in determining the area of a circle, which is given in Book XII, Proposition 2:

> Circles are in that proportion the one to the other, that the squares of their diameters are.
>
> Heath XII, 2: Circles are to one another as the squares on the diameters.[92]

Proposition XII, 2 says that area of the circle *ABCD* to the area of the circle *EFGH* is in the same proportion as the square of the diameter *BD* to the

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[91] See *TH* 1993, 5: 1218a.

[92] Quoted from Billingsley 1570, folio, 354 *verso*, which is wrongly numbered as page 357 in the original text. Heath's text is quoted from Heath 1956: 3, 371.

square of the diameter *FH* (see Figure 13). The proof is once again by *reduction ad absurdum*, and at its core applies the result of Euclid X, 1. However, this time it is shown that the difference between the area of a circle and the area of its inscribed polygons can be made smaller than any given arbitrarily small area.[93]

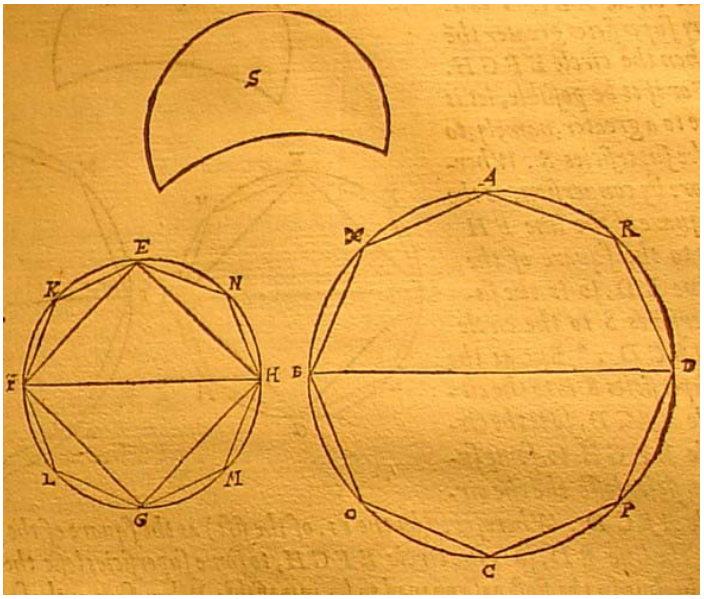

Figure 13. Diagrams in Book XII, 2 of Billingsley's *Elements*, folio 355 *recto*. Stuart Collection, Rare Books Division, The New York Public Library, Astor, Lenox and Tilden Foundations.

Though one may not find the original proof easy to follow, the Chinese translation, however, is accurate. In fact, all of the passages on incommensurable magnitudes and other contents related to the infinite in the *Elements* are accurately translated. The success of this work may be explained by the talent of the translators, Alexander Wylie and LI Shanlan, as well as by the lengthy and excellent commentaries inserted by Billingsley in his English version of the *Elements,* upon which Wylie and LI were

[93] For detailed proof, see Billingsley 1570 *folio*, 354 *verso*, and 355 *recto*; and Heath 1956, 3: 371-373.

drawing. Thanks to Ricci and XU, Wylie and LI, Chinese readers by the end of the 1850s could read the entire corpus of the Euclidean *Elements*. But before long, Chinese mathematicians would soon learn a new language by which to address such matters as the method of exhaustion by which the *Elements* treated the problem of finding the area of the circle, or volumes like the sphere. The problem of the infinite in mathematics would be approached from an entirely different direction, with far greater returns for Chinese mathematics. This occurred when the infinitesimal calculus was first introduced to Chinese readers, also at the end of the 1850s, thanks to another collaborative effort of Alexander Wylie and LI Shanlan. But this is the story to be told in the chapter which follows.

Chapter 5: Introduction of the Calculus

> I call [the calculus], and aptly enough if I am not mistaken, the *analysis of indivisibles and infinites,* which having been once revealed, whatever of this kind I had formerly wondered about seems only child's play and a jest.
>
> —Gottfried Wilhelm von Leibniz[1]

> Those [problems] unsolvable in the past now can be solved…These were thought very difficult to work out now can be easily done. Mathematics has now reached its pinnacle and no more can be added to it.
>
> —LI Shanlan 李善蘭[2]

The discovery of the inverse relation between what have come to be known as the differential and the integral calculus was made by Isaac Newton and Gottfried Wilhelm von Leibniz, respectively, in the second half of the seventeenth century, and represents the beginnings of a new mathematical branch: the calculus. Although Isaac Barrow had proved the essential connection between the problems of drawing tangents to curves and determining the area under that same curve (under suitable conditions) in what was a geometric version of the fundamental theorem of calculus, he accorded it no special status and did not see it as part of a systematic treatment of infinitesimal methods, as did both Newton and Leibniz.[3] Although their subsequent, simultaneous, yet apparently

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[1] See Leibniz, "Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas, nec irrationales quantitates moratur," *Acta Eruditorum*, 1864, 466-73. The English translation is by Evelyn Walker, and quoted from David Smith [1929]1959, 626. For basic surveys of the history of the calculus, see Boyer 1959, Baron 1969, and Edwards 1979. See also Whiteside 1960-62.

[2] Translated from LI Shanlan's preface to the Chinese translation of the *Elements of Analytical Geometry and of the Differential and Integral Calculus* by Elias Loomis. The translation is entitled "*Daiweiji shiji* 代微積拾級," and was published by the London Missionary Society in Shanghai in 1859.

[3] On Barrow's mathematics, and especially the extent to which he failed to appreciate the importance of the connection between the method of tangents and finding areas under curves, see Mahoney 1990.

independent discoveries of the calculus (in Newtonian and Leibnizian versions) led to what A.R. Hall has called a case of "Philosophers at War," this is not the focus of concern here.[4] By the time the calculus was introduced to China in the late nineteenth century, the priority disputes were as forgotten as Newton's fluxional approach to the calculus. In the course of the eighteenth century, an essentially Leibnizian version of the calculus had come to prevail, especially his notation of differentials *dx* and integrals ∫. The French had been particularly active in these developments, and in the nineteenth century concerns for teaching and rigor had inspired new approaches to the calculus, especially those associated with Lagrange, Cauchy, Riemann, and Weierstraß (among many others).[5] But again, what concerns us here is the calculus as it was first known in China, and for reasons to be discussed here, it was in a rather surprising version of the calculus due to an American, and not the calculus in any of its European versions, that this new subject became widely known in China in the late nineteenth century.

A first sketch of what later would come to be known as the calculus was first published by Leibniz in the *Acta Eruditorum* in 1684. In the revolutionary article, *Nova methodus pro maximis et minimis* ... (New Methods for Maxima and Minima…), Leibniz aptly describes the calculus as "*analysis of indivisibles and infinites*,"[6] The Calculus indeed provides new concepts and methods, some as controversial now as then, to deal with infinitesimals and the infinite. This chapter discusses the ways in which this subject was first introduced into China, analyses the translations of the most crucial and

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[4] Hall 1980.

[5] The literature on the calculus in this period is vast, but among the best introductions are works by Bruno Belhoste (Belhoste 1985, 1991); Judith V Grabiner (Grabiner 1981); and Ivor Grattan-Guinness (Grattan-Guinness 1970); among many others.

[6] David Smith [1929] 1959, 626, See also note 1.

fundamental concepts and methods, and investigates their impact on the research of Chinese mathematicians.

5.1 Prelude

When Matteo Ricci, the pioneer Jesuit missionary in China, who with XU Guangqi translated the first six books of Euclid's *Elements* into Chinese, died in 1610, he was buried in the West of Beijing at a place specially granted by the Ming dynasty Emperor Shen Zong 神宗 (reigned 1573 - 1619). Following in Ricci's footsteps, more missionaries, especially educated Jesuits, also went to China and tried to win the hearts of the ruling Manchus and the Chinese *literati* by introducing Western sciences and arts in hopes, ultimately, of converting them, as well as the ordinary people, to Christianity. Among these early Catholic missionaries was the French Jesuit, Pierre Jartoux (1669 - 1720), who arrived in China on September 9, 1701. Subsequently, he was better known by his Chinese name, DU Demei 杜德美.

Jartoux worked in China for nearly two decades. Primarily, his work was devoted to geological surveys and the drawing of maps. But he also introduced three formulas for 1) calculating the length of the circumference of a circle given its diameter; 2) determining the sine of an arc if the length of the arc is known; and 3) finding the versed sine of an arc if the length of the arc is provided. These formulas may be represented symbolically as follows:

$$c = 3d(1 + \frac{1^2}{4 \cdot 3!} + \frac{1^2 \cdot 3^2}{4^2 \cdot 5!} + \frac{1^2 \cdot 3^2 \cdot 5^2}{4^3 \cdot 7!} + \frac{1^2 \cdot 3^2 \cdot 5^2 \cdot 7^2}{4^4 \cdot 9!} + ...) \qquad 1)$$

$$r \sin \frac{a}{r} = a - \frac{a^3}{3!r^2} + \frac{a^5}{5!r^4} - \frac{a^7}{7!r^6} + \frac{a^9}{9!r^8} - \ldots \qquad 2)$$

$$rvers \frac{a}{r} = \frac{a^2}{2!r} - \frac{a^4}{4!r^3} + \frac{a^6}{6!r^5} - \frac{a^8}{8!r^7} + \frac{a^{10}}{10!r^9} - \ldots \qquad 3)$$

where *c* is the circumference of a circle, *d* its diameter, *r* its radius, and *a* a sector of an arc.

These formulas were first recorded in 1761 in Chinese in a treatise entitled *Chishui Yizhen* 赤水遺珍 (*Pearls Recovered from the Red River*) by MEI Juecheng 梅瑴成 (1681-1763), a mathematician in his own right and the grandson of MEI Wending. Though not originated by Jartoux himself, they are still called the "Three Formulas of DU" in Chinese, despite the fact that LI Yan has identified the first formula as discovered by Isaac Newton in 1676, and the other two by James Gregory in 1667.[7] Because MEI Juecheng did not explain how Jartoux got "his" results, these formulas aroused considerable interest among Chinese mathematicians who sought to supply proofs for them. MING Antu 明安圖, a Mongolian, was trained at and later served in the Qing court. He also worked together for a time with Jartoux and other missionaries. Not only was he able to prove all three of the "DU" formulas, he also provided six "new" ones.[8] MING Antu's work, which was completed in 1774 and published posthumously in 1839, together with the three formulas introduced by Jartoux, stimulated further Chinese interest in methods for the summation of series and division of the circle during the first half of the nineteenth century. This is a subject of considerable interest, and one that has

[7] LI Yan [1927-1928] 1998: 7, 296.

[8] See Jami 1990, and LUO Jianjin 1998.

been thoroughly studied by LI Yan 李儼, HE Shaogen 何紹庚, LI Zhaohua 李兆華, LUO Jianjin 羅見今, Cathérine Jami, and most recently by Tegus 特古斯.[9]

Although the "Three Formulas of DU" are related to the calculus, they are not the core of its concern with derivatives and integrals of functions, and the reciprocal nature of differentiation and integration. The calculus in this form was actually introduced to China in the 1750s, as the correspondence between Antoine Gaubil and his colleagues in Paris suggests. In a letter Gaubil wrote in October or November of 1752 to Guillaume-François Berthier (1704-1782), editor of the *Mémoires de Trévoux*, it reads:

> Pour les mathématiques nous manquons d'un ou deux livres essentials. Je vois que dans tout ce qui regarde l'astronomie on emploie les formulas algébraiques, on suppose souvent une connoissance assez grande du calcul différential et intégral; bien de ceux qui viendront ne sauront peut-être que l'algèbre ordinaire jusqu'au second degré et seront neufs sur le reste, et sur les calculs différential et intégral, faits depuis. Ils prevent difficilement lire les nouveaux livres surtout. Cela étant je voudrois voir ici:
> 1° quelques livres ou soient les méthods les plus faciles pour l'algébre, commencant par les premiers éléments.
> 2° les régles suivies et faciles du calcul différential et intégral; et si dans tous les livres on suppose qu'on ne sait rien, tant mieux. Notre P. Procureur pourra vous fournir la somme necessaire pour l'argent de ces livres, vous êtes au fait sure tout cela, si on lui écrivolit sur ce point il l'oublieroit peut-être, et n'en voyant pas bien comme vous l'utilité et la nécessité, il feroit achepter peut-être quelques livres en ce genre qui seroient inutiles. Pardon de la peine, mais[10]
>
> For mathematics, we lack one or two essential books. I see that for everything that concerns astronomy, they use algebraic formulas. Therefore one can assume that there is a wide knowledge of differential and integral calculus. Many of the ones who will come later will not know more than ordinary algebra, maybe to the second degree. They will be new to the rest, such as differential and integral calculus. It will be hard for them to read the new books. What I would like to see here are the following:
> 1) Some books that contain the easiest methods of algebra starting with the very basic elements.
> 2) The regular and easy rules of the differential and integral calculus. And if, in the books, they assume that readers do not know much, so much the better. Our benefactor should provide us with the necessary money for those books. Are you

[9] See LI Yan 1927-1928; HE Shaogeng 1984, 1989; LI Zhaohua 1987; LUO Jianjin 1988, 1989, 1990, 1998; Jami 1988, 1990; and Tegus 1995, 1996, 2000.

[10] Gaubil 1970, 717. Similar contents of the letter also can be found on page 719 of the same reference.

> sure if we write him on this matter, he will not forget and not seeing, like you do how to use those books, he will probably ask to buy books that are not so useful. Sorry for the pains, but....

In his reply to Gaubil's letter, Joseph-Nicolas DeLisle (1688-1768) writes:

> Suivant une lettre aue vous avez écrite au P. Berthier et qu'il m'a communiqué, j'ai vu que dvous souhaitiez d'avoir les meilleurs livres qui traits du calcul différential et intégral, pour mettre en état d'entendre les formula algébriques que l'on emploie á present dans l'astronomie; c'est pour que je vous envoie 1° l'algèbre de M. Clairaut qui est élémentaire et la meilleure que nous ayons en françois, c'est un volume in-8°. 2° L'analyse des infiniment petits du marquis de l'Hopital, avec le commentaire de M. de Crouzas, in 4°. 3° Le traité du calcul intégral de M. Stone, traduit en français depuis quelques années.[11]
>
> According to a letter that you wrote to Father [Guillaume-François] Berthier and that he sent to me, I see that you would like to have the best books that deal with the differential and integral calculus, so that you can learn the algebraic formulas that are presently used in astronomy. Thus I am sending you: firstly, the algebra of Clairaut which is elementary and the best we have in French (it is one volume in octavo); secondly, the analysis of infinitesimals by the Marquis de L'Hôpital, with the commentary by Mr. de Crou[saz], in quarto; thirdly, the treatise on integral calculus by Mr. Stone, translated into French some years ago.

DeLisle's letter mentions two books on the calculus in particular, one by Jean-Pierre de Crousaz: *Commentaire sur l'Analyse des infiniment petits*, published in Paris in 1721; and the other a work by Edmund Stone: *Analyse des infiniment petits*, which also appeared in Paris in 1735. Both of these are closely related to the first textbook on the calculus written by the Marquis de l'Hôpital (1661 - 1704), *Analyse des infiniment petits*.[12]

Antoine Gaubil (1689-1759), who arrived in China about two years after Jartoux died, was another French Jesuit; his Chinese name was SONG Junrong, or SUN Kiun-yung, 宋君榮. Like Jartoux, Gaubil also worked in the Qing court, and served as the

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[11] Gaubil 1970, 718.

[12] The first, as its title notes, is based explicitly on the Marquis de l'Hôpital's book. The second, by Edmund Stone, is a French translation of a work originally written in English, in two parts. The first part, however, was an English translation of de l'Hôpital's *Analyse des infiniment petits* (and was not translated back into French). The second part was Stone's own account of "The Method of Fluxions, both Direct and Inverse," and the French translation was only of this second part.

Director of the Imperial Astronomical Bureau (*qin tian jian* 欽天監) in Beijing.[13] Assuming that Gaubil indeed received the above-mentioned books, it is not clear to what extent his research was actually affected by any of them. What is almost certainly the case, he did not convey the calculus to MING Antu or to any other of his Chinese colleagues, for no mention is made by any Chinese mathematicians at this time with respect to the infinitesimal calculus.

Nevertheless, Gaubil made many great contributions to the exchange of knowledge between East and the West. Besides sending numerous reports and correspondence back to Europe, he also translated the *History of Genghis Khan*, parts of the *Annals of the Tang Dynasty*, and the *Book of History* into French.[14] Moreover, he authored a "*Traité de l'Astronomie Chinoise* (A Treatise on Chinese Astronomy)," in which he provided a reasonably detailed account of the history of Chinese astronomy and related mathematical sciences. Jean E. Montucla (1725-1799 CE), based largely on this volume by Gaubil, wrote his own account of the "Histoire des Mathématiques chez les Chinois (History of Chinese Mathematics)" as part of his famous *Histoire des Mathématiques*.[15] Gaubil's contributions to China would have been even greater had he introduced the calculus.

Gaubil was not the only missionary, however, who had some knowledge of the calculus and also had access to books on the subject. According to the *Catalogue de la Bibliothèque du Pét'ang*, a catalog of the Jesuit library in Beijing which was compiled by

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[13] For a list of the Jesuit directors of the Imperial Astronomical Bureau, see Dehergne 1973, 307-308. See also BO 1978.

[14] *Shu Jing*, Paris: N. M. Tilliard, 1770.

[15] See Montucla 1799, 448-480.

Hubert Verhaeren and published in 1949, no fewer than five books on the calculus or related subjects had made their way to China by 1773, the year in which the Society of Jesus was suppressed by Pope Clement XIV and the Jesuits were forced to leave China.

1. *Analyse des infiniment petits* by the Marquis de l'Hôpital (1768).[16]
2. *Introductio in analysin infinitorum* by Leonhard Euler (1748).[17]
3. *Methode pour la mesure des surfaces, la dimension des solides, leurs centres de pesanteur, de percussion et d'oscillation Par l'Application du Calcul integral* by Louis Carré (1700).[18]
4. *Élements de la géométrie de l'infini* by Bernard de Fontenelle (1727), the secretary of the Académie des Sciences from 1697 until his death.[19]
5. *The Doctrine of Fluxions, Founded on Sir Isaac Newton's Method, Published by Himself in His Tract upon the Quadrature of Curves* by James Hodgson (1736).[20]

Even though such important calculus books as the first one ever written, by l'Hôpital, and that by the great mathematician, Leonhard Euler, were physically in China, no single Chinese scholar seems to have been aware of their existence, let alone their contents, or to have made any use of them. We cannot even be certain if any of the Catholic missionaries in Beijing ever consulted these works, or to what extent. But we cannot blame missionaries such as Gaubil for not having introduced the calculus. For the Jesuits, introducing the Western sciences was regarded as a tool, *not* as a goal of their

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[16] Verhaeren [1949]1969, 118. The entry number for this book title is #440. See also Martzloff 1997, 356.

[17] Verhaeren [1949]1969, 454. The entry number for this book title is #1563.

[18] Verhaeren [1949]1969, 44. The entry number for this book title is #161.

[19] Verhaeren [1949]1969, 3-4. The entry number for this book title is #10.

[20] Verhaeren [1949]1969, 1198. The entry number for this book title is #4079.

missionary activities in China. Perhaps they thought introducing the calculus would not have served their interests as Ricci had hoped his translation of Euclid would definitely serve his missionary ambitions to convert more Chinese *literati* to Christianity. But when the situation in China changed, and Chinese scholars themselves became interested in learning more about western mathematics, then their interest in the calculus, and indeed in many of the other branches of modern western mathematics, became substantial.

5.2 Chinese Translations of the Calculus

Between the last three decades of the eighteenth century and the first four of the nineteenth century, virtually no missionaries entered China because of the suppression of the Society of Jesus and hostile polices of the Qing court towards missionaries in general. But the situation changed suddenly and dramatically as a direct consequence of the so-called Opium Wars in the 1840s and 1850s. Missionaries, this time the majority of them Protestant, flooded in. Among the first to arrive was Alexander Wylie. In the previous chapter, we discussed Wylie and his Chinese collaborator, the mathematician LI Shanlan, who together translated the last nine books (VII-XV) of Euclid's *Elements* into Chinese, thereby completing the work left unfinished by Matteo Ricci and Xu Guangqi in 1607.

It was also Wylie and LI who worked together to introduce the calculus to Chinese readers for the first time. Wylie conceived the ideas of translating a calculus book perhaps soon after he met LI Shanlan in Shanghai in 1852. In the preface to his own book, the *Shuxue Qimeng* 數學啓蒙 (*Compendium of Arithmetic*), which was published in 1854, Wylie writes:

> 余自西土遠來中國，以傳耶穌之道為本。余則兼習藝能，爰述一書，曰《數學啓蒙》，凡二卷。舉以授塾中學徒，由淺入深，則其知之也易。譬諸小兒，始而匍匐，繼而扶墻，後乃能疾走。茲書之成，姑教之匍匐耳，扶墻徐行耳，若能疾走，則有代數、微分諸書在，余將續梓之。[21]
>
> I came to China from the far West, and my main mission is to propagate the gospel of Jesus. I also learned useful crafts, and have written this book, which is entitled *Compendium of Arithmetic*, in 2 volumes. Using it to teach students from very basic to some difficult [arithmetic], one can easily make them understand. This is like a baby who crawls first on the floor, and then walks with the help of the hands on a wall, and eventually [learns to] walk fast. This book is written to teach "crawling" and "walking with the help of the hands on a wall." If [a student] can walk fast, then there are books on algebra and the differential calculus which I am going to publish soon.

This preface of Wylie's suggests clearly that he already had a textbook in mind on the calculus, one he intended to prepare for those who wished to learn more advanced mathematics. About the context and his motivation for introducing the calculus to potential readers, Wylie had more to say in the English preface to the calculus textbook he wrote and eventually published in Chinese in 1859:

> The present work, which is a translation of [Elias] Loomis' [*Elements of*] *Analytical Geometry, and* [*of the*] *Differential and Integral Calculus*, is issued in pursuance of a project formed some time since, as the continuation of a course of mathematics, the first of which, a *Compendium of Arithmetic*, was published by the undersigned [A. Wylie] in 1854. The next in order is a *Treatise on Algebra*, which should have preceded this, but in consequence of unavoidable delays in the publication, it will not be issued till some weeks later. A tolerable acquaintance with the last named treatise, will put the student in a position to understand the work now presented to the public….A spirit of inquiry is abroad among the Chinese, and there is a class of students in the empire, by no means small in number, who receive with avidity instruction on scientific matters from the West. Mere superficial essays and popular digests are far from adequate to satisfy such applicants; and yet when anything beyond that is attempted, the want of a common medium of communication at once appears as an insuperable obstacle; and it is evident that how clearly soever we may be enabled to lay results before the native mind, yet until they understand something of the processes by which such results are obtained, thinkers of the above class can scarcely be supposed to appreciate the achievements of modern science, to repose absolute confidence in the results, or to rest satisfied till they are in a position to

[21] Quoted from the Preface to the *Shuxue Qimeng* 數學啓蒙. See Wylie 1854.

> some extent to verify the statements which are laid before them. It is hoped that the present translation will in some measure supply what is now a desideratum.[22]

Wylie here states plainly that his book on the calculus is meant to be a textbook, and that the readers he has in mind are those students who have already learned some Western science and are eager to learn more on their own. Accordingly, this book meant to introduce the calculus was not prepared for "general intellectuals and working scientists and technicians," as HU Mingjie 胡明傑 argues in his recent study of various translations of the calculus.[23]

As explained in Wylie's English preface, the calculus textbook he had chosen for translation was the *Elements of Analytical Geometry and of the Differential and Integral Calculus* by Elias Loomis. Loomis (1811 - 1889) had graduated from Yale College in 1830, and was a Professor of Mathematics and Natural Philosophy at a newly-founded university: the University of the City of New York (now NYU), where he taught from 1844 to 1860, except for one year that he spent teaching at Princeton University. From

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[22] Quoted from the copy of the 1859 edition of the first Chinese translation of Elias Loomis' *Elements of Analytical Geometry, and of the Differential and Integral Calculus* preserved at the Library of the Tohoku University, Sendai, Japan. This copy is a rare one because it contains the two prefaces by Wylie, one in English and the other in Chinese. Amazingly, the contents of the two prefaces are also very distinctive. The author wishes to thank FENG Lisheng 馮立升 for providing a copy of this English preface before he saw its facsimile in Martzloff 1997, 381.

There are some different opinions concerning the English preface and in which editions of the Chinese translation of Loomis' *Calculus* it appears. Jean-Claude Martzloff holds that it was an added "special feature" in the 1872 Japanese reprint of the 1859 Chinese edition; see Martzloff 1997, 382. But FENG Lisheng says that in the Library of the Tohoku University, he discovered that the 1859 Chinese edition preserved there also contains the English preface; see FENG Lisheng 1999, 45. Wylie in his *Memorials of Protestant Missionaries to the Chinese* specifically mentions that there is an English preface beside the Chinese one to his translation of the calculus. Wylie writes "A few of the copies have also an English preface, and an index of technical terms, pp. vii." See Wylie 1867, 174. Moreover, Wylie in the same book also points out that "there is an English preface to some of the copies [of *Compendium of Arithmetic*]." See Wylie 1867, 173. we indeed find that there are both distinct dual prefaces to the *Compendium of Arithmetic* in the type-set printed version and the lithographic version published in 1886.

[23] HU Mingjie 1998, 195.

1861 until his death in 1889, he was Professor of Natural Philosophy and Astronomy at Yale University in New Haven, Connecticut. He was also a member of numerous academic societies, including the U.S. National Academy of Sciences. During his long and successful career, Loomis worked in various areas of the sciences in addition to mathematics, including astronomy, geophysics, and meteorology. The most original and important contributions he made are in the field of meteorology. Loomis not only created the first synoptic weather map, but helped to provide a solid foundation for the later more accurate development of weather forecasting.[24] In mathematics, Loomis is now best remembered for a series of textbooks he wrote while he was teaching in New York. These textbooks were so successful that they brought him not only fame as a well-known mathematician around the world, but also a substantial income, which in turn provided him with needed resources and time for pursuing his scientific explorations.[25]

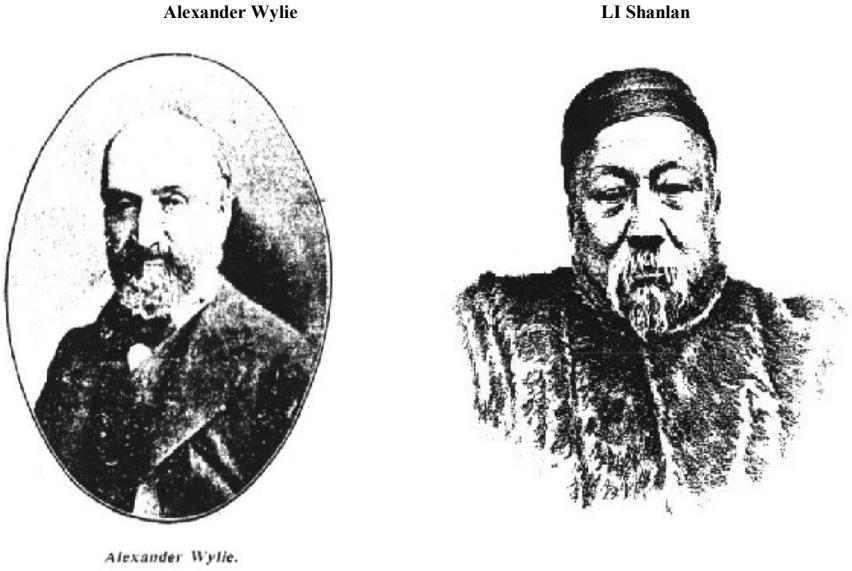
The *Elements of Analytical Geometry and of the Differential and Integral Calculus* is the sixth in a series of textbooks Loomis wrote on mathematics. It was first published in New York in 1851. In his preface, Loomis wrote that the book was intended primarily for "the mass of college students of average abilities."[26] He claimed in particular that his treatment of the subject was "better adapted to the apprehension of common minds than any other."[27] He also hoped that it would be used as "a standard

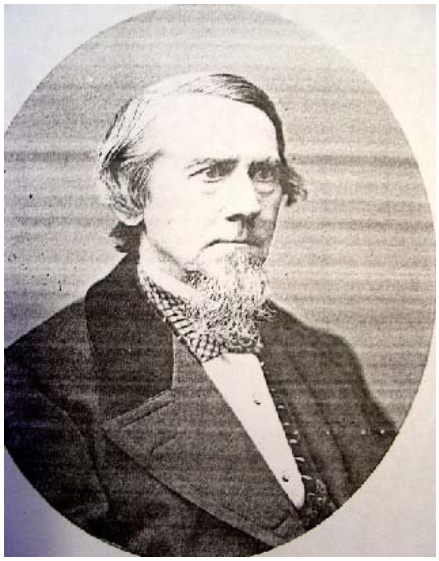
[24] For details of Loomis' life and work, see Newton 1890; and Kutzbach 1973.

[25] Newton 1890, 5. For a list of the mathematics textbooks Elias Loomis authored, see ZHANG and Dauben 1994, 288-289.

[26] Quoted from Loomis 1851, iii.

[27] Quoted from Loomis 1851, iv.





Elias Loomis (1811-1889)

Figure 14. Photos of Alexander Wylie (top left), LI Shanlan (top right), and Elias Loomis (bottom)
The Photo of Loomis is a courtesy of the Manuscripts and Archives Department of the Beinecke Rare Book and Manuscript Library at Yale University

study for all the students of our colleges."[28]

The success of this textbook not only met, but in fact, exceeded Loomis' expectations. It was reprinted numerous times and by 1874, when a revised edition was issued, the calculus textbook had sold more than 25,000 copies![29] It was also immediately translated into Spanish.[30] With such obvious success as a textbook of wide-spread interest, it is easy to understand why Wylie chose this particular source for his translation.

The Chinese translation of the book is entitled *Daiweiji shiji* 代微積拾級, for which Loomis' name was rendered phonetically to LUO Mishi 羅密士. The immediate impact of this translation on Chinese readers turned out to be more "ideological" than technical (see the next section). If some Chinese scholars had previously found "evidence" that all Western mathematics, including geometry and algebra, had originated in China, they now had to admit that at least the calculus was an exception. And indeed, this is exactly what LI Shanlan says in his preface to the translation:

> 中法之四元即西法之代數也。諸元乘方、諸互乘積；四元別以位次、代數別以記號，法雖殊，理無異也。我朝康熙時西国來本之、奈端二家又創立微分、積分二術。其法亦借徑於代數，其理實發千古未有之奇秘。[31]
>
> [In essence] the Chinese method of the "Four Elements" is the same as Western algebra. For multiplication of an element by itself or products of different elements, [one can use] the [method of the] Four Elements to differentiate them by placing them in difference places or note them by symbols with algebra. The methods are different, but the principles (*li* 理) are the same. During the Kangxi period, Leibniz and Newton in the West created, respectively, the differential calculus and the integral calculus. Their methods rely on algebra, but their

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[28] *Ibid.*

[29] See Loomis 1874, iii.

[30] See David Smith 1927-1936, 398-399.

[31] Quoted from LI Shanlan's preface to the first Chinese translation of the calculus.

> principles reveal a secret which was hidden from human beings for thousands of years.

LI Shanlan even goes so far as to say that having reached the calculus, "mathematics has now reached its pinnacle and no more can be added to it."[32]

LI Shanlan's high regard for the calculus paved the way for its warm reception among its Chinese readers.[33] Since its appearance in 1859, for the next fifteen years it remained the only calculus textbook to which Chinese readers had access. In 1874 a second work on the calculus appeared as a result of the Self-Strengthening Movement.[34] The goal of the Movement is "to defend the traditional society against barbarian onslaught and internal subversion…To reach this goal, [it needs] sufficient military power to defend China, was to imperil the cherished order the statesmen sought to preserve."[35] As a physical move to achieve that end, the Jiangnan Arsenal was created in Shanghai in 1865 hoping to manufacture weapons, the qualities of which can march to that of used by the Europeans. Realizing that Western sciences and technologies were essential to but well beyond the military strength of China at the time, a Translation Department was soon founded at the Arsenal to make basic scientific texts available in Chinese.

A number of specialists, Chinese, English and American, were employed in the arsenal's Translation Department.[36] Among those who were on the payroll over a

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[32] See Li Shanlan's preface to the first Chinese translation of the calculus.

[33] For a detailed discussion of the Chinese reception of the calculus, see Chapter Four, Section III, of HU Mingjie's dissertation: "The Chinese Views of the Calculus," in HU Mingjie 1999, 214-230.

[34] About the background of the Self-Strengthening Movement, see Wright 1957, 196-221. For the role that the Movement played in the emergence of modern science in China, see Dauben 2002, 254-256, 258-260.

[35] Quoted from Wright 1957, 196.

[36] For details, see TSIEN 1954, 317.

substantial period of time, and who were also responsible for the majority of the translations of mathematical texts produced at the Arsenal, were HUA Hengfang 華蘅芳 (1833 - 1902) and Wylie's British countryman, John Fryer (1839 - 1928), whose Chinese name is FU Lanya 傅蘭雅.[37] From 1871, when the first translation was completed and published at the Arsenal until 1912, when the Translation Department of the Arsenal came to its end, some 241 translations were undertaken, of which 193 were published.[38] These translations cover a variety of subjects from astronomy to zoology. For mathematics, books or encyclopedia articles on algebra, probability, trigonometry, and the calculus were all translated at the arsenal.[39]

The material on the calculus translated by HUA and Fryer was based on a lengthy article entitled "Fluxions" in the eighth edition of the *Encyclopedia Britannica,* published in London in 1855. The article, written by William Wallace,[40] was given the Chinese title: *Weiji suyuan* 微積溯源. Due to the differences between a textbook and an encyclopedia article, Loomis' approach to and treatment of the calculus were substantially different from what Wallace wrote, and the corresponding differences are naturally reflected in the Chinese versions of the two works. In the following section, how the concepts and methods related to infinitesimals and infinity were translated into Chinese is the focus of our analysis, as well as the differences in treatment of the calculus in the respective translations by LI and Wylie, and by HUA and Fryer.

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[37] For John Fryer's life and work, see Bennet 1967, and WANG Yangzong 2000.

[38] WANG Yangzong 1995, 18.

[39] For a list of the translated titles of mathematics books and their sources, see Bennet 1967, 84-85; LI and DU 1987, 260; and WANG Yangzong 1994, 36-37; 1995, 6-7.

[40] For a brief biography of William Wallace, see Broadbent 1976.

It should be added that the revised edition of Loomis' textbook on the calculus was rendered into Chinese twice at the beginning of the twentieth century. First the American Protestant missionary Alvin Pierson Parker 潘慎文 (1850 - 1924) translated Loomis' book, with the help of his Chinese convert XIE Honglai 謝洪賚 (1873 - 1916).[41] Their translation was entitled *Weiji Xue* 微積學, and was published by the Commercial Press of Shanghai in 1905. The second translation, with the same Chinese title, was coincidently published in the same year by The American Presbyterian Mission Press. This version was translated independently for the first time by a Chinese working on his own: LIU Guangzhao 劉光照.

5.3 Concepts of the Infinite Introduced Through the Calculus in Chinese Translations

The calculus, as Leibniz once called it, is the "*analysis of indivisibles and infinites*."[42] As such, it served to introduce novel ideas and methods that depend intrinsically on the infinite, especially in terms of the infinitely small or infinitesimals, to China towards the end of the nineteenth century. How this was accomplished through the translations of the textbook by Loomis and the encyclopedia article by Wallace is the subject of this section.

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[41] XIE Honglai, also known as HSIEH Hung-Lai, or Ch'ang-Hou, or H. L. Zia, was a prominent Chinese Protestant. He was the Secretary of the National Committee and Director of the Publications Department of YMCA in China between 1904 and 1916. Before taking such positions, he taught physics and chemistry at the Anglo-Chinese College in Suzhou, where Alvin P. Parker was the President. XIE translated numerous books including several on algebra, geometry and trigonometry. His translations emphasize a style which is more readable and less scholarly but in the meantime also command the respect of the educated. For his life and work, see Gregg 1946, and The *Biographical Dictionary of Republican China*, vol. 2, pp. 100-101 (New York: Columbia University Press, 1971).

[42] See note 1 of this chapter.

Loomis' textbook, *Elements of Analytical Geometry and of the Differential and Integral Calculus*, treats the calculus in two separate parts, "Differential Calculus" (pages 113-216), and "Integral Calculus" (pages 217-286), a total of 173 out of 286 pages, the rest being devoted to analytical geometry. The style of the book is lucid and since it is a textbook, exercises are also provided. At the end of the book, an entire section is devoted to "Examples for Practice," providing both questions and answers (pages 267-286).[43]

Loomis starts his account of the differential calculus by defining *variables*, *constants, functions*, *explicit* and *implicit functions*, and *increasing* and *decreasing functions*. He then defines one of the most important concepts of the calculus, "*limit*," as follows:

> The *limit* of a variable quantity is that value which it continually approaches, so as, at last, to differ from it by less than any assignable quantity.[44]

In symbols, s is said to be the limit of $f(x)$ if, given a positive constant ε, however small, for x sufficiently close to a certain number, $\left| f(x) - s \right| < \varepsilon$. This definition in essence is the same as that given first by Cauchy in his *Cours d'analyse* of 1821. Loomis provides four specific examples illustrating this definition, one of which is the classic treatment of the area of the circle as the limit of the areas of its inscribed polygons:

> [I]f we have a regular polygon inscribed in a circle, and we inscribe another polygon having twice the number of sides, the area of the second will come nearer to the area of the circle than that of the first. By continuing to double the number of sides, the area of the polygon will approach nearer and nearer to that

[43] For a more detailed description of the *Elements of Analytical Geometry and of the Differential and Integral Calculus*, see ZHANG Dianzhou 1992; ZHANG and Dauben 1994; and HU Mingjie 1998, 184-185.

[44] Loomis 1851, 115.

> of the circle, and may be made to differ from it by a quantity less than any finite quantity. Hence the circle is said to be the limit of all its inscribed polygons.[45]

The Chinese translations of the definition and the example by Alexander Wylie and LI Shanlan are as follows:

> 凡變數有限者，其數為變數所漸近，而永不能過，或必不能過，故謂之限。如圓内作一多等邊形，于此形外倍其邊，再作一多邊形。如此遞倍遞作，則其積漸近平圓積。然兩積之較可至甚微，而終不能等于〇，則圓積為多邊形積之限。[46]

"Limit" is rendered as "有限" or simply "限." The translation says that if a variable has a limit, then the limit as a number can be approached closer and closer by the variable but cannot and must not exceed that number. There is, in fact, a subtle difference between this translation and the original text, which emphasizes that the difference between the successive values attributed to a variable and a fixed constant is smaller than any assignable number. The original definition does not mean that the values attributed to the variable cannot be greater than its limit, as the Chinese translation implies. This difference, however, does not apply to the translation of the four examples, including the one quoted above.

After defining the concept of *limit*, Loomis does not continue to follow Cauchy to define *derivative* as the central concept of the calculus, but instead takes Leibniz's approach and treats the *differential* as the most fundamental concept. Loomis defines this as: "The rate of variation of a function or of any variable quantity is called its *differential*."[47] In symbols, this definition may be expressed as $dx = (x + h) - x$ for a variable; and $du = f(x + h) - f(x)$ for a function $u = f(x)$. This definition, however, is

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[45] Loomis 1851, 115.

[46] Quoted from Wylie and LI 1959, *juan* 10, 2b-3a. Punctuation is added.

[47] Loomis 1851, 120.

not a rigorous one even by the standards of 1850, and in particular, it confuses the distinction between differential and increment.

The Chinese translation of this definition is even worse, for it not only inherits the original sin, but also adds a new one. The Chinese translation: "函數與變數之變比例，俱謂之微分,"[48] may be rendered back to English as: "The changing proportions of a function and a variable are both called differentials." As this indicates, the Chinese translation interprets "rate of variation" as "changing proportion," but this does not rightly convey the original meaning of "differential."

Based on his definition of "*differential*," Loomis goes on to define the "*differential coefficient*" as follows:

> [T]he ratio of the rate of variation of the function to that of the independent variable is called the differential coefficient.[49]
>
> In symbols this may be written as: $\frac{du}{dx} = \frac{f(x+h) - f(x)}{(x+h) - h}$. From the

formula, strictly speaking, this definition is precise. But as one reads Loomis' account further, it turns out that what Loomis meant was that the ratio is actually a *limit* of ratios:

> The method here exemplified is applicable to the determination of the differential coefficient of any function of a single variable, and is expressed in the following
>
> Rule
>
> *Give to the variable any arbitrary increment* h*, and find the corresponding value of the function; from which subtract its primitive value. Divide the remainder by*

[48] *Ibid*.

[49] *Ibid.*

> *the increment* h*, and find the limit of this ratio, by making the increment equal to zero; the result will be the differential coefficient.*[50]

Due to omission of the key word "limit" as well as the confusing language such as "rate of variation" in the original text, the definition of differential coefficient is rendered into Chinese as:

> 變數之變比例約函數之變比例，等于函數之微係數也。[51]

The translation again uses "變比例" (literally, "changing proportion") for "rate of variation;" and "約" (literally, "divide" or "simplify") for "ratio." Clearly this fails to convey adequately the true meaning of differential coefficient.

How these mistranslations may have affected Chinese readers' understanding of the two essential concepts of differential and differential coefficient is difficult to assess. On the one hand, the translators offered no comments or notes in their translation, and the original text did not discuss these concepts again elsewhere. On the other hand, the Chinese translation of the procedures for finding differentials and differential coefficients may have sufficed to give readers a correct working sense of what the two concepts entail.

Following his definitions of *limit*, *differential* and *differential coefficient*, Loomis went on to provide several formulas for determining differential coefficients, including:

a. $d(x^n) = nx^{n-1}dx$

b. $d[cu(x)] = cd[u(x)]$

c. $d[u(x) \pm v(x)] = d[u(x)] \pm d[v(x)]$

d. $d[u(x) \cdot v(x)] = d[u(x)] \cdot d[v(x)]$

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[50] Loomis 1851, 122

[51] Wylie and LI 1859, *juan* 10, 9a.

e. $d[\frac{u(x)}{v(x)}] = \frac{d[u(x)]}{d[v(x)]}$

f. $d(\sqrt{x}) = \frac{1}{2\sqrt{x}}dx$

g. $d[(ax + x^2)^n] = n(ax + x^2)^{n-1} \cdot (a + 2x)dx$

All of these formulas as well as examples of their applications are faithfully translated in Chinese, though the corresponding symbols invented by the translators are cumbersome. For instance, the differential symbol, *d*, is substituted by the Chinese character, "彳"; "+" by "⊥"; "-" by "⊤."[52]

Based on the differential coefficient, Loomis proceeded to discuss successive differentials. Since the differential of a function containing a variable raised to certain powers may be considered as a new function, its differential may be determined accordingly. Thus, one obtains the second differential coefficient, and the process may be repeated, resulting in successive differentials of higher orders. In dealing with successive differentials, Loomis introduced both Maclaurin's theorem as well as Taylor's theorem. Loomis pointed out that in applying Maclaurin's theorem, if the variable is made equal to 0, the function or some of its differential coefficients may become infinite. Under such circumstances, the function cannot be developed by Maclaurin's theorem.[53] Similarly, in a Taylor series, for certain values of the variable, its differential coefficients may also become infinite. Loomis provided several examples to illustrate these points. For

[52] In his English preface to the Chinese translation, Alexander Wylie provides a table corresponding symbols in the original version of Loomis' book and the ones invented or used in the Chinese translation. The table can be found in Martzloff 1997, 121. For a detailed discussion of the rationale behind choosing the Chinese characters "彳" and "禾" for the symbols of the differential and integral, respectively, see HU Mingjie 1998, 188-190; 195-197; and 201-203.

[53] Loomis 1851, 141.

instance, consider the function: $u = ax^{\frac{1}{2}}$, whose first differential coefficient is $\frac{du}{dx} = \frac{a}{2x^{\frac{1}{2}}}$. But when *x* is equal to 0, the differential coefficient becomes infinite. Therefore, this function cannot be developed by Maclaurin's theorem.[54] Here, the Chinese translations are accurate, and the fact that the limit of a ratio whose denominator becomes 0 is infinite is rightly understood.[55]

Loomis also provided formulas for the differentials of transcendental functions, including exponential functions, logarithmical functions, and trigonometric (which Loomis called circular) functions, as well examples of their applications. Moreover, methods of applying the differential to find maxima and minima of a function; to determine the lengths of a tangent, subtangent, normal and subnormal; or to analyze curved lines, are also given. In order to see how the concepts of the infinite in these contexts were introduced, it may be helpful to compare two representative passages in the original text with their Chinese translations. The first passage deals with the famous relation between the tangent and the first differential coefficient (see Figures 15 and 16).

Loomis discussed the relation between tangents and the first differential coefficient in the format of a theorem. The substance of his proof goes as follows: Suppose *P* is a point on a curve, and its coordinates are (*x*, *y*); increase *x* by a small increment *h*, which is represented by *RR*′. Draw a line at *R*′ perpendicular to the line *CR*′, intercepting the curve at *P*′. In the right-angled triangle *DPP*′, $\tan P = \frac{DP'}{DP}$. Since *DP*′ is the increment of *y*, namely $y - y'$, and *DP* is equal to *h*, then $\tan P = \frac{y - y'}{h}$. As *P*′ gets

[54] *Ibid*.

closer and closer to *P*, the limit of the ratio *y* - *y'* and *h* is the first differential coefficient. In the meantime, the line *P'S* approaches *TP*, which is the tangent to the curve at the point *P*. Therefore, the theorem is proved.

Overall, the Chinese translation of Loomis' proof is accurate. For instance, the original sentence, "in order to find the differential coefficient of *y* with respect to *x*, we must find the limit of this ratio by making the increment equal to zero,"[56] was rendered as "求微係數，令辛=〇，以求比例之限。"[57] This translation rightly conveys the procedures of finding a differential coefficient by setting the increment of a variable equal to zero to find the limit of the ratio of the increment of a function to that of the variable. The key term "approach" is accurately rendered as "*jian jin* 漸近" (gradually close). Another important term, "increment," is translated as "*zhang shu* 長數" (increased number). This translation, though it fails to express the idea that an increment could be a geometrical magnitude, rightly introduces the concepts of differential coefficient and limit.

The second passage we consider here comes at the end of the section devoted to analyzing curved lines:

> From the preceding propositions its will be seen that, in order to trace out a curve from its equation, we first discover the most remarkable points by putting *x* and *y* successively equal to zero or infinity, and also the first and second differential coefficients equal to zero or infinity. Then, to trace the curve in the neighborhood of the points thus determined, when they appear to present any peculiarity, we increase one of the co-ordinates by a very small quantity, and observe the effect upon the other co-ordinate. Having determined the singular points, and examined

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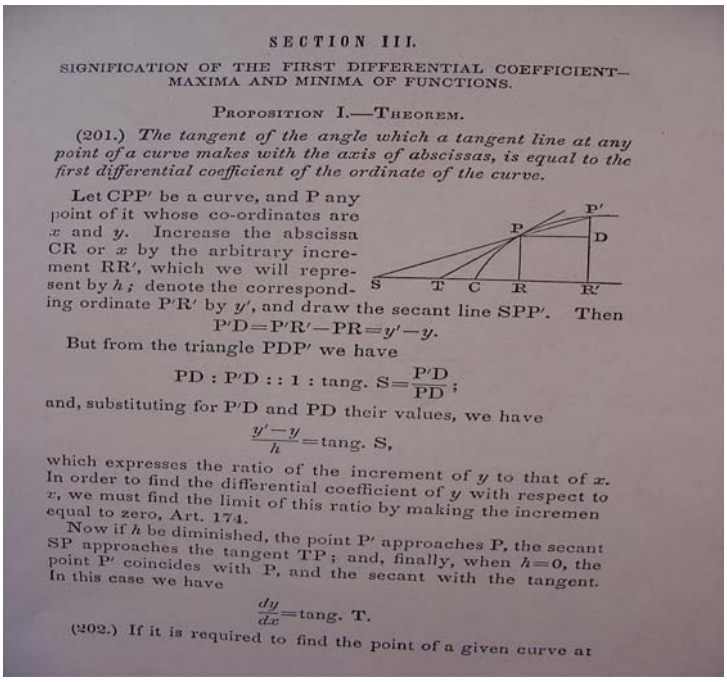
[55] Wylie and Li 1859, *juan* 11, 2-9.

[56] Loomis 1851, 147.

[57] Wylie and LI 1859, *juan* 12, 1b.



SECTION III.

SIGNIFICATION OF THE FIRST DIFFERENTIAL COEFFICIENT—MAXIMA AND MINIMA OF FUNCTIONS.

PROPOSITION I.—THEOREM.

(201.) *The tangent of the angle which a tangent line at any point of a curve makes with the axis of abscissas, is equal to the first differential coefficient of the ordinate of the curve.*

Let CPP′ be a curve, and P any point of it whose co-ordinates are x and y. Increase the abscissa CR or x by the arbitrary increment RR′, which we will represent by h; denote the corresponding ordinate P′R′ by y', and draw the secant line SPP′. Then

$$P'D = P'R' - PR = y' - y.$$

But from the triangle PDP′ we have

$$PD : P'D :: 1 : \text{tang. } S = \frac{P'D}{PD};$$

and, substituting for P′D and PD their values, we have

$$\frac{y'-y}{h} = \text{tang. } S,$$

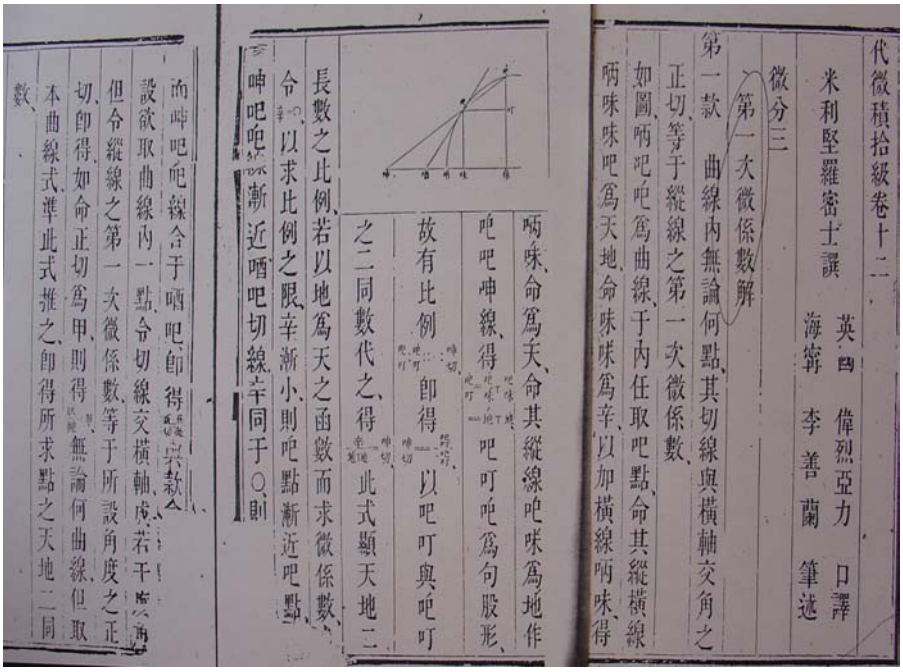
which expresses the ratio of the increment of y to that of x. In order to find the differential coefficient of y with respect to x, we must find the limit of this ratio by making the incremen equal to zero, Art. 174.

Now if h be diminished, the point P′ approaches P, the secant SP approaches the tangent TP; and, finally, when $h=0$, the point P′ coincides with P, and the secant with the tangent. In this case we have

$$\frac{dy}{dx} = \text{tang. } T.$$

(202.) If it is required to find the point of a given curve at

Figure 15. Elias Loomis' treatment of the relation between the tangent and the first differential coefficient. From Elias Loomis' *Elements of Analytical Geometry and of the Differential and Integral Calculus* (New York: Harper & Brothers, 1851), p. 147



代微積拾級卷十二

美利堅羅密士譔　英偉烈亞力　口譯

海寧　李善蘭　筆述

微分三

第一次微係數解

第一款　曲線內無論何點、其切線與橫軸交角之正切、等于縱線之第一次微係數、

如圖、丙叱叱爲曲線、于內任取叱點、命其縱橫線丙味味叱爲天地、命味味爲辛、以加橫線丙味、得丙味、命爲天、命其縱線叱味爲地、作叱叱神線、得叱叱可叱爲句股形、故有比例卽得以叱可與叱可之二同數代之、得此式顯天地二長數之比例、若以地爲天之函數而求微係數、令○以求比例之限、辛漸小、則叱點漸近叱點、神叱叱漸近酉叱切線、辛同于○、則而神叱叱線合于酉叱、卽得切線、與款△設欲取曲線內一點、令切線交橫軸成若干角、但令縱線之第一次微係數、等于所設角度之正切、卽得、如命正切爲甲、則得無論何曲線、但取本曲線式、準此式推之、卽得所求點之天地二同數、

Figure 16. Chinese translation of Elias Loomis' treatment of the relation between tangent and the first differential coefficient. From the *Daiweiji Shiji* 代微積拾級(*Elements of Analytical Geometry and of the Differential and Integral Calculus*) (Shanghai: The London Missionary Press, 1859), *juan* 12, 1-2a.

> the course of the curve in their immediate vicinity, we can easily trace the remainder to the curve, by assigning to *x* and *y* arbitrary values at pleasure.[58]

This English text may be translated into Chinese as follows:

> 觀上諸款，則知依式作曲綫圖，當先求諸獨異點。法令天與地等于〇、等于無窮，又令一次、二次微係數等于〇、等于無窮，即得諸獨異點。諸點中，若有可疑者，則略增損本從横線之一，驗其所得餘一線若何，即可定矣。諸獨異點既定，點之相近曲線狀亦既定。即可任置天地之同數，曲線之餘點俱可得。而作曲線易易矣。[59]

This translation not only accurately conveys the techniques of using infinitesimals and the infinite to draw curved lines, but at a certain point it is even more accurate than the original text. One sentence in the above-quoted English text reads: "Then, to trace the curve in the neighborhood of the points thus determined, when they appear to present any peculiarity, we increase one of the co-ordinates by a very small quantity, and observe the effect upon the other co-ordinate." Here, "increase" was translated as "增損," "increase or decrease," which in fact is more accurate.

Having established the theory of the differential calculus, Loomis then went on to lay out his account of the integral calculus. At the opening of this account, Loomis reiterated the point of the differential calculus:

> The object of the Differential Calculus is to determine the ratio between the rate of variation of the independent variable, and that of the function into which it enters.[60]

To introduce the concept of the integral and the notation introduced by Leibniz, Loomis offered the following explanation: "Leibnitz considered the differentials of functions as indefinitely small differences, and the *sum* of these indefinitely small

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[58] Quoted from Loomis 1851, 215-216.

[59] Quoted from Wylie and Li 1859, *juan* 16, 13a.

[60] Loomis 1851, 118.

differences he regarded as making up the function….This sign $\int$ is still retained even by those who reject the philosophy of Leibnitz."[61] The key concept of the integral calculus was its inverse relation to the differential calculus, and on this matter the Chinese translation clearly expressed the inverse connection between the differential calculus and the integral calculus: "積分為微分之還原，其法之要在識別微分所由生之函數。"[62] (The operation of the integral calculus is the inverse of the differential calculus. The key method is to identify the function from which the differential is derived).

As for the passage devoted to Leibniz, it appears as:

> "來本之視微分若函數諸小較之一，諸小較并之，即成函數……來氏說今西国天算家大率不用，而惟用此禾字，取其一覽了然也。"[63]

This Chinese translation may be rendered back into English as follows: "Leibniz considers the differential as a function's small difference, all small differences combined result in a function…Leibniz's theory is not used nowadays by most mathematicians in the West, except the sign $\int$, which looks straightforward." HU Mingjie has pointed out the obvious discrepancy between the last sentence and its corresponding English text, and thought that this difference indicates Wylie's preference for the Newtonian system of the calculus.[64] But the discrepancy may be caused by the various meanings of the word "philosophy." This word in the above context could mean Leibniz's own technical work on the calculus, or the entire Leibnizian approach, in contrast to the Newtonian approach to the calculus in terms of fluents and fluxions, and a correspondingly very different

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[61] *Ibid*.

[62] Wylie and LI 1859, *juan* 16, 1a.

[63] Wylie and LI 1859, *juan* 16, 1b.

[64] HU Mingjie 1998, 194.

notation. Perhaps Wylie took the first meaning of "philosophy," and interpreted it to LI Shanlan, who phrased it in Chinese more generally than either Loomis or Wylie may have intended.

What is of greater interest here, however, is the translation of "indefinitely small differences." Loomis in his textbook never used the word "infinitesimal," and did not take an infinitesimal approach to the calculus. The Chinese translation "函數諸小較之一" for "indefinitely small differences" actually means something closer to "function's small difference." But it is very hard to tell whether a reader could form any meaningful concept of infinitesimal from the proofs of theorems, their applications and other specific examples given in Loomis' textbook.

In his treatment of the integral calculus, Loomis provided formulas for integration of binomial differentials, series, and polynomial functions. He also applied the integral calculus to rectifying curves, finding areas and volumes, as well as areas of surfaces of revolution, and curvatures of solids of revolution. All these methods introduced in the textbooks are faithfully translated into Chinese.

How did all of these methods and concepts related to the infinite introduced in Alexander Wylie and LI Shanlan's translation of Loomis' textbook differ from the ones introduced in the second Chinese translation of the calculus fifteen years later? This "second" transmission of the calculus, in fact, as mentioned in the previous section, was entitled "*Weiji suyuan* 微積溯源," and was based on a lengthy article by William Wallace devoted to "Fluxions" in the eighth edition of the *Encyclopedia Britannica,* published in 1855. HU Mingjie has discussed the general structure and content of this article, and

given as well a brief comparison with Loomis' textbook, in his dissertation.[65] Here we confine our consideration specifically to how John Fryer and HUA Hengfang treated concepts of the infinite in their translation of Wallace's article.

The most striking difference in the translation of the article on fluxions, which introduces a clearly Newtoninan form of the calculus, is the fact that it devotes a section entirely to infinitesimals. This section is in *juan* four and is entitled "*lun wuqiong xiaoshu* 論無窮小數之理" (Of the Method of Infinitesimals).[66] It first introduces a proposition which says: "A quantity is not infinite so long as it admits of augmentation."[67] Here the idea is that if there is a quantity $x + a$, when x becomes infinite, a added to x does not make it bigger. In other words, a can be disregarded. Due to the fact that $\frac{a}{x}$ decreases as x increases continually, eventually it becomes smaller than any assignable quantity; an infinitesimal is defined as such a quantity (say a as in the ratio $\frac{a}{x}$), with respect to an infinite x.[68]

From a Leibnizian point of view, the differentials of quantities are the ratios to which the ratios of the increments of the variables continually approach. Since these increments may have any magnitude whatever, Leibniz supposes them to be infinitesimal. With the introduction of infinitesimals, this naturally leads to a succession of different orders of infinitesimals. For example, in a circle the ratio of the diameter to the chord of an arc is the same as the ratio of the chord to the versed sine. If the chord is infinitely

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[65] HU Mingjie 1998, 184-188.

[66] See Fryer and HUA 1874, 4: 25a-30b. The original English text is in Wallace 1855, 713b-714b.

[67] Quoted from Wallace 1855, 713b.

[68] See Wallace 1855, 713b-714a.

small with respect of the diameter, or an infinitesimal of the first order, the versed sine will be infinitely small with respect to the chord. Accordingly, the versed sine will be an infinitesimal of the second order with respect of the diameter.

Following introduction of infinitesimals, the rules for differentiation are easily introduced. For instance, if $u = xy$, considering the differentials *du*, *dx*, and *dy* as infinitely small, it follows that:

$$u + du = (x + dx)(y + dy) = xy + xdy + ydx + dxdy;$$

since $u = xy$, then

$$du = xdy + ydx + dxdy;$$

since *du*, *xdy*, *ydx* are all infinitesimals of the first order, and *dxdy* is an infinitesimal of the second, it is therefore incomparably smaller, and with respect to the first order infinitesimals, it may be ignored. Therefore:

$$du = xdy + ydx.$$

Observing the distinct character of this infinitesimal approach to the differential calculus, Fryer and HUA inserted one of the rare notes to be found in their translation:

> 以上各款之法，乃為流數之法。此節所論者，乃真為微分之法。因舊譯《代微積拾級》，既以流數為微分，則不便更改以淆讀者之耳目。所以不得不以微分為無窮小數。讀是書者，會通其意，勿泥其名可也。
>
> The method used in the previous sections is the method of fluxions, but the current one is really about the method of the differential. Since the *Daiweiji shiji* 代微積拾級 has used the term "*weifen*" for "fluxion," we retain it; otherwise readers may be confused by different terminologies. Therefore, we render this real differential [infinitesimal] as *wuqiong xiaoshu* 無窮小數. Readers should learn the contents, and should not be fooled by the different terms used.[69]

[69] The Chinese is quoted from Fryer and HUA Hengfang 1874, *juan* 4, 25a. For an alternative translation of this note, see HU Mingjie 1998, 194.

In addition to introducing new concepts such as infinitesimals, the "second" introduction of the calculus to China through the Chinese translation by Fryer and HUA also provided new terms for concepts already introduced in the first translation, for instance, the concept of *differential*. As discussed at the beginning of this section, Wylie and LI Shanlan rendered the definition of differential as something like: "The changing proportions of a function and a variable both are called differentials." Perhaps finding this too confusing, Fryer and HUA used another term "*yilü* 溢率," which literally means "exceeding rate."[70] This new term does clarify the status of the differential, and avoids the imprecision of the term employed by Wylie and LI Shanlan.

Both the article by Wallace and the textbook by Loomis were written before the rigorous formulation of the calculus by Karl Weierstraß in the second half of the nineteenth century. Indeed, the revolution that was taking place in European mathematics with respect to the theory of functions in the nineteenth century, beginning with Cauchy and continuing in the works of the German school in particular, represented by Riemann and Weierstraß, and then by the French, especially Lebesgue at the turn of the century, brought new rigor and new concepts to the calculus. But these developments, which had taken several centuries in Europe to reach their full maturity, would eventually coalesce into a much more fundamental understanding of the calculus among a new generation of Chinese mathematicians in the 1920s. Until then, however, the first two attempts to introduce the calculus to China in the late nineteenth century at least served to provide Chinese readers with new perspectives and tools for comprehending this new subject. They paved the way as well for Chinese mathematicians to accept and

[70] See Fryer and HUA 1874, *juan* 1, 14a-15a. For a detailed discussion of the meanings of this Chinese term as well as the influence of traditional Chinese mathematics upon selecting this term, see HU Mingjie

appreciate the rigorous treatment of the calculus when it was finally introduced in the 1920s.

1998, 207-211.

Chapter 6 Introduction of Mathematical Logic

> Russell's logic is not normal or traditional logic, but a new one, namely mathematical logic.[1]
>
> ZHANG Shenfu 張申府

Whether the *Mohist Canon* or the *Book of GONGSUN Long* contain the basic concepts of formal logic, or whether formal logic is wholly alien to the ancient Chinese way of thought in the pre-Qin periods, has been widely debated. But when China was defeated by a handful of English troops from a remote island country as a result of the so-called Opium Wars in 1842, and forced to re-open its doors to the west as a result of a number of treaties thereafter, Chinese thinkers and scholars alike began to ponder what had gone wrong for a once mighty China? Among numerous theories ventured at the time was the suggestion that the Chinese lacked a logical way of thinking or reasoning, which perhaps due in part to the continuing influence of Matteo Ricci and his Jesuit colleagues, was widely believed to be the corner stone of modern Western science. An introduction of Western logic, therefore, was considered by some as a compelling necessity. With this in mind, one of the most prominent student pioneers, YAN Fu 嚴复, who had studied mathematics, chemistry, physics, and naval science at the Greenwich Naval College in Great Britain in the late 1870s, devoted himself to introducing modern Western logic to China. He did so first by translating John Stuart Mill's *A System of Logic, Ratiocinative and Inductive: Being a Connected View of the Principles of Evidence, and Methods of Scientific Investigation* into Chinese: *Mule mingxue* 穆勒名學 (literally, *The Logic of Mill*,

[1] Translated from ZHANG Shenfu 1993, 66.

1905), and then William Stanley Jevons's *Elementary Lessons in Logic: Deductive and Inductive*, *Mingxue qianshuo* 名學淺説 (*Simple Explanation of Logic*, 1909).[2] Chinese interest in logic was immediately aroused, and was soon extended to the more technical study of mathematical logic, which was widely promoted in China thanks to Bertrand Russell. It was through his lectures in China and translation of Russell's books that Chinese mathematicians and philosophers were directly introduced to this subject which slowly but steadily gained its foothold in China.

6.1 Bertrand Russell's Lectures in China

In the 1910s, China experienced drastic political and social changes. The Qing dynasty rule of the Manchus ended in 1911 after two-hundred-and-sixty-eight years of turbulent history. The Republic was created in the following year, but most of China was really under the control of several powerful warlords, each of whom was backed by different Western governments. The power struggles among the warlords left the county in turmoil. The elites in China assumed social responsibilities, and advocated science and democracy for the stability and modernization of the nation. One of the most effective ways to achieve this noble goal, they thought, was to invite foreign scholars to lecture in China. The Progressive Party (*Jinbudang* 進步党), lead by the well-known scholars LIANG Qichao 梁啟超 and ZHANG Dongsun 張東蓀, also supported this idea. The Progressive Party, established in 1913, advocated, among others, "an open minded awareness of world trends."[3] To that end, the Party created a program entitled "*xueshu*

[2] For a biography of YAN Fu, see Boorman 1971b. For the contributions of William Stanley Jevons to logic, see Grattan-Guinness 1991; 2000, 56-60.

[3] Levenson 1967, 175.

jiangyan hui 學術講演會 (The Chinese Lecture Association)," and in 1919 started to invite high profile Western scholars to visit China, in cooperation with other institutions including the National University of Beijing. The first to be invited was the American Philosopher John Dewey of Columbia University.[4] Bertrand Russell was the second.[5]

Russell arrived in Shanghai on 12 October, 1920, and was welcomed as the second Confucius.[6] As a result of intensive media coverage throughout China, Russell became a celebrity overnight; even the founding father of the Republic, SUN Zhongshan [SUN Yat-sen] 孫中山, invited him for dinner.[7] During his first three weeks in China, Russell delivered innumerable public lectures in Shanghai, Hangzhou, Hankou, and Changsha on topics ranging from Einstein's relatively theory to education and social problems. Early in November, Russell arrived at his ultimate destination: The National University of Beijing.

The Beijing University Daily 北京大學日刊 announced on November 3 that Russell was to lecture there about 1) principles of social reform and 2) the analysis of mind, beginning November 3 until the end of April, 1921.[8] But because he had been in China for less than a month, Russell felt it would have been irresponsible to address such issues as social reform after as yet so brief a stay in China, and thus he instead chose to lecture on problems of philosophy, one of his own specialties.[9] Over the next four months,

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[4] For John Dewey's lectures in China and their subsequent impact on the thought of the Chinese, see Clopton and OU 1973, 1-39.

[5] ZHAO Yuanren 1977, 57.

[6] YANG 1921, 7; Russell 1968, 197.

[7] Russell 1968, 180.

[8] Beijing University 1981: 6, November 3, 1920.

[9] Beijing University 1981: 6, November 5, 1920.

Russell also lectured on two additional topics: the analysis of matter and the structure of society. At the beginning of March, he was invited jointly by The Society for Mathematics and Physics at Beijing University and by The Society for Mathematics, Physics, and Chemistry at Beijing Teachers College to lecture on mathematical logic. It was this course of lectures that was translated as *Suanxue de lunlixue* 算學的論理學 (literally, the science of mathematical reasoning). According to *The Beijing University Daily*, Russell agreed to give four lectures in all, the first two at the University of Beijing, the last two at the Teachers College.[10] The first lecture was given in the evening on Tuesday, March 8, to an audience of about one hundred and fifty professors and college students. Russell began by explaining the meanings of the different symbols he would use, and then introduced both the calculus and algebra of propositions.[11]

The second lecture was announced for March 15, at the same time and place.[12] But just one day before, after delivering a speech at Yude 育德 High School in Baoding City, about ninety miles from Beijing, Russell came down with pneumonia and almost died. For nearly six weeks he was confined to his bed. Thanks to the Rockefeller Institute in Beijing, he eventually recovered, but was still very weak when he left Beijing on July 10, 1921.[13] Because of his sudden illness, Russell's subsequent lectures on mathematical logic were all cancelled.

Russell's one lecture on mathematical logic, like the ones he had given on philosophy, was orally translated by ZHAO Yuanren [Yuan Ren Chao] 趙元任. ZHAO

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[10] Beijing University 1981: 7, March 4, 1921; March 5, 1921; March 7, 1921; and March 8, 1921.

[11] Russell 1921; 1921b.

[12] Beijing University 1981: 7, March 12, 1921.

[13] Russell 1968, 186-190.

had been educated in the United States with financial support from the Boxer Indemnity. After receiving his Bachelor degree in 1915 from Cornell University, where he studied mathematics and physics, ZHAO went to Harvard University to study mathematical logic and philosophy. There he wrote his dissertation which is entitled "Continuity: a Study in Methodology," under the supervision of Harry M. Sheffer in 1918.[14] One chapter of his dissertation even discusses "infinity," though not in the sense of a completed, static infinite, or as transfinite numbers, but as a variable.[15] Thus ZHAO was thoroughly conversant with the subjects Russell intended to cover, and Russell could hardly have hoped for a better and more well-informed translator. Thanks to ZHAO's efforts, Russell's lectures were conveyed as accurately as possible to his Chinese audiences.

Among those who attended Russell's lecture were WU Fanhuan 吳範寰 and WANG Shiyi 王世毅. Both of them kept detailed notes of the lecture. Wang's notes were immediately published in the *Magazine of Beijing University for Mathematics and Natural Science*.[16] WU's notes, nevertheless, were published in 1921 by the New Knowledge Press of the National University of Beijing as a pamphlet entitled: *Shuli luoji* 數理邏輯, an expression that soon became the standard Chinese translation for "mathematical logic."[17] Although Russell's lectures on mathematical logic in Beijing unfortunately ended so abruptly, his Chinese audience was nevertheless inspired and interest he kindled in the subject began to grow. Two young scholars in particular

[14] ZHAO Yuanren 1918; 1975, 83-84. ZHAO's dissertation has been recently published in volume 14 of his completed works, see ZHAO 2004, 263-415.

[15] ZHAO Yuanren 1918.

[16] Russell 1921b; and ZHANG Zugui 1995, 912.

[17] Russell 1921a.

devoted themselves to studying Russell's books on mathematical logic and made up their minds to translate one of them into Chinese. The book they selected was *Introduction to Mathematical Philosophy*.

6.2 Translations of *Introduction to Mathematical Philosophy*

Russell wrote his *Introduction to Mathematical Philosophy* (hereafter referred to as *IMP*) while he was in prison serving a six-month sentence for an article he had published on January 3, 1918, in the weekly newspaper, *The Tribunal*, where he actually said that "American soldiers would be employed as strike-breakers in England, an occupation to which they were accustomed when in their own county."[18] In the Preface to *IMP*, Russell said that his major reason for writing the book was to present "The main results of mathematical logic in a form [re]quiring neither a knowledge of mathematics nor an aptitude for mathematical symbolism."[19] Later in his autobiography, he wrote that the book was "a semi-popular version" of his earlier book, *The Principles of Mathematics*, published in 1903.[20] Because of its simple and lucid style, *IMP* was popular among young students who were interested in learning mathematical logic in the 1920s.[21]

The two Chinese mathematicians who set themselves the task of translating Russell's *IMP* were FU Zhongsun 傅种孫 and ZHANG Bangming 張邦明. FU was a pioneer in the modernization of Chinese mathematics, and he was a long-time faculty

[18] Russell 1968, 33. For the entire article, which is entitled "The German Peace Offer," see Russell 1968, 79-81.

[19] Russell 1919, xii.

[20] Russell 1968, 30.

[21] Broadbent 1975, 15.

member in the Department of Mathematics at Beijing Teachers College, the forerunner of Beijing Normal University. FU is now best remembered for his translation of Russell's work under discussion here, and for his contributions to mathematics education in China.[22] In contrast to FU, ZHANG is an otherwise obscure figure who is now remembered only for his collaboration with FU.

Valuable information about how FU and ZHANG went about making their translation may be found in a letter by them to Russell (see Figures 17-19).[23] Their letter was written on March 23, 1921, when Russell was ill with pneumonia and literally struggling for his life, but from the letter, it is apparent that the authors did not know how serious Russell's situation was. In the letter, FU and ZHANG explained that they were translating his *IMP*, based on the second edition of 1920, and that they had both attended his first lecture on mathematical logic and planned to attend the remaining three. They also intended to publish their lecture notes as an appendix to the translation of the book. At the end of the letter, they also asked whether Russell had any recent information about his gifted student, Ludwig Wittgenstein, about whose life Russell had shown concern in a footnote in *IMP*.[24] The main purpose of their letter, however, was to ask Russell to clarify some questions they had while preparing their translation of *IMP*. The letter also indicates that earlier FU and ZHANG had already put some questions to Russell before,

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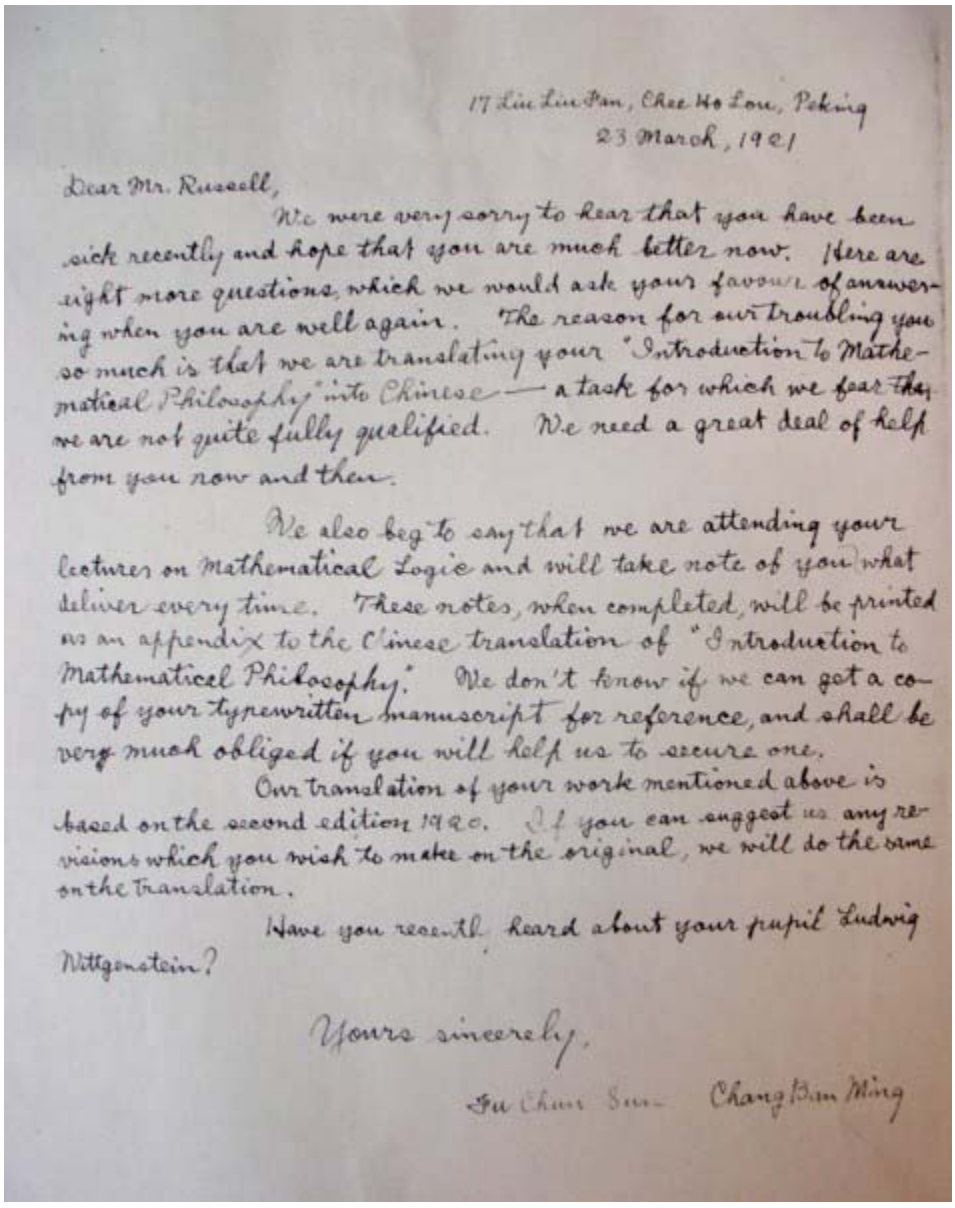
[22] ZHAO and BAI 1996, 117-124.

[23] The letter from FU Zhongsun and ZHANG Bangming to Bertrand Russell is currently preserved at the Bertrand Russell Archives at McMaster University, Ontario, Canada. Hereafter the letter is referred to as "FZ's Letter." The author is grateful to Dr. Carl Spadoni and Ms. Sheila Turcon for providing copies of this letter as well as other relevant documents preserved at the Bertrand Russell Archives. Permission to quote from, and reproduced, the letter is gratefully acknowledged.

[24] Russell 1920, 205.

but it is unclear whether Russell ever replied to any of these questions or not (see Figure 17).



17 Lin Lin Pan, Chee Ho Lou, Peking
23 March, 1921

Dear Mr. Russell,

We were very sorry to hear that you have been sick recently and hope that you are much better now. Here are eight more questions, which we would ask your favour of answering when you are well again. The reason for our troubling you so much is that we are translating your "Introduction to Mathematical Philosophy" into Chinese — a task for which we fear that we are not quite fully qualified. We need a great deal of help from you now and then.

We also beg to say that we are attending your lectures on Mathematical Logic and will take note of you what deliver every time. These notes, when completed, will be printed as an appendix to the Chinese translation of "Introduction to Mathematical Philosophy". We don't know if we can get a copy of your typewritten manuscript for reference, and shall be very much obliged if you will help us to secure one.

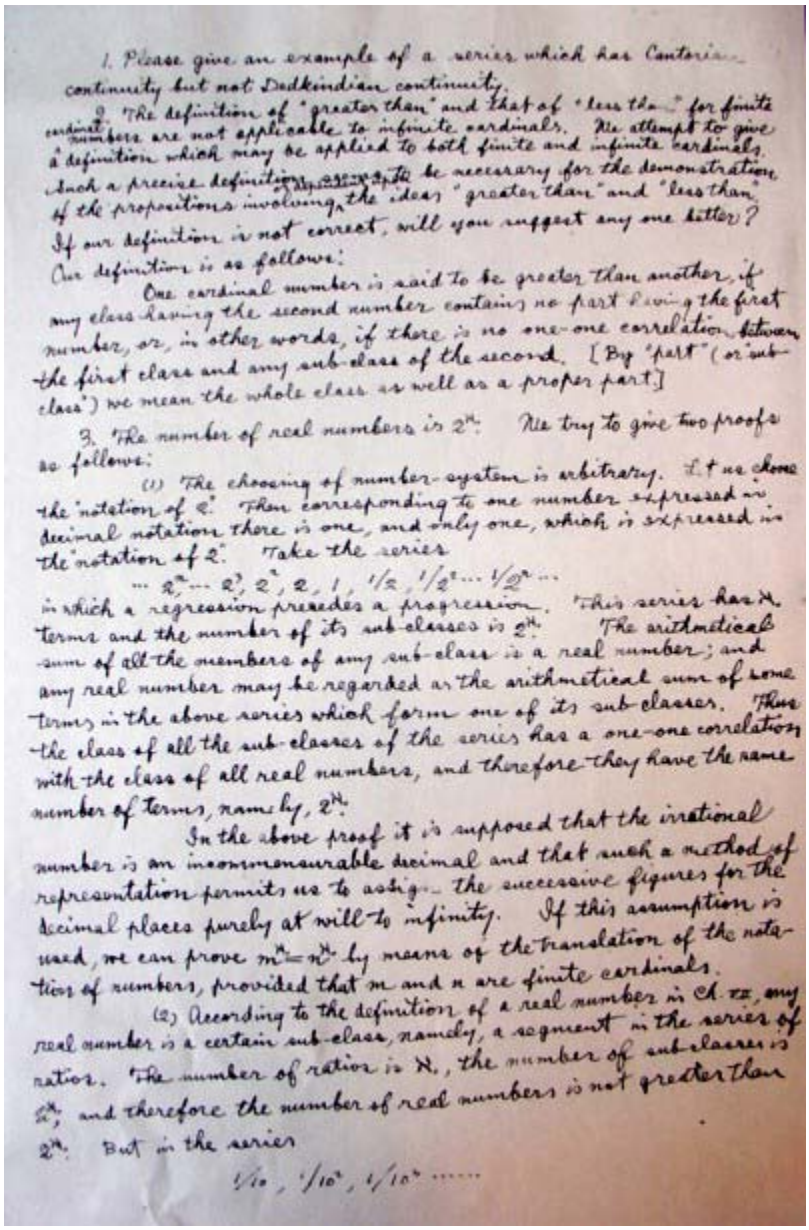
Our translation of your work mentioned above is based on the second edition 1920. If you can suggest us any revisions which you wish to make on the original, we will do the same on the translation.

Have you recently heard about your pupil Ludwig Wittgenstein?

Yours sincerely,

Fu Chun Sun Chang Bun Ming

Figure 17. Page 1 of the letter dated March 23, 1921, from FU Zhongsun and ZHANG Bangming to Bertrand Russell. The Bertrand Russell Archives at McMaster University, Ontario, Canada.



1. Please give an example of a series which has Cantorian continuity but not Dedkindian continuity.

2. The definition of "greater than" and that of "less than" for finite cardinal numbers are not applicable to infinite cardinals. We attempt to give a definition which may be applied to both finite and infinite cardinals. Such a precise definition seems to be necessary for the demonstration of the propositions involving the ideas "greater than" and "less than". If our definition is not correct, will you suggest any one better?

Our definition is as follows:

One cardinal number is said to be greater than another, if any class having the second number contains no part having the first number, or, in other words, if there is no one-one correlation between the first class and any sub-class of the second. [By "part" (or "sub-class") we mean the whole class as well as a proper part.]

3. The number of real numbers is $2^{\aleph_0}$. We try to give two proofs as follows:

(1) The choosing of number-system is arbitrary. Let us choose the notation of 2. Then corresponding to one number expressed in decimal notation there is one, and only one, which is expressed in the notation of 2. Take the series

$$\ldots 2^n \ldots 2^3, 2^2, 2, 1, 1/2, 1/2^2 \ldots 1/2^n \ldots$$

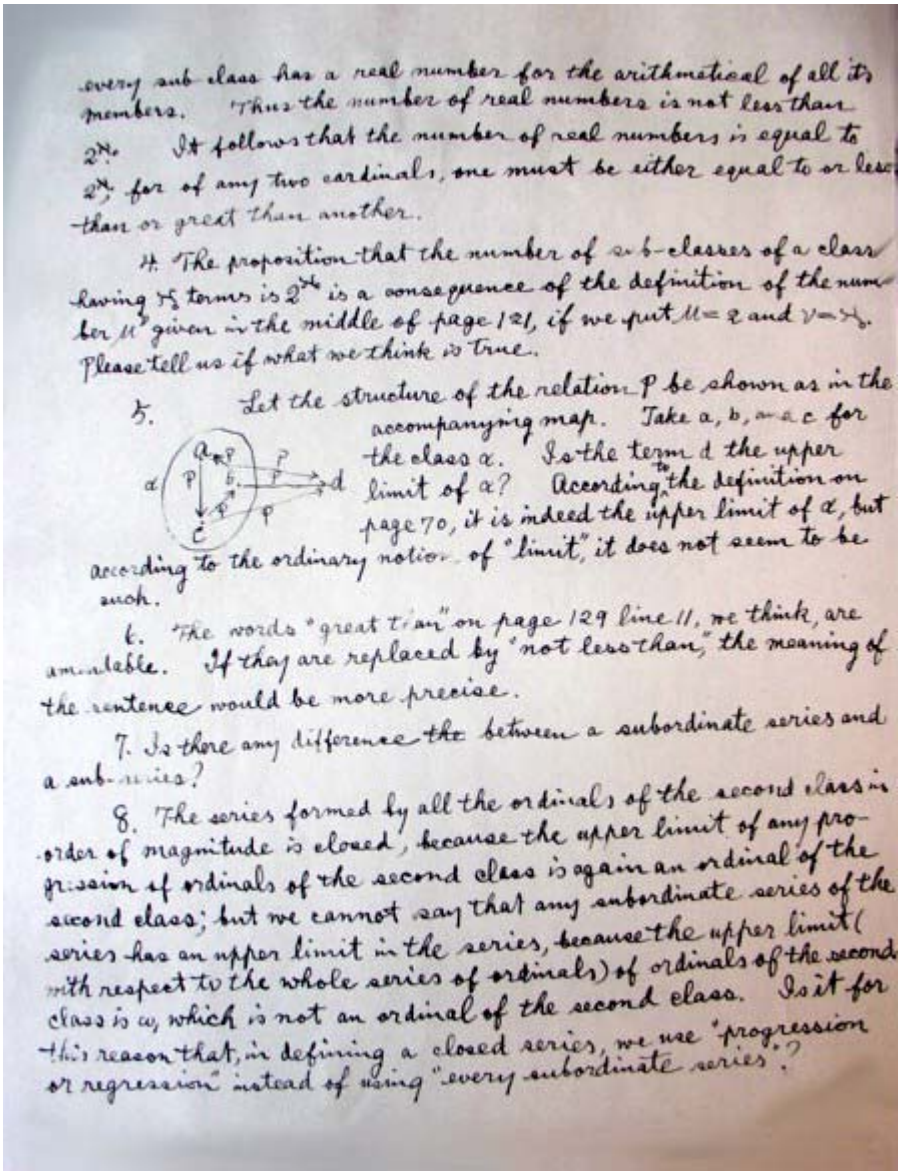
in which a regression precedes a progression. This series has $\aleph_0$ terms and the number of its sub-classes is $2^{\aleph_0}$. The arithmetical sum of all the members of any sub-class is a real number; and any real number may be regarded as the arithmetical sum of some terms in the above series which form one of its sub-classes. Thus the class of all the sub-classes of the series has a one-one correlation with the class of all real numbers, and therefore they have the same number of terms, namely, $2^{\aleph_0}$.

In the above proof it is supposed that the irrational number is an incommensurable decimal and that such a method of representation permits us to assign the successive figures for the decimal places purely at will to infinity. If this assumption is used, we can prove $m^{\aleph_0} = n^{\aleph_0}$ by means of the translation of the notations of numbers, provided that m and n are finite cardinals.

(2) According to the definition of a real number in Ch. IV, any real number is a certain sub-class, namely, a segment in the series of ration. The number of ration is $\aleph_0$, the number of sub-classes is $2^{\aleph_0}$, and therefore the number of real numbers is not greater than $2^{\aleph_0}$. But in the series

$$1/10, 1/10^2, 1/10^3 \ldots\ldots$$

Figure 18. Page 2 of the letter dated March 23, 1921, from FU Zhongsun and ZHANG Bangming to Bertrand Russell. The Bertrand Russell Archives at McMaster University, Ontario, Canada.



every sub class has a real number for the arithmetical of all its members. Thus the number of real numbers is not less than $2^{\aleph_0}$. It follows that the number of real numbers is equal to $2^{\aleph_0}$, for of any two cardinals, one must be either equal to or less than or great than another.

4. The proposition that the number of sub-classes of a class having $\aleph_0$ terms is $2^{\aleph_0}$ is a consequence of the definition of the number μ^ν given in the middle of page 121, if we put $\mu = 2$ and $\nu = \aleph_0$. Please tell us if what we think is true.

5. Let the structure of the relation P be shown as in the accompanying map. Take a, b, c for the class α. Is the term d the upper limit of α? According to the definition on page 70, it is indeed the upper limit of α, but according to the ordinary notion of "limit", it does not seem to be such.

6. The words "great than" on page 129 line 11, we think, are amendable. If they are replaced by "not less than," the meaning of the sentence would be more precise.

7. Is there any difference between a subordinate series and a sub-series?

8. The series formed by all the ordinals of the second class in order of magnitude is closed, because the upper limit of any progression of ordinals of the second class is again an ordinal of the second class; but we cannot say that any subordinate series of the series has an upper limit in the series, because the upper limit (with respect to the whole series of ordinals) of ordinals of the second class is ω, which is not an ordinal of the second class. Is it for this reason that, in defining a closed series, we use "progression or regression" instead of using "every subordinate series"?

Figure 19. Page 3 of the letter dated March 23, 1921, from FU Zhongsun and ZHANG Bangming to Bertrand Russell. The Bertrand Russell Archives at McMaster University, Ontario, Canada.

The first question raised by FU and ZHANG concerns different definitions of continuity. In chapter 10 of the *IMP*, Russell discussed two definitions of continuity, one given by Richard Dedekind and the other by Georg Cantor. Russell concluded that: "Cantor's definition [of continuity] is narrower than Dedekind's: a series which had Cantorian continuity must have Dedekindian continuity, but the converse does not

hold."[25] The first question FU and ZHANG asked Russell was to provide "an example of a series which has Cantorian continuity but not Dedekindian continuity" (see Figure 18).

In the chapter entitled, "Rational, Real, and Complex Numbers," Russell considered how to define "greater" and "less" among ratios or fractions.[26] FU and ZHANG realized that Russell's discussion did not provide a strict definition of the two terms, and thought it might be better to provide one, which would be suitable for comparing any cardinals, finite or infinite. Accordingly, they ventured their own definition, as follows: "One cardinal number is said to be greater than another, if any class having the second number contains no part having the first number, or, in other words, if there is no one-one correlation between the first class and any sub-class of the second (by "part" (or "sub-class") we mean the whole class as well as a proper part)" (see Figure 18). Their second question was about Russell's opinion of their proposed definition.

FU and ZHANG then gave two alternative proofs that the number of real numbers is $2^{\aleph_0}$, and asked Russell whether their proofs were right. The proofs they offered are worth quoting in their entirety (the words in parentheses are quoted from the original letter, their modifications and added words are in square brackets):

> (1) The [choice] (choosing) of number system is arbitrary. Let us choose the "notation of 2." Then corresponding to one number expressed in decimal notation there is one, and only one, which is expressed in the "notation of 2." Take the series
>
> ... 2^n, ... 2^3, 2^2, 2, 1, 1/2, $1/2^2$, ... $1/2^n$...

[25] Russell 1920, 100; and Dauben [1979]1990, 48-50, 110-111.

[26] Russell 1920, 65-66.

> in which a regression precedes a progression. This series has $\aleph_0$ terms and the number of its sub-class is a real number; and any real number may be regarded as the arithmetical sum of some terms in the above series which form one of its sub-classes. Thus the class of all the sub-classes of the series has a one-one correlation with the class of all real numbers, and therefore they have the same number of terms, namely, $2^{\aleph_0}$.
>
> In the above proof, it is supposed that [an] (the) irrational number is an incommensurable decimal and that such a method of representation permits us to assign the successive figures for the decimal places [arbitrarily] (purely at will) to infinity. If this assumption is [correct] (used), we can prove $m^{\aleph_0} = n^{\aleph_0}$ by means of the translation of the notation of numbers, provided that m and n are finite cardinals.
>
> (2) According to the definition of a real number in Ch. VII [of Russell's *IMP*], any real number is a certain sub-class, namely, a segment in the series of ratios. The number of ratios is $\aleph_0$, the number of sub-classes is $2^{\aleph_0}$, and therefore the number of real numbers is not greater than $2^{\aleph_0}$. But in the series
>
> $1/10, 1/10^2, 1/10^3$ ...
>
> every sub-class has a real number for [arithmetic] (the arithmetical) of all its members. Thus the number of real numbers is not less than $2^{\aleph_0}$. It follows that the number of real numbers is equal to $2^{\aleph_0}$, for [of] (if) any two cardinals, one must be either equal to or less than or great[er] than another (see Figures 18 and 19).

Russell in a chapter devoted to the multiplicative axiom, or the axiom of choice, said that the product of two classes, one having μ terms, the other ν terms, is μ^{ν}.[27] FU and ZHANG asked, in their fourth question, whether it was right to draw the conclusion that the number of sub-class of a class having $\aleph_0$ terms is $2^{\aleph_0}$ by letting $\mu = 2$ and $\nu = \aleph_0$?

The fifth question they raised concerned the definition of "upper limit" and that of "limit." FU and ZHANG constructed a class α, consisting of three elements *a*, *b*, and *c*, each having the relationship *P* with each other. Each of the elements also had the relationship *P* with another element *d* which did not belong

[27] Russell 1920, 121.

to the class α. FU and ZHANG thought that the element *d* should be precisely the upper limit, but not the limit, of the class α according to Russell's definition of "upper limit" and "limit" in *IMP*.[28]

FU and ZHANG then went on to point out that the words "greater than" (on line 11 of Russell 1920, 129) in the *IMP* might be replaced by "not less than" for the sake of making the meaning of the original text more precise.[29] Immediately after that, FU and ZHANG asked their seventh question: Was there any difference between a subordinate series and a sub-series? Their eighth and the last question concerned the second class of ordinals:

> The series formed by all the ordinals of the second class in order of magnitude is closed, because the upper limit of any progression of ordinals of the second class is again an ordinal of the second class; but we cannot say that any subordinate series of the series has an upper limit in the series, because the upper limit (with respect to the whole series of ordinals) of ordinals of the second class is ω, which is not an ordinal of the second class. Is it for this reason that, in defining a closed series, we use "progression or regression" instead of using "every subordinate series"? (See Figure 19).

The above questions, though not of much theoretical significance, clearly show that FU and ZHANG were both very serious about their roles as translators of the *IMP*. Their two proofs about the number of real numbers show considerable ingenuity, and their replacement of "bigger than" by "no less than" is appropriate. The *IMP* has been reprinted many times, most recently by Routledge in 1993, but its contents remain the same as the second edition of 1920 used by FU and ZHANG. Thus, Russell never seems to have responded to these

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[28] Russell 1920, 70.

[29] Russell 1920, 129.

questions raised by FU and ZHANG as he might have had he ever issued a revised new edition of his book. Moreover, it is not known whether Russell ever replied to their letter. What can be said is that FU and ZHANG did incorporate some points from their letter in the translation they made of the *IMP*. For instance, the proofs provided in questions four and five of their letter can be found in three of the translators' notes to chapter eight;[30] and the correction they suggested in question six (substituting "not less than" for "greater than") can be found in the second note added by the translators in chapter twelve.[31]

FU and ZHANG's translation of the *IMP* introduced Chinese readers for the first time to:

1) Giuseppe Peano's system of logic built upon three primitive ideas and five primitive propositions;

2) An array of fundamental concepts such as "set," "class," "order," "relation," "infinity," and "type," among others;

3) Strict definitions of "number," including natural, rational, real and complex numbers;

4) Georg Cantor's transfinite set theory;

5) Logic and use of relations;

6) Logic of descriptive functions;

7) The axiom of choice, its equivalent propositions, and the axiom of infinity;

8) Russell's type theory, which was meant as an antidote to the logical paradox Russell himself discovered in 1903; and

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[30] Russell 1931, 141-144.

[31] Russell 1931, 212, 215-216. For recent studies of Bertrand Russell, his logic and mathematics, see Garciadiego 1992; Moore 1993; and Grattan-Guinness 2000, 333-410.

9) Russell's logicism, which asserts that mathematics is logic and that all mathematical theories can be reconstructed in terms of logic, *et cetera*.

In addition to each of the above, FU and ZHANG briefly introduced most of the important concepts and theories of mathematical logic known up to that time.

FU and ZHANG's translation appeared in August of 1922, and was published by the Commercial Press in Shanghai (then, as now, one of the most prestigious presses in China) under the Chinese title: *Luosu suanli zhexue* 羅素算理哲學, which literally means *Russell's Mathematical Philosophy*. The translation itself was very successful; it was reprinted two years later, and in 1930 it reappeared in a series of world classics issued by the same Press under a slightly different title: *Suanli zhexue* 算理哲學 (*Mathematical Philosophy*), moving Russell's name from the title to that of author. In April of 1931, the translation was also included in another series— *Gongxueshe Luosu Congshu* 共學社羅素叢書 (The Russell Series of the Communal Education Society).[32] The Chinese translation, like its original in the West, provided Chinese scholars and students alike in the 1920s and 30s with an excellent introduction to mathematical logic.

FU and ZHANG's translation of the *IMP* was initially a success, but as time went by, the literary style they had used, which was very much influenced by the movement for the reform of writing begun in the mid-1910s, was regarded as awkward by the new generation. In addition, certain terminologies in the *IMP* were not translated as accurately as they should have been. When HE Lin 賀麟, the most prominent Chinese expert on Hegel, called upon the Chinese Philosophical Society in 1941 to form a Committee in Kunming, Yunnan Province, for Translating Western Classics of

[32] Russell 1931.

Philosophy (*Zhongguo zhexuehui xiyang zhexue mingzhu bianyi weiyuanhui* 中國哲學會西洋哲學名著編譯委員會), it was suggested that the *IMP* be retranslated. Eventually, YAN Chengshu 晏成書, a graduate of the Department of Philosophy of Central University in Nanjing, and then a junior member of the Department of Philosophy at Beijing University, was invited to make the retranslation. Although she completed this task by the late 1950s, it was not published until 1982, and when it did appear, it was under a direct transliteration of Russell's original title: *Shuli zhexue daolun* 數理哲學導論 (*Introduction to Mathematical Philosophy*).[33] The delay in publication was due in large measure to the Cultural Revolution (1966-1976). Besides the *IMP*, YAN was also asked to translate Hermann Weyl's *The Open World* for the Committee. She finished the first three chapters on "God and the Universe," "The Relation of Cause and Result," and "Infinity," and left behind a manuscript translation of about thirty thousand Chinese characters when she died on April 11, 1995, at the age of 71.[34]

The most telling difference between the two translations of the *IMP* is their literary styles. Compared with the translation by FU and ZHANG, YAN's translation is easier to understand and more accurate. Consider, for example, the translations of the term "function," first rendered as "*han shu* 函數" in 1859 by the British missionary Alexander Wylie and the Chinese mathematician LI Shanlan 李善蘭 when they translated Elias Loomis' *The Elements of Analytical Geometry and That of Differential and Integral Calculus*, and Augustus De Morgan's *Elements of Algebra*, into Chinese. The basic

[33] Russell 1982.

[34] The information about YAN Chengshu and her translation work provided here is based on an interview with TIAN Fangzheng 田方增, YAN's widower and Professor of mathematics at the Institute of Mathematics of Chinese Academy of Sciences, in August of 1999, and subsequently personal communication.

meaning of the Chinese character "*han* 函" is "contain," and that of "*shu* 數," "number." Therefore, the "*han shu* 函數" means the "numbers contained." This is a very good translation. Nevertheless for a very good reason that will be apparent in a moment, FU and ZHANG chose not to adopt this translation, but created instead another Chinese term, "*cong yuan* 從元."[35] Accordingly "descriptive function," "intensional and extensional functions," "predicative functions," and "propositional function" were then rendered as "*moshu congyuan* 摹述從元," "*shuxing congyuan ji waifan congyuan* 屬性從元及外範從元," "*shici congyuan* 實辭從元," and "*mingti congyuan* 命題從元," respectively.[36] Clearly FU and ZHANG understood that Russell's "function" was not restricted to number. The character "*yuan* 元" was borrowed from the "*tian yuan shu* 天元術 (celestial method)," a Chinese algebra popularized at least by the tenth century, and possibly earlier. In this method, "*yuan*" represents the unknown, in just the same way as "*x*" does in modern algebra.[37] As for the meaning of "*cong yuan* 從元," "*cong* 從" literally means "follow, or derive from," and "*yuan* 元" means "any unknown numbers." Therefore, "*cong yuan* 從元" has the meaning of "derived from unknown," which unfortunately failed to reflect the full meaning of "function" in the *IMP*. YAN Chengshu, however, rendered "function" as "*han xiang* 函項."[38] Here "*xiang* 項" means "term," or "item." "*Han xiang* 函項," therefore, not only retains the meaning of "*han*" in the "*han shu* 函數," of Wylie and LI,

[35] Russell 1931, 72.

[36] Russell 1931, 308.

[37] FU also had an interest in the history of Chinese mathematics, and in 1918 published an article on a traditional Chinese method for solving indeterminate problems. See ZHAO and BAI 1996, 118.

[38] Russell 1982, 46.

but it also does not restrict variables to number. Thus, the term as translated by YAN has the exact same meaning as that of "function" in the *IMP*.

One more example may serve to illustrate this point more directly. The term "successor," one of Peano's three primitive ideas, was rendered "*ji shu* 繼數" by FU and ZHANG.[39] The basic meaning of "*ji* 繼" is "to follow," and that of "*shu* 數" is "number." When used together, the two combined characters have the meaning of "number followed, or successor to another number." The term in YAN's translation, however, was rendered "*hou ji* 後繼."[40] The literal meaning of "*hou* 後" is "after," or "late." Thus the meaning of "*hou ji*" is "follow," or "followed by." Like the term "function," the term "successor" in the *IMP* is not restricted to numbers but can be applied to anything. Accordingly, the translation "*hou ji*" is more general than "*ji shu*."

YAN's retranslation of the *IMP* was reprinted in 1999 and continues to provide an excellent introduction to mathematical logic for the current generation of young mathematicians and logicians in China. Chinese students in the 1920s, however, were introduced to mathematical logic solely through FU and ZHANG's translation of the *IMP*. But before long, some serious students, especially those who had majored in mathematics or philosophy, were eager to learn how mathematical propositions are built up on a few axioms and concepts. It was quite natural that Russell's system of mathematical logic and his monumental work, *Principia Mathematica*, which was mentioned or cited in the *IMP* many times, became the focus of the continuing introduction of mathematical logic in China.

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[39] Russell 1931, 7-13.

[40] Russell 1982, 11-15.

6.3 Introduction of *Principia Mathematica*

Principia Mathematica (hereafter referred to as *PM*) was a result of the collaboration between Russell and his mentor Alfred North Whitehead. The first volume of *PM* appeared in 1910, and was soon followed by volume II (1912) and volume III (1913).[41] Although most of its propositions could be found elsewhere, *PM* was instantly recognized by experts in this nascent field as a milestone for its systematic construction of mathematics in terms of the concepts of logic, for the grand theory of types Russell introduced, and for its encyclopedic scale.[42] Unlike the *IMP*, *PM* took an axiomatic-deductive approach. Starting from a few undefined concepts and axioms, following the example of Euclid's *Elements*, *PM* went on to deduce propositions from which more fundamental propositions were inferred. Although a portion of the results of *PM* may be found in the *IMP*, more important was the extent to which *PM* showed how a consistent logical system might be built.[43]

The first Chinese to study *PM* at least in part was perhaps ZHAO Yuanren while he was a graduate student in the Department of Philosophy at Harvard University.[44] ZHAO, nevertheless, was not responsible for introducing mathematical logic to China except indirectly by serving as an oral translator for Russell, as mentioned above. Nor was YU Dawei [YU Ta-wei, David Yule] 俞大維, who was ZHAO's fellow student at Harvard. YU earned his Ph.D. in philosophy in 1922, having devoted his thesis to *Theories of Abstract Implication: A Constructive Study*. After finishing a research paper

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[41] Whitehead and Russell 1910-1913.

[42] Gandy 1973, 342.

[43] For recently historical studies of *Principia Mathematica*, see Grattan-Guinness 2000, 333-410.

[44] ZHAO Yuanren 1975, 83; 1977, 58.

on set theory which he later published in the German journal, *Mathematische Annalen*, YU soon turned his interests to politics and eventually rose to a high-ranking position as Minister of Defense in the Nationalist's government in Taiwan between 1954 and 1964.[45] The first substantial introduction of mathematical logic to China in a more serious way, however, was due to a scholar trained in political science.

JIN Yuelin [CHIN Yueh-Lin] 金岳霖 was educated first at the University of Pennsylvania, and then at Columbia University, where he received his Ph.D. in 1920 for a thesis on the political theory of the British philosopher, Thomas Hill Green.[46] After graduating from Columbia, JIN went to Europe where he continued his studies. His intellectual interests, however, soon switched from political science to philosophy. During the time he spent in London, Paris, and Berlin between 1921 and 1925, JIN enthusiastically and systematically studied mathematical logic, which he considered a major component of philosophy, primarily through Russell's *PM*. After returning to China in November of 1925, JIN soon became a member of the faculty at Qinghua University, and also a founding member of the Department of Philosophy there. A convert to Russell's *PM*, JIN believed that mathematical logic represented the latest development of traditional logic, and that students of philosophy should study mathematical logic in order to have a better understanding of contemporary philosophy in general, and of logic in particular. Accordingly, in 1927 JIN offered a course devoted to mathematical logic in the newly created Department.

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[45] See YU Dawei 1926; and Boorman 1971a, 43a.

[46] JIN Yuelin 1920.

JIN's lectures on the subject, first given at Qinghua and later at the National University of Beijing, were based on Russell's *PM*. The specific contents of his lectures are reflected, more or less, in his lecture notes, which were informally reproduced by the Printing Department of Qinghua University in 1935. Entitled *Luoji* 邏輯 (*Logic*), JIN's textbook was formally published the following year by the Commercial Press. Chapter three was devoted to introducing, in JIN's own words, "a grand system melting logic and mathematics together," namely, Russell's logical system.[47]

The chapter on mathematical logic is divided into three sections: 1) Deductions of Primitive Propositions (*wei jixi de mingti de tuiyan* 未解析的命題的推演); 2) From Primitive Propositions to the Calculus of Classes and Relations (*you wei jixi de mingti dao lei yu guanxi de tuiyan* 由未解析的命題到類与關係的推演); and 3) Calculus of Classes and Relations (*lei yu guanxi de tuiyan* 類与關係的推演). The first section, after explaining the symbols Whitehead and Russell used in *PM*, provided three definitions and the first six primitive propositions (on pages 12 and 13 of the *PM*, respectively) that were to be employed in the propositional calculus which followed, occupying a large portion of the section. JIN presented sixty-six propositions taken from Part I, Section A, subsections 2, 3, and 4 of the first volume of *PM*, each followed by a proof. The proofs, however, are not always the same as those given by Whitehead and Russell because many of the propositions in *PM* were left out, some of which happened to be used in the proofs of the sixty-six propositions JIN chose from the original text. For instance, a new proof for

[47] JIN Yuelin 1978, 146.

proposition (*2·43) in *PM*, ⊢: p ∩ ε ∩ pεq : ε ∩ pεq, was needed and given by JIN himself.[48]

Section two covers mainly the contents of Part I, Section B, of the first volume of *PM*. It divides further into fives subsections, the first of which is devoted to the theory of propositions containing one apparent variable. Here fifteen propositions from subsection 10 of *PM* are quoted. Explanations are also provided for each proposition but without proofs (which were not provided in the following sections either). The second subsection is devoted to the theory of two apparent variables, for which eighteen propositions from subsection 11 of *PM* are included. The third subsection is on identity. In *PM*, definitions and propositions on identity were based on the predicative function and axiom of reducibility, neither of which, however, is introduced. Seventeen propositions from subsection 13 of *PM*, however, are quoted. This subsection provides seventeen propositions on descriptions from subsection 14 of *PM*. The last subsection is on appearance of classes and relations. It describes what class and relation mean, but does not provide rigid definitions, which in *PM* are based, once again, on the predicative function and axiom of reducibility. It does emphasize that not only may an individual be a class, but also classes, but that under no circumstances may a class be an element of itself. The former was considered as a matter of mathematics foundations, and the latter a principle necessary to avoid contradictions.[49]

The third and last section is on the calculus of classes and relations, taken from Part I, Section C, of the first volume of *PM*. This section is divided into four subsections,

[48] JIN Yuelin 1978, 158-159.

[49] JIN Yuelin 1978, 199.

the first of which is on general properties of classes and lists seventeen propositions from subsection 20 of *PM*. The following subsection provides sixty-five propositions on the calculus of classes, which are in turn taken from subsection 22 of *PM*. The third subsection is on general properties of relations, and here twenty propositions from subsection 21 of *PM* are quoted. The last subsection states sixty-five propositions on the calculus of relations, all of which can be found in subsection 23 of *PM*.

JIN's introduction of *PM* only covers selected contents from the first twenty-three sections of the first volume of *PM*, and does not include the trademark of this book, type theory, but it does present the most basic and fundamental features of mathematical logic.[50] Its importance rests on the introduction of a logical system in a relatively systematic way for the first time to Chinese college students. The selection of the contents from *PM* reflects JIN's own knowledge of mathematical logic and his awareness of its latest trends. As mentioned above, the axiom of reducibility was omitted. This is the axiom in *PM* designed to reduce propositional functions of higher orders to predicative functions, and is an axiom needed to ensure such general properties of numbers as mathematical induction. But the admission of this axiom is hardly a logic result. JIN may have been aware of this flaw in *PM*. The theory of types had been criticized immediately upon publication of *PM* in part because of the axiom of reducibility, and by the 1930s there was a trend to abandon it completely, which was actually achieved in 1937 by Willard Quine, professor of philosophy at Harvard and an early proponent of Russell's logicism. JIN Yuelin devoted himself to logic beginning in the late 1920s. In addition to teaching, he was an active researcher and published a

[50] MO 1987, 260.

number of papers in such journals as *The Monist* and *Zhexue pinglun* 哲學評論 (*The Philosophical Review*, a Chinese journal created in 1927 in Beijing).[51] JIN also spent his sabbatical year, 1931-1932, at Harvard, and would therefore doubtless have been aware of Quine's work and the growing interest in symbolic logic. He may also have been involved in or aware of the debate over the axiom of reducibility.

In his introduction to *PM*, JIN illuminated and even modified certain points made in *PM*. For instance, he pointed out that "primitive propositions are not only premises for deducing other propositions, but also ways of inference."[52] In other words, JIN understood that primitive propositions could be used both in an object language as preconditions for the deduced results, and in a meta language as means of inference. *PM*, however, does not differentiate the two usages, and fails to make this clear. In addition, JIN pointed out that the definition of identity presented in *PM* is quite narrow, and is only valid for specific individuals, expressed symbolically as: '|X x = x'. JIN thought that the definition of identity should be extended to include all things, which are identical in terms of predicates, concepts, and relations. Thus any propositions expressible in the form "|X pεp, or |X p≡p" should be considered as identical.[53]

Like FU and ZHANG's translation of *IMP*, JIN's textbook was an instant success, and was immediately reprinted in 1937. But unlike the Chinese translation of *IMP*, JIN's book was used in such prestigious universities as Qinghua and Beijing. In fact, before publication of the book, portions of *Principia Mathematica* had already been used at both

[51] JIN Yuelin 1934.

[52] JIN Yuelin 1978, 149.

[53] JIN Yuelin 1978, 189-190.

universities.[54] It was largely through Russell's system of logic as presented in *PM* that Chinese students beginning in the late 1920s, could study mathematical logic in a systematical way, and this helped to lay solid foundations for their future development. Among those who did so and later were to become famous logicians in their own right were SHEN Youding (SHEN Yuting) 沈有鼎, WANG Xianjun 王憲鈞, and HU Shihua (HOO Tzu-Hua) 胡世華. After graduating from Qinghua in 1929, SHEN went to Harvard to study logic with Henry M. Sheffer. With a Masters degree in philosophy from Harvard, SHEN then went on to the University of Heidelberg to continue his study of logic. Following SHEN, in 1936 WANG and HU both went to Europe after they had completed their graduate studies at Qinghua and Beijing Universities, respectively. WANG studied at the University of Berlin and the University of Vienna, where he was a student of the great logician Kurt Gödel. HU studied first at the University of Vienna in Austria, then at Munster University in Germany, where he received his Ph.D. in mathematical logic in 1941. All three subsequently went back to China where they helped to make mathematical logic a subject not only taught in universities, but a field of growing academic research as well.[55] The late prominent Chinese logician WANG Hao 王浩 was also inspired by JIN Yuelin's textbook when he was a senior in high school, and later WANG not only studied logic under the guidance of JIN, but under his two former students SHEN Youding and WANG Xianjun as well.[56] In 1946 WANG went to Harvard University, where he received his Ph.D. under the guidance of Willard Quine in the remarkably brief period of one year and eight months.

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[54] ZHANG Shenfu 1993, 108.

[55] LIN and ZHANG 1983, 175-176; LI, ZHOU and ZHOU 1989, 8-13; SONG 1999, 139-142.

JIN Yuelin's college textbook on logic, however, was not the only one available in China that introduced Russell's system of logic in the 1930s. A number of other books on logic written or compiled by Chinese scholars also covered mathematical logic. Chronologically, these include *Luoji yu shuxue luoji lun* 邏輯與數學邏輯論 (*Logic and Mathematical Logic*, 1927) by WANG Dianji 汪奠基; *Xiandai luoji* 現代邏輯 (*Contemporary Logic*, 1933) by SHEN Youqian 沈有乾; *Xiandai luoji* 現代邏輯 (*Contemporary Logic*, 1937), also by WANG Dianji; and *Luoji dianfan* 邏輯典範 (*Logic*, 1940) by MOU Zongsan 牟宗三. Interestingly enough, all of these books adopted Russell's system and the logical symbols he used to present the basic elements of mathematical logic, and each emphasized the importance of mathematical logic over traditional Western formal logic.[57] Nevertheless, in contrast to JIN's textbook on logic, none of these books succeeded in presenting Russell's system in a systematic, deductive way. But these books, along with JIN's, collectively made Russell's logic popular in China in the 1930s and 1940s.

Chinese interests in mathematical logic were, to a large extent, inspired by Bertrand Russell. ZHANG Shenfu 張申府, an ardent advocate of Russell's philosophy in China and the translator of Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*, explained the impact Russell had in his own case as follows: "Why I praised and admired Russell was mainly because of his great contributions to philosophy through mathematical logic, which deeply interested me. I was fond of mathematics when I was young, and then studied philosophy and logic. Russell's logic, however, is not normal or

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[56] ZHANG Dianzhou 1994, 97-98.

[57] LI, ZHOU and ZHOU 1989, 7.

traditional logic, but a new one, namely mathematical logic."[58] It was indeed through Russell's visit to China that interest in mathematical logic among Chinese scholars like ZHANG Shenfu, FU Zhongsun and ZHANG Bangming, was kindled. It was also through FU and ZHANG's translation of *Introduction to Mathematical Philosophy* that more scholars and students alike could get an excellent introduction to mathematical logic. And thanks to translation of Russell's *Principia Mathematica*, mathematical logic was finally taught in China's most prestigious universities. It is also true that mathematical logic was advocated in China through other channels. Students who returned to China after having studied abroad, such as ZHU Gongjin 朱公謹, wrote their own articles or translated works by other scholars to promote this very subject in China.[59] But in retrospect, it was Bertrand Russell himself—through his own visit and lectures in China, and by virtue of his published works—helped accelerate the introduction of mathematical logic in China, and served to establish it as an important subject of study, teaching and research.

6.4 Concepts of Infinity in Mathematical Logic

Concepts of infinity such as those which arose in the 19th century in the contexts of rigorous treatments of the real numbers (as provided by Richard Dedekind and Georg Cantor), in Cantorian transfinite set theory, or in various axiomatizations such as Zermelo-Frankel set theory, were introduced to China not only through translations of Russell's books, but also through the works of a number of pioneering Chinese logicians.

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[58] ZHANG Shenfu 1993, 66.

[59] ZHU Gongjin 1932; 1934; 1936.

Almost immediately following Russell's lectures in China, some Chinese students went to Europe and America to study mathematics, and in some cases this included an emphasis upon logic. Among the first and most notable of such students were ZHU Gongjin and TANG Zaozhen 湯躁真.

ZHU Gongjin, also known as CHU Kun-ching, or ZHU Yanjun, was a graduate of Qinghua University in Beijing. He went to Europe in 1922 and studied mathematics at the University of Göttingen with David Hilbert, Edmund Landau and Richard Courant, among others. After having received his doctoral degree in 1927 for a thesis written under Courant on an existence proof for solutions of certain types of ordinary functional equations, ZHU was appointed Professor of Mathematics at Guanghua University in Shanghai, and was also vice president of the University from 1932-1937.[60] In order to promote advanced mathematics in China, ZHU wrote a series of articles including several on mathematical logic and the theory of real numbers. In the latter, his approach to the real numbers was based on Dedekind's *Stetigkeit und Irrationale Zahlen*.[61] In this book, published in 1858, Dedekind redefined irrational numbers in terms of his famous Dedekind cut, as follows:

> Now, in each case when there is a cut (A_1, A_2) which is not produced by any rational number, then we create a new, irrational number a, which we regard as completely defined by this cut; we will say that this number a corresponds to this cut, or that it produces this cut.[62]

[60] ZHU's dissertation: "Über den Existenzbeweis fűr die Lösungen gewisser Typen von gewöhnlichen Funktionalgleichungen," was written under the supervision of Richard Courant. For a brief account of ZHU's life and work, see LIU Jingde 1994.

[61] See ZHU Yanjun 1928. The article on the theory of real numbers and others he authored between 1927 and 1932 were published together as a pamphlet in 1935. See ZHU Yanjun 1935.

[62] Quoted from Dedekind 1963, 98.

Understanding the importance of Dedekind's definition of the real numbers, ZHU later extended his efforts and produced a Chinese version of Dedekind's book: *Shishu tanyuan* 實數探源 (*Exploring the Origins of Real Numbers*), published in 1940.[63] On mathematical logic, ZHU contributed three articles to the most influential journals then in China: *Zhexue pinglun* 哲學評論 (*The Philosophical Review*), *Like jikan* 理科季刊 (*Journal of Wuhan University for Natural Sciences*), and *Shuxue zazhi* 數學雜誌 (*Chinese Journal of Mathematics*, published by the Chinese Mathematical Society). Besides introducing the propositional and predicate calculi, ZHU also focused on infinite sets and transfinite numbers. Especially new to Chinese readers were the proofs that whereas the sets of all rational numbers, and of all algebraic numbers, are denumerable, the set of real numbers is not, thereby establishing as well the existence of transcendental numbers.[64]

Like ZHU Yanjun, TANG Zaozhen, also known as TANG Tsao-Chen, studied in Germany as well. After graduating from Department of Mathematics of Beijing Teachers College in 1919, TANG taught mathematics at Beijing Teachers College for Girls. In late of 1923, he went to Germany to pursuit his further study in mathematics.[65] During the next two and a half years, TANG studied primarily differential geometry and mathematical logic at University of Berlin and University of Göttingen. When he returned in 1926, he was appointed as Professor of mathematics at the National Wuchang University (the predecessor of Wuhan University). At the University, he used Wilhelm J.

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[63] Besides translating Dedekind's work on the theory of real numbers, ZHU Yanjun also translated his former adviser's account on the calculus: *Vorlesungen űber Differential-and Integralrechnung*. The Chinese translation is in two volumes, vol. I published in 1949; and vol. II in 1952. See LIU Jingde 1994, 9-10.

[64] See ZHU Yanjun 1932; 1934; and 1936.

[65] For a brief account of TANG Zaozhen's life and work, see TANG Xiangsen 1994.

E. Blaschke's *Vorlesungen über Diffretialgeometrie* vol. I as the textbook for teaching his course on differential geometry. Mathematical logic was not on the curriculum of the University, but TANG did not lost his interests in it. Although his focus was on Clarence Irving Lewis's calculus of strict implication, and was not directly related to aspects of the infinite, it does show how quickly Chinese mathematicians began to respond to this new area for their own research. TANG published three articles on the subject in the *Bulletin of the American Mathematical Society*. In the first of these, "The Theorem '*p*—3*q*.=.*pq*=*p*' and Huntington's Relation Between Lewis's Strict Implication and Boolean Algebra," he shows that the theorem " *p*—3*q*.=.*pq*=*p*" holds true in Lewis's system, and that it strengthens a previous result of Edward Huntington.[66] Based on this result, TANG went on to show that where any implication "*p*—3*q*" is asserted, then "*p*—3*q*.=.*i,*" from which if follows that any two asserted implications are strictly equivalent, and, in particular, that any two of Lewis's first eight postulates can be deduced from each other.[67] TANG also studied algebraic postulates for Boolean rings.[68]

Under TANG's influence, XIAO Wencan 蕭文燦, assistant professor of mathematics at Wuhan University, was also interested in mathematical logic.[69] XIAO later authored a series of articles that served to introduce mathematical logic, especially those on the concept of the infinite into China. Among the papers XIAO wrote was a lengthy one on set theory, which also appeared in the *Like jikan*, in four installments.

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[66] See TANG 1936a. For the development of Lewis's theory of strict implication, see Curley 1975.

[67] For the reviews of the two articles by Paul Henle, see *Journal of Symbolic Logic* 2, no. 1(1937): 58.

[68] TANG 1938. For a review of the article by Charles A. Baylis, see *Journal of Symbolic Logic* 4, no. 1(1939): 27.

[69] For a short biography of XIAO Wencan, see DU and XIAO 2000.

The major contribution of these articles was the careful presentation they made of Cantorian set theory in a systematic way.[70] In addition, XIAO also authored an article discussing orders of infinity. The mathematical infinite, as XIAO points out, is not just *one*, but in fact infinitely many. It has different levels. The least of the transfinite numbers is the cardinal number of the set of integers or natural numbers, which Cantor denoted by the Hebrew letter aleph, as $\aleph_0$ (aleph-null). All sets of power aleph-null are denumerable, i.e. denumerably infinite. However, the real numbers represent a higher order of infinity, which is non-denumerably infinite, and thus of a higher cardinality than $\aleph_0$. At first Cantor used the letter *c* to denote the cardinality of such continuous sets. Among the questions that immediately arose: Is there any set whose cardinal number is greater than $\aleph_0$ but less than *c*? Cantor's answer to this question was negative, and his conjecture has become widely known as the *Continuum Hypothesis*.[71]

XIAO's papers on transfinite set theory were among the first devoted to the subject in China. He also wrote an article on irrational numbers, in which he discussed not only Dedekind's definition, but also Cantor's, which was based upon fundamental series and very similar to the approach Weierstraß had taken to rigorously defining the reals.[72]

XIAO's and ZHU's articles, together with Chinese translations of Russell's work, kindled further interests in mathematical logic in China. As a result, in the 1930s a number of Chinese students of mathematical logic were able to carry out their own

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[70] XIAO 1933-34. This paper was reprinted as a book: *Jihe lun chubu* 集合論初步 (*Basic Set Theory*), in 1939 by Shangwu chubanshe in Shanghai. See XIAO 1939. For Cantorian set theory, see Dauben [1979] 1990.

[71] See Dauben [1979]1990, 101-111.

research. By the time TANG Zaozhen's third article was published in America, another Chinese student had written his dissertation on mathematical philosophy and the theory of sets at the University of Paris. This was ZENG Dinghe 曾鼎鉌, also known as TSENG Ting-Ho. The major parts of ZENG's thesis dealt with set theory and transfinite numbers.[73] In retrospect, one may regard ZENG's thesis as superficial and sketchy. It nevertheless represents the beginning of serious and important work that Chinese logicians would soon make to mathematical logic.

In 1936, two of JIN Yuelin's best students, WANG Xianjun and HU Shihua, went to Europe determined to study mathematical logic. JIN had traveled in Europe but did not obtain an advanced degree there, since he already had a Ph.D. degree in Philosophy. Upon his return to China, his lectures on logic inspired his two students to continue their studies in Europe. WANG first studied at the University of Berlin, and then in 1937 went to the University of Vienna where he studied logic with Kurt Gödel. Before returning to China to take a position as a lecturer at the Southwest Union University in Kunming, WANG visited HU at Westfalische Wilhelms—Universität Münster (HU had first studied in Vienna as well, but then went to Münster).[74] HU eventually wrote his dissertation on pseudo-Boolean algebra and topological foundations under the supervision of Heinrich Scholz and K. Köthe, for which he was awarded a doctoral degree in 1941.

In his dissertation, HU developed a novel concept of "incomplete point" in a topological space, which some have interpreted as containing a germ of the concept of

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[72] XIAO 1932.

[73] See ZENG 1938. A review of the thesis can be found in *Journal of Symbolic Logic* 8, no. 2 (1943): 56-57.

[74] WANG's educational background presented here is drawn from his *Curriculum Vitae*, which is inserted in WANG Xianjun 1998.

"non-standard point" in non-standard analysis, which was conceived much later by Abraham Robinson in the 1960s.[75] Whatever the merits of this claim may be, HU was indeed an original scholar. In the 1940s and 50s, he was especially interested in multi-valued propositional calculi and applied his ideas within mathematics. His research results were published in a series of articles, one of which, "*m*-valued Subsystem of (*m*+*n*)-valued Propositional Calculus," was published in the *Journal of Symbolic Logic*.[76] This is just one sign that Chinese mathematicians were beginning to work at a level comparable to that of their western colleagues. In addition, HU also made valuable contributions to recursion theory, and is one of the major figures credited for the development of computer science in China.[77]

Compared with HU, WANG's contributions are less technical and more philosophical. His accounts, however, contain more relevant material for us to consider with respect to Chinese understanding of the concept of infinity. WANG's discussion of the infinite was set firmly in the context of his study of the history of mathematical logic. He categorized set theory as the second stage in the development of mathematical logic, and discussed it under the following headings: 1) classification of infinite sets; 2) the continuum in multi-dimensions; 3) larger infinites; 4) Cantor's theorem; 5) the well-ordering theorem and the continuum hypothesis; and 6) actual and potential infinity.[78]

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[75] See LU Zhongwan *et al*. 1996, 269. HU's dissertation remains unpublished, but for an abstract see *Xueshu Jikan* 學術季刊 (*Academic Quarterly*) 1(3)(1943).

[76] See HU Shihua 1949.

[77] For HU Shihua's life and work, see LU Zhongwan *et al*. 1996. For a review of the three articles on recursive algorithms HU published in 1960, see V. Vuckovic in the *Journal of Symbolic Logic* 37(3)(1972), 612.

[78] See WANG Xianjun [1981] 1998, 292-304.

On the first of these subjects, "classification of infinite sets," WANG pointed out that this originated from Cantor's study of whether or not a continuous function can be uniquely represented by a trigonometric series. Cantor wrote a number of articles devoted to this subject, beginning with a paper of 1870, in which he proved that the uniqueness of the representation holds on the assumption that the series is uniformly convergent. This proof was extended to allow a finite number of exceptional points in a paper of 1871, followed by an even more general result, published in 1872, in which Cantor was able to show that the uniqueness result held even in the case where the set of exceptional points of the representation is infinite.[79] In this case, however, the uniqueness of the representation held only for certain kinds of exceptional sets, which Cantor called sets of the "first species." WANG pointed out how this led Cantor to make a classification of infinite sets in order to explain the characters of these infinite sets of exceptional points. The classification is based on the concept of derived sets and Cantor's own definition of the real numbers.

This was not the only approach that would lead Cantor to his later study of set theory. Intrigued by properties of the continuum and especially his derived sets of the "second species" (for which, given an infinite set P, all of its derived sets P^{ν} were infinite, i.e. there was no finite index ν such that $P^{\nu}=\varnothing$), Cantor's second means of classifying infinite sets was based on the concept of one-to-one correspondence. WANG traced this idea back to Proclus, then Galileo, Leibniz, Cauchy, and eventually down to Bernhard Bolzano. WANG recognized that Bolzano also used one-to-one correspondences to

[79] See WANG Xianjun [1981] 1998, 293-294. For a more detailed technical and historical discussion of Cantor's study of trigonometric series, see Dauben [1979] 1990, 30-46.

differentiate sets, but not as the *only* criterion as Cantor does.[80] It is interesting to note that WANG regarded Cantor's emphasis on one-to-one correspondences as a means of studying infinite sets to be a breakthrough in intellectual history. WANG also attributed this to Cantor's spirit of materialism, which he explained as follows:

> 如果以"和真部分一一對應"為悖論，就必須否認實無窮。相反地，如果數學實踐說明了實無窮的必要性，而實無窮又確實有此特性，那麼就必須承認这是無窮和有窮的本質差別，只有這樣才能用一一對應作爲對無窮進行分類的標準。無窮和有窮本來就是矛盾的兩方面，我們不能勉強要求無窮具有有窮的一切性質。[81]
>
> If the statement "one-to-one correspondence with real things" is a paradox, then [one] must reject the absolute infinite. On the other hand, if mathematical practice explains the necessity of the actual infinite, and the actual infinite indeed has such characteristics, then [one] must recognize that this is the essential difference between the infinite and finite. Only with this [understanding] can the [concept of] one-to-one correspondence be used as a criterion for classifying the infinite. Infinite and finite are the two sides [of the coin] of contradiction. We cannot demand that the infinite has all the [same] properties as the finite has.

WANG's account of the history of mathematical logic was written in the late 1970s, and was first published in 1981. It is very hard to tell whether the above comments were written with political correctness in his mind, or if they truly represent his personal views based upon lengthy study and research on the subject. In any case, attributing Cantor's use of "one-to-one correspondence" as the criterion for classifying infinite sets to his "spirit of materialism" is certainly questionable, if not totally wrong.

After distinguishing these two infinite sets, the set of natural numbers and the set of real numbers or continuum, Cantor began, as WANG explained, to think about whether there is any infinite set greater than the continuum of real numbers. To answer this question, Cantor at first thought the plane might contain more points than the linear

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[80] See WANG Xianjun [1981] 1998, 296.

[81] Quoted from WANG Xianjun [1981] 1998, 296.

continuum, and that higher dimensions might contain correspondingly greater hierarchies, absolutely, of infinite point sets. But as he tried to correspond the points of the plane with the points on a straight line, he found to his great surprise that he could produce a one-to-one correspondence between the two (although the mapping was not continuous, which was to be a decisive factor with respect to the problem of the invariance of dimension that this result immediately raised). But Cantor's result, as WANG put it: "not only does not destroy the concept of dimensions [as Paul de Bois-Reymond, Leopold Kronecker and other critics also pointed out], but stimulated further study on the relation between dimension and the concept of one-to-one correspondence."[82] Jacob Lüroth's proof in 1879 that the one-dimensional continuum cannot be corresponded by means of a continuous one-to-one correspondence with a two-dimensional continuum is but one example of an early reaction to Cantor's result. A fully satisfactory proof of the invariance of dimension was not given, however, until that of L.E.J. Brouwer in 1911.[83]

As for infinite sets larger than the set of real numbers, WANG explained how Cantor constructed these in two different but related ways. The first was through transfinite ordinal numbers, which Cantor showed led to hierarchies of sets of transfinite ordinal numbers of distinctly larger power or cardinality than sets of either power aleph-null or aleph-one, in an ascending sequence of transfinite ordinal and cardinal numbers. The second means of generating sets of greater power or cardinality than either the set of integers or the set of real numbers was through the construction of power sets. Given a set N, its power set is the set 2^N of all its subsets, and in 1891 Cantor showed that for any

[82] WANG Xianjun [1981]1998, 297.

[83] See Brouwer 1911.

N, even for transfinite powers, $2^N > N$. Since Cantor could show that the set of real numbers was equivalent in power to the set $2^{\aleph_0}$, this meant that $2^{\aleph_0} > \aleph_0$. This also permitted Cantor to express, algebraically, his Continuum Hypothesis, that $2^{\aleph_0} = \aleph_1$. Although Cantor was unable to determine whether the Continuum Hypothesis is true or not, he had succeeded in establishing, by several different methods, that there was indeed a hierarchy of distinct and increasingly large transfinite ordinal and cardinal numbers.

In his account, WANG not only described Cantor's views on the nature of the infinite, but recognized that the distinction between the actual and the potential infinite was still a fundamental issue in the foundations of mathematics. For those who believed in actual infinity, WANG wrote: "the infinite is real, completed, and an existent entity. It can be comprehended." But for these who held the opposite, "the infinite is not a completed totality, but it is a process, and thus, is potential (but not actually infinite)."[84] WANG summarized Cantor's arguments about the actual infinite as follows:[85]

1. Mathematical theories must acknowledge the actual infinite.

a) The history of mathematics indicates that the actual infinite or infinite sets have been applied in mathematical proofs. Examples can be found in the works of Joseph-Louis Lagrange, Adrien-Marie Legendre, Peter G. L. Dirichlet, Augustin-Louis Cauchy, Karl Weierstraß, and Bernhard Bolzano. Cantor also provided an example of his own, which roughly says: dividing an infinite set of points into a finite number of subsets, (at least) one of these must be an infinite set.
b) In a letter to GiulioVivanti in 1886, Cantor writes that the actual infinite must be recognized because the definitions of irrational numbers, due to both himself and Dedekind, are based on it. Moreover, acknowledging the potential infinite must lead to acknowledging the actual infinite. This is because if a variable can take infinitely many values, there must be such a "domain" where (all such) values can be taken. This "domain" must be actually infinite, and exists before the variable can take on any values.
c) In a letter to A. Eulenberg of 1886, Cantor provides several examples of the

[84] WANG Xianjun [1981] 1998, 301.

[85] WANG Xianjun [1981] 1998, 301-304. For a detailed and balanced discussion of Cantor's philosophy of the infinite, see Dauben [1979] 1990, 122-125.

actually infinite: the set of positive integers, the points on a circle, and the monads which make up matter.

2. One should not impose properties of the finite on the infinite. The infinite has its own intrinsic qualities.
   WANG quoted a letter from Cantor to Gustav Eneström, dated November, 1885, in part to show that infinite numbers have their own rules of operation, and should not be expected to conform to those which hold for finite numbers.[86]

3. Although human capacity for understanding is limited, it can comprehend the infinite, both philosophically and mathematically.
   WANG explained that Cantor said that the infinite, like the finite, can be expressed by, or understood, through definite, clear, and different quantities. In a way, it may be said that humans do have "infinite power" to formulate bigger and bigger sets, to produce larger and larger cardinal numbers. The mathematical infinite is different from that of the religious infinite. The mathematical infinite can be defined, but the religious infinite, God, cannot because any limitation to Him is a negation.

WANG's historical account of Cantorian set theory and Cantor's philosophy of the infinite, though patchy and lacking coherence, does outline the major results of set theory and its philosophy. But due to the limited number of comments provided within, it is very difficult to discuss WANG's own views on the concepts of infinity in any detail. But one thing is certain—that the translations or paraphrases he provided of the original texts are quite accurate. In fact, the majority of Chinese experts on mathematical logic before 1950 had studied either in Europe or America. Fluent in at least one of the three major foreign languages: English, French and/or German, they studied and consulted articles and books in these languages directly. If Chinese accounts were lacking in any way with respect to concepts of the infinite, this can in no way be interpreted as their not being aware of the concepts, or worse, that they did not understand them. Meaningful discussion of modern mathematical concepts of the infinite in China became possible only when original contributions began to be made by Chinese researchers. Indeed, this

[86] For English translation of the letter, in part, see Dauben [1979] 1990, 125.

was not long in coming.

Soon after WANG joined his former teacher JIN Yuelin by taking a position at Southwest Union University, he found among his students a twenty-eight-year old young man by the name of WANG Hao. WANG Hao's major was mathematics, but he also had a strong interest in logic. Besides taking courses offered by the Department of Mathematics, he also took logic courses taught by faculty members in the Department of Philosophy. After receiving a Bachelors degree in 1943, he enrolled in the Philosophy Department as a graduate student and studied with JIN and WANG Xianjun. In 1946 WANG Hao won a government scholarship and went to Harvard University, with the definite intent to study mathematical logic. Within two years, WANG had received his Ph.D. from Harvard, and was soon to become a renowned logician and philosopher.

WANG Hao was a first-rate mathematician and one of the most prolific logicians in the second half of the twentieth century, and is well-known for his studies and interpretations of the work of Kurt Gödel.[87] WANG himself has made many contributions to mathematical logic, computer science, and philosophy. But he also made important contributions to the introduction of modern logic in China. His most important work in this regard is *A Survey of Mathematical Logic*. What follows is intended as only a general overview of WANG's accomplishments in this regard.

A Survey of Mathematical Logic was published by Science Press in Beijing in 1962, and in the next year as a volume co-published with North-Holland in the well-known series on the *Studies in Logic and the Foundations of Mathematics*, edited by L. E. J. Brouwer, E. W. Beth, and A. Heyting. As a collection of the research results WANG

[87] For WANG Hao's life and work, see ZHANG Dianzhou 1994; and Parsons 1996. For a more or less completed bibliography of WANG Hao, see Grossi *et al.* 1998.

Hao made between 1947 and 1960, most had been published previously in American journals. But as WANG explained in his Preface, "I have added from unpublished notes [and] discussion of a number of standard topics and replaced a few papers by somewhat more general descriptions of related matter."[88] Thus there was still something new to be found here, even for those already familiar with WANG's work.

The book is divided into five parts, under the following headings: General Sketches, Calculating Machines, Formal Number Theory, Impredicative Set Theory, and Predicative Set Theory. The first part provides a general expository sketch which covers such results as Gödel's completeness and incompleteness theorems, and decidability and undecidability results. It also contains a summary of important aspects of research on the foundations of mathematics done since the 1870s, including the axiomatizations of arithmetic undertaken by Dedekind and Frege. The second part is on a theorem-proving machine, WANG's version of which turns out to be closely related to Turing's, but is more economical in its basic operations. Here WANG also discusses the use of logical systems and techniques in the analysis of the structure and behavior of automata. Perhaps the most important and interesting chapter in this part is: "Toward Mechanical Mathematics," wherein WANG Hao describes how to design a program which can prove all the theorems of propositional logic in *Principia Mathematica* in a few minutes. Another program that would likewise prove most of the theorems of predicate logic in *Principia Mathematica* is also discussed.

Part three first takes up the predicate calculus and many-sorted predicate calculus. Gödel's reduction of metamathematics to number theory is presented, along with several

[88] See WANG Hao 1963, Preface.

applications, including Gödel's famous theorems on consistency and independence of the Continuum Hypothesis and Axiom of Choice, Kleene's normal form for recursive functions, and Bernays's lemma as a sharpening of the completeness theorem of the predicate calculus. WANG follows all of this with an exposition of Ackermann's consistency proof.

Part four is on axiom systems and their relations. Among the systems discussed are Zermelo-Fraenkel theory (*ZF*), Bernays theory (*NB*), Quine systems (*NF*) and (*ML*), the simple theory of types (*T*), a system of negative types (*NT*), Frege's axiom system, Ackermann's system, and several subsystems of *ZF* and *NB*. This part contains WANG Hao's most original contributions to *ML*. W.V. Quine, WANG Hao's former advisor at Harvard, announced his own axiom system *NF* in his *Mathematical Logic* published in 1940. But J. Barkely Rosser soon discovered flaws in *NF,* which Quine tried to fix but without success. By modifying one postulate in *NF,* WANG not only provided a revised system which has since become known as *ML*, but showed that *ML* is consistent with *NF*.

In the last part of his book, WANG Hao mainly introduces his own theory schema Σ, and constructs a transfinite sequence of ramified systems indexed by ordinals to replace Russell's axiom of reducibility. This section includes an opening discussion of certain predicates defined by inductive schemata. In the system *Z* of Hilbert-Bernays, certain predicates of natural numbers which cannot be expressed explicitly within the framework of number theory can nevertheless be introduced by such schemata. This requires a general notion of definability by induction which can be applied to all the known predicates of a similar kind. WANG proves that in a system L_1 which forms an extension of number theory, all predicates which are definable by induction according to

his definition are explicitly expressible. He then goes on in the second part to construct several undecidable sentences based on such semantic paradoxes as the Epimenides paradox and the Richard paradox. In his account here of "The Formalization of Mathematics," WANG introduces his own theory schema Σ, and constructs a transfinite sequence of ramified systems indexed by ordinals. From this transfinite sequence, the real number system can be derived without using Russell's controversial axiom of reducibility. This result was highly praised by Abraham Fraenkel, who said it "revived Russell's theory of reducibility."[89] In a final two sections, WANG provides some further details about this theory.

All in all, WANG's book covers several important aspects of mathematical logic, especially axiomatic set theory, but predicative and impredicative set theory as well. Concepts of the infinite are at the core of these theories, and on these and related matters the book introduced Chinese readers to the most updated results on these subjects as of the 1960s.

In their overview of mathematical logic in China, LIN Xiaoshui and ZHANG Shangshui write "Among those who have made contributions to the development of mathematical logic in our country, we should especially mention Professor WANG Hao. In 1961 he wrote a book entitled *A Survey of Mathematical Logic* for our country. The book was published in English by the Science Press. From 1972 on, he came back and lectured five times, introducing the latest results (in logic) and exchanging scientific ideas with scholars in this country. In 1977 he delivered six wide-ranging and popular lectures in Beijing (at the Institute of Computing Technology) thanks to an invitation from the

[89] Fraenkel and Bar-Hillel,1958, 157.

Chinese Academy of Sciences. His lecture notes were published both in China and the United States in 1981 under the title: *Popular Lectures on Mathematical Logic*. In the meantime, the Science Press has published a Chinese version. Professor WANG Hao's lectures undoubtedly stimulated recovery and the (further) development of mathematical logic in China after suffering serious setbacks during the Cultural Revolution."[90] Looking back, WANG Hao's contributions to mathematical logic in China began with his book: *A Survey of Mathematical Logic*, and were further augmented and popularized more widely through his *Popular Lectures*.

It was only forty years from Russell's trip to China that first stimulated Chinese interest in mathematical logic to the year WANG Hao was asked to write a book on the subject for a wider Chinese public. During this short period of time, not only did Chinese mathematicians come to appreciate fully the axiomatic way of thinking which traditional Chinese mathematics lacks, but they also began to make some important original contributions of their own. Modern mathematical concepts of the infinite were alien to the Chinese in such forms as the calculus and transfinite set theory, but following the introduction of the calculus in the 1850s, the infinite became increasingly familiar in Chinese mathematics, and this may help to explain the receptivity of Chinese mathematicians to accept it as a necessary foundation for modern logic, as did WANG Hao. Beginning in the 1920s, a number of prominent foreign mathematicians and logicians went to China to teach and lecture, and many more Chinese students went abroad to study. Between the two World Wars, an indigenous group of mathematicians

[90] LIN and ZHANG 1983, 181.

in China became increasingly active and professionalized, not only in universities, but through the activities of the Chinese Mathematical Society. The early years of the People's Republic of China, under the leadership of MAO Zedong, placed special emphasis on applied mathematics, and during the upheavals of the Cultural Revolution, pure mathematics was all but nonexistent. However, following the Cultural Revolution, the infinite was to find its way back into mathematics with unexpected applications in both theory and practice, especially in the teaching of mathematics and questions raised about foundations in particular. This was reflected in the great interest logicians showed in nonstandard analysis and the rigorous introduction of infinitesimals it made possible. But these are the subjects of the following Part III of this thesis devoted to "The Politics of Infinity," and the remarkable story of how the works of Karl Marx came to have a significant bearing on Chinese mathematics during and after the Cultural Revolution.

Part III: The Politics of Infinity

Chapter 7 Ideology, Politics, and Mathematics

> Mathematical infinity is taken from reality although unconsciously, and consequently also can only be explained from reality and not from itself, from mathematical abstraction.
>
> -Frederick Engels[1]

> Practice, knowledge, again practice, and again knowledge.... Such is the whole of the dialectical-materialist theory of knowledge, and such is the dialectical-materialist theory of the unity of knowing and doing.
>
> -MAO Zedong 毛澤東[2]

The Chinese Communist Party (CCP) took over power from the Nationalist Party in Mainland China in October of 1949, and founded the People's Republic of China following the socialist model of the Soviet Union. In the first half of the1950s, the new China adopted as much as she could from the USSR in virtually every area, chiefly due to admiration for the great achievements accomplished by the world's first socialist country, but also due in part to the international political realities of the Cold War. Following the Bolsheviks, the CCP made the Party's ideology the State's ideology, and demanded that culture generally, including the arts and sciences, must serve the Party, as well as socialist construction.[3] As a result, Soviet Marxist philosophy of the natural sciences, namely dialectical materialism, was swiftly adopted in China on a large scale. This philosophy, as a part of orthodox ideology together with politics at home, exerted an immediate and lasting influence on the development of science and education in China, but reached its

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[1] Quoted from Engels 1940, 319.

[2] Quoted from MAO 1975, 308.

[3] For the influence of Marxist ideology on Chinese intellectuals in general between 1949 and 1955, see Goldman 1987a, 234-42.

climax during the Cultural Revolution.

This chapter considers the ideological and political contexts within which Chinese mathematicians, mathematics educators, and philosophers of mathematics alike had to operate in the first seventeen years of the new China. It also sets the stage for topics to be discussed in the concluding part of this thesis. The first section of this chapter focuses on the establishment of a Marxist philosophy of mathematics in China under Soviet influence, and outlines Marxist views of the nature of mathematics and its historical development. Section two discusses MAO Zedong's writings on dialectical materialism and their guiding role for Chinese scientists in general and mathematicians in particular. Section three explains how both ideology and politics served to determine the priorities of mathematical research and the content of mathematics education. A concluding section is devoted to how the infinite mathematically was interpreted and treated in the context of Chinese dialectical materialism.

7.1 Introduction to Marxist Philosophy of Mathematics in China

Marxist philosophy of mathematics is a part of dialectical materialism, which was developed from Hegelian philosophical principles by Karl Marx. Like Hegel's dialectic philosophy, dialectical materialism holds that everything in the world is constantly changing, and that such change is due to contradictions inherent in "the thing itself," the famous Hegelian "*Ding an sich*." But unlike Hegel, Marx believed that matter was the source of everything and that thoughts were no more than reflections of matter. As conceptualized by Marx, dialectical materialism was systematically studied and applied to mathematics and the natural sciences by his lifetime friend and collaborator, Frederick

Engels.[4] In his book entitled *Herr Eugen Dühring's Revolution in Science* (*Anti-Dühring*), Engels outlined his understanding of mathematics from the point of view of dialectical materialism by criticizing Eugen Dühring, who was a radical lecturer on philosophy and political science at the University of Berlin, and who held that pure mathematics was *a priori*, in the sense intended by Immanuel Kant.[5] Mathematics, Engels asserted, was neither *a priori* nor the product of the free creativity of the human mind. It was derived, instead, from experience or practice in the physical world. Primitive and fundamental mathematical concepts and ideas, for example the idea of numbers and that of geometrical forms, were abstractions from the external world. Based on these abstracted concepts and ideas, mathematical theories developed. These theories subsequently were applied to the external world for testing and verification, from which new or revised theories were produced. As these procedures went on and on, mathematics itself advanced.[6] This, essentially, is Engels' view of the nature of mathematics and of its general development.

As for the methodologies of mathematics, Engels claimed that dialectics or dialectical logic was the most effective method for dealing with higher mathematics of variable quantities, and he downplayed the role of formal logic itself, which in Engels' view was another name for syllogistics, in the domain of elementary mathematics. According to Engels:

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[4] The term "dialectical materialism" was used neither by Marx nor by Engels, but was coined in 1891 by Georgii V. Plekhanov, who is considered the father of Russian Marxism.

[5] The book, written in German and first published in 1878, was a collection of articles serialized in the Leipzig *Vorwärts* in 1877. The second edition was made in 1885, followed by a third in 1894. The book was translated into English by Emile Burns in 1939. See Engels 1966, 10, 12, 19-20. The quotations from the book throughout this dissertation are from the 1966 edition of Burns' translation. For a synopsis of mathematical *apriorism* in various forms, and a criticism thereof, see Kitcher 1984, 13-87.

[6] See Engels 1966, 44-47.

> Elementary mathematics, the mathematics of constant magnitudes, moves within the confines of formal logic, at any rate taken as a whole; the mathematics of variable magnitudes, whose most important part is the infinitesimal calculus, is in essence nothing other than the application of dialectics to mathematical relations.[7]

"The relation between the mathematics of variable and the mathematics of constant magnitudes is in general," Engels says, "the same as the relation of dialectical to metaphysical thought."[8] By this, Engels inexplicably links formal logic with metaphysics.

Contradiction, according to Engels, is one of the basic principles of mathematics, both at elementary and advanced levels.[9] The square root of negative numbers, straight lines and curves, and even the very concept of the topic of this dissertation, infinity, are full of contradictions. About the latter, Engels says:

> Infinity is a contradiction, and is full of contradictions. From the outset it is a contradiction that an infinity is composed of nothing but finites, and yet this is the case. The finiteness of the material world leads no less to contradictions than its infiniteness, and every attempt to get over these contradictions leads, as we have seen, to new and worse contradictions. It is just because infinity is a contradiction that it is an infinite process, unrolling endlessly in time and in space. The removal of the contradiction would be the end of infinity.[10]

Infinity and the finite are contradictions. Space and time are infinite. These assertions were to establish the framework within which Marxist philosophy of mathematics, cosmology, physics, and the other natural sciences was to develop in China. We shall discuss these in detail later.

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[7] Engels 1966, 148.

[8] Engels 1966, 134.

[9] See Engels 1966, 133-134.

[10] Engels 1966, 59.

Besides *Anti-Dühring*, Engels, in manuscripts published posthumously under the title *Dialectics of Nature*, discussed his philosophy of science relatively systematically.[11] In addition to the points on philosophy of mathematics outlined above, Engels emphasized that mathematics, like the natural sciences, is subject to the following three laws of dialectical materialism:

1. The law of the transformation of quantity into quality and *vice versa*;
2. The law of the inter-penetration of opposites; and
3. The law of the negation of the negation.[12]

One example of the application of the last property, negation of negation, in mathematics was illustrated by Engels in the case of "zero":

> *Zero*, because it is the negation of any definite quantity, is not therefore devoid of content. On the contrary, zero has a very definite content. As the borderline between all positive and negative magnitudes, as the sole really neutral number, which can be neither positive nor negative, it is not only a very definite number, but also in itself more important than all other number[s] marked off from it. In fact, zero is richer in content than any other number.[13]

Engels' philosophy of natural science subsequently was sanctioned and further developed by Vladimir I. Lenin. In his *Materialism and Empirio-criticism,* Lenin drew upon the latest developments in the natural sciences, particularly physics, to illustrate points made by Engels.[14] This book and the above-mentioned works of Engels together

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[11] Most of the manuscripts were written in German between 1872 and 1882. Because Engels shifted his focus to editing Marx's *Das Capital* when Marx died in 1883, the manuscripts are incomplete. After Engels himself died in 1895, the manuscripts were in the control of the German Social Democratic Party. The manuscripts were first published in a dual language edition (German-Russian) in Moscow in 1925. An English translation was made by Clemens Dutt and published by International Publishers in New York in 1940. The passages cited in this dissertation are from this English edition. See John B. S. Haldane's preface to the *Dialectics of Nature*, in Engels 1940, vii-xvi, esp. ix.

[12] Engels 1940, 26.

[13] Engels 1940, 251.

[14] See Lenin 1964.

provide an outline of the basic Marxist philosophical and epistemological views of mathematics.[15]

As the Bolsheviks successfully overthrew the Czar and founded the Soviet Union in 1917, Marxism also began to be disseminated in China. Accordingly, when the CCP was founded in 1921, the works of Engels and Lenin subsequently were introduced to Chinese readers as part of the introduction of Marxism.

Anti-Dühring was the first of many Marxist books translated into Chinese. This was actually done by QIAN Tieru 錢鉄如 in 1929, based upon a Japanese translation. A revised version was published the following year in Shanghai. In the meantime, another Chinese translation was prepared by WU Liping 吳理屏. Parts of the *Dialectics of Nature*, including "The Part Played by Labor in the Transition from Ape to Man," "Dialectics," and "Introduction," among others, were also translated. A translation of the *Dialectics of Nature* in its entirety (based on the first Russian-German edition) was made by DU Weizhi 杜畏之 and published in 1932. Like Engels' works, Lenin's *Materialism and Empirio-criticism* was rendered into Chinese more than once in the 1930s. The first translation was made by DI Qiu 笛秋 and ZHU Tiesheng 朱鉄笙 in 1930; the second by FU Zidong 傅子東 four years later.[16]

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[15] Jean van Heijenoort in his article "Friedrich Engels and Mathematics" has carefully examined Engels' comments on mathematics and philosophy of mathematics, and concludes that "Mathematics is undoubtedly the field in which Engels is at his weakest," and his remarks are worthless. See Heijenoort 1985. The quotation is from p. 150.

[16] See Engels 1930a; 1930b. "The Part Played by Labor in the Transition from Ape to Man" was translated by LU Yiyuan 陸一遠 and published as a pamphlet under the title "*Makesi zhuyi renzhong youlai xueshuo*" 馬克思主義人種由來學説 (Shanghai: Chunchao shuju, 1928); "Dialectics" was translated by Qing Rui 青鋭 in 1929, and republished as an article entitled "Ziran kexue yu bianzhengfa" 自然科學與辯證法 (*Kexue luncong*, 4 (1935)); The "Introduction" was translated by DU Weizhi and published as an article under the title "Bianzheng weiwulun de yuzhouguan yu jindai ziran kexue zhi fazhan" 辯證唯物論的宇宙觀與近代自然科學之發展 (*Dongli*, 1, 2 (1930)); See SHEN Zhenyu 1983, 18-20. DI Qiu was the pen name of CHEN Shaozou 陳韶奏. For the translation he made with ZHU Tiesheng, see Lenin 1930. For FU Zidong's translation, see

In the meantime, other works related to philosophy of mathematics by Engels had also been rendered into Chinese. For instance, *Ludwig Feuerbach and the Outcome of German Classical Philosophy* was translated no less than five times by different individuals and groups between 1929 and 1938.[17] Moreover, the pamphlet entitled *Engels and Science* by the British Marxist John D. Bernal was published in 1935.[18] Books and articles on Marxist philosophy of the natural sciences in general, written by such influential Soviet Marxist theorists as Abram M. Deborin, Borisovich M. Mitin and Boris M. Hessen, were also introduced into China, including Deborin's *Dialectics and Natural Science*, Mitin's *Dialectical and Historical Materialism*, and Hessen's well-known article, "The Social and Economic Roots of Newton's '*Principia*'."[19] Even such a book as *Dialectics and Mathematics* by the Russian historian of mathematics Vladimir N. Molodshii, which

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Lenin 1934. For more detailed accounts of the introduction of these works, see SHEN Zhenyu 1983, 18-49; CHEN Shaozou 1983, 138-140; and GONG 1996, 6.

[17] LIN Chaozhen 林超真, trans., *Foaibahei yu deguo weixin zhexue de weisheng* 佛愛巴黑與德國唯心哲學的尾聲 (Shanghai: Hubin shuju, 1929); PENG Kang 彭康 (whose pen name was PENG Jiasheng 彭嘉生), trans., *Feierbaha lun* 費爾巴哈論 (Shanghai: Nanqiang shuju, 1929); YANG Dongchun 楊東蓴 and NING Dunren 寧敦仁, trans., *Jixielun de weiwulun pipan* 機械論的唯物論批判 (the first part) (Shanghai: Kunlun shudian, 1932); QING Li 青驪, trans., *Feierbaha lun* (Shanghai: Shehui zhuyi yanjiushe, 1938); And ZHANG Zhongshi 張仲實, trans., *Feierbaha yu deguo gudian zhexue de mori* 費爾巴哈論與德國古典哲學的末日 (Shanghai: Shenghuo shudian, 1938); See SHEN Zhenyu 1983, 20.

[18] See Bernal 1935.

[19] Deborin's *Dialectics and Natural Science* was translated by LIN Boxiu 林伯修 (the pen name of DU Guoxiang 杜國庠) and published under the Chinese title W*eiwu bianzhengfa yu ziran kexue* 唯物辯證法與自然科學 (Shanghai: Guanghua shuju, 1936). Mitin's *Dialectical and Historical Materialism* was rendered by SHEN Zhiyuan 沈志遠 under the heading *Bianzheng weiwulun he lishi weiwulun* 辯證唯物論和歷史唯物論 vol. I. (Shanghai: Shangwu yinshuguan, 1936). Hessen's "The Social and Economic Roots of Newton's '*Principia*'" was translated by HE Feng 何封 (pseudonym of ZHANG Hanfu 張漢輔) and published as a pamphlet under the title: *Niudun yuanli pipan* 牛頓原理批判 (Shanghai: Xingzhi Shudian, 1937). See GONG 1996, 7. For a list of the books, pamphlets, and articles on dialectics or philosophy of natural sciences published in Shanghai before 1949, see SHEN Zhenyu 1983, 43-48.

was devoted entirely to a discussion of the history of mathematics from a materialist point of view, was also immediately translated into Chinese.[20]

It was mainly through such translations that Marxist philosophy of the natural sciences in general, and that of mathematics in particular, were introduced into China. The translators of these books and articles were either members of the CCP, or partisans of Communist ideology. In order to promote Marxism in China, the CCP began in 1930 to organize the Alliance of Chinese Social Scientists (*Zhongguo shehui kexuejia lianmeng* 中國社會科學家聯盟) in major cities such as Beijing (then called Beiping, or Peking) and Shanghai. The main task of the Alliance was to propagate Marxism by lecturing at colleges and by publishing journals and magazines. The Alliance in Shanghai was especially active in disseminating and studying dialectical materialism.[21] Some members of the Alliance occasionally gathered to report on and discuss what they had read, translated, and written by themselves on dialectical materialism. Among their publications, mathematics was not ignored, even though no member of the Alliance was a mathematician by training or profession.

WU Shichang 吳仕昌 (believed to be a pen name of AI Siqi 艾思奇, one of China's major Marxist theorists), for instance, wrote an article entitled "Dialectics of Mathematics" (*Shuxue de bianzhengfa* 數學的辯證法)."[22] In this article, AI defended Engels' explanation

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[20] Vladimir N. Molodshii, *Effektivizm v matematike* (*Dialectics and Mathematics*) (Moscow: Sotsekgiz, 1938). The book was translated by CHEN Xiaoshi 陳曉時 (the pen name of CHEN Guiru 陳珪如) under the Chinese title: "Lun weiwu shiguan shuxueshi de wenti," 論唯物史觀數學史的問題 in *Zhexue Zazhi* 哲學雜誌, no. 2, 1940. See SHEN Zhenyu 1983, 23, 41.

[21] For a brief account of the activities of the Alliance in Shanghai on dialectical materialism, see SHEN Zhenyu 1986, 32-34; and GONG 1996, 7-8.

[22] As stated above, numerous books and articles were published under pseudonyms in the 1930s. This was to avoid harassment, arrests, or even persecutions by the Nationalist government. We will see many more pseudonyms in the next chapter when we focus on the 1970s. But that was for entirely different reasons, one of which was to emphasize the role of groups rather than individuals. WU Shichang's article was published

of dialectics made in *Anti-Dühring* about one of the basic arithmetic operations, that of subtraction or negation. One positive number, say *a*, becomes negative *a* when it is negated. The negative *a* in turn changes back into a positive number a^2 when it is negated again.[23] By criticizing objections made by several Japanese writers (and by the "revisionist" Friedrich Adler) to Engels' application of the law of the negation of the negation to this example, AI argued that it was wrong to think that only "higher" mathematics could be a subject for study in terms of dialectical materialism. "The development of mathematics from arithmetic up to the integral and differential calculus," AI wrote, "is nothing but the dialectical development of numbers."[24]

AI's colleague and one of the organizers of the groups studying dialectical materialism, LIAO Shuqian 廖庶謙, also studied dialectics in mathematics. In his pamphlet entitled *Lectures on Mathematics* (*Shuxue jianghua* 數學講話), LIAO reiterated Engels' views on the nature of mathematics and illustrated the properties of dialectics by using examples from elementary mathematics. Much of the contents of the pamphlet also appeared separately as articles in the *New China Daily* (*Xinhua Ribao* 新華日報).[25]

Because dialectical materialism is an integral part of Marxism and Communist orthodox ideology, the introduction of dialectical materialism immediately stirred debate in China. The focus of the debate at first was on the relation between dialectical logic and formal logic, which meant syllogistics rather than mathematical logic. The two

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in the founding issue of the magazine, *Knowledge Monthly* (*Renshi yuekan* 認識月刊, July 1937), and reprinted in AI 1983, 270-276.

[23] See Engels 1966, 150.

[24] See WU 1983, 273.

[25] See LIAO Shuqian 1937.

protagonists in the early phase of this debate were YE Qing 葉青 and ZHANG Dongsun 張東蓀. Prior to 1928, YE Qing was a member of the CCP, but then switched to the Nationalist Party after being captured in 1928; he argued for dialectics. ZHANG Dongsun, who was a professor of philosophy at Yanjing University 燕京大學 in Beijing, argued for formal logic. Marxist theorists such as AI Siqi soon joined the debate because they regarded the arguments raised by both YE and ZHANG as unacceptable: one distorted materialist logic, whereas the other downplayed it. The core of this debate was trying to identity formal logic as metaphysics, to treat formal logic as a methodology of subjective idealism and a kind of world outlook. The arguments raised by the Chinese Marxists were heavily influenced by a philosophical trend then led by Leon Trotsky in the Soviet Union. The whole debate did not cease until Trotsky lost his influence in the USSR in 1940. The debate was eventually won by the Marxists, but in retrospect with several harmful consequences in the new China for the development of logic in general and of formal logic in particular. Firstly, most Chinese Marxist debaters (like their Soviet comrades) misidentified dialectical logic with dialectical materialism. Secondly, they underplayed the role of formal logic in both mathematical and scientific reasoning. And thirdly, they emphasized the class characteristics of both dialectical logic and formal logic: the first serving the proletariat and the latter the bourgeoisie.[26] Whether formal logic was neutral or class based, what its objective foundations were, and what its relation to dialectical logic might be, were among the many related issues that would be debated again in the 1950s and 1960s. The debate itself was complicated, very interesting, and deserves to be studied

[26] For the Deborin School and the controversy it provoked in the Soviet Union, see Joravsky 1961, in particular chapter four entitled "The Great Break, 1929-1932," 233-271. For a summary of the debate in China, see GUAN Zhu 1999, 64-131, particularly the section entitled "Sanshi niandai chuantong luoji

historically in its own right. But here we emphasize only that this debate in turn generated more weighty discussion of dialectical materialism.

In the late 1930s and 1940s Marxist philosophy of mathematics, as an inseparable part of dialectical materialism, was further spread into Yanan 延安 where the headquarters of the CCP was located from 1937 to 1947. For political reasons, dialectical materialism was mandatory among the Communist cadres in Yanan. In February of 1940, the "Yanan Society for Natural Sciences (*Yanan ziran kexue yanjiuhui* 延安自然科學研究會)" was set up. One of its primary goals was to unite, or integrate, the natural sciences with dialectical materialism, namely to study dialectical materialism based on achievements or results of the natural sciences, and in turn to pursue the natural sciences under the guidance of dialectical materialism.[27] Accordingly, Engels' *Anti-Dühring* and *Dialectics of Nature*, along with other Marxist works, were required reading, and consequently, in this way dialectical materialism was not only transmitted to Chinese Communist Party members, but they also began to think in terms of dialectical materialism.

The introduction of a Marxist philosophy of the natural sciences in general, and that of mathematics in particular, to China in the 1930s and 40s was in one way or another associated with the CCP. The influence of those introductions was largely confined to political and philosophical circles, and had very little, if any, influence on working

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hengzao pipan yu fouding 三十年代傳統邏輯橫遭批判與否定 (Criticism and negation of traditional logic in the 1930s in China)," 86-93.

[27] See YU Guangyuan 1983, 5. In order to meet the demand for reading materials, YU Guangyuan 于光遠, a student of dialectical materialism and later one of the top authorities on this subject in new China, retranslated the chapters entitled "Introduction" and "The part played by labor in the transition form Ape to Man" in the *Dialectics of Nature* into Chinese. These were published in the newspaper *Liberation Daily* (*Jiefang ribao* 解放日報) in October of 1941. The translations also appeared in the magazine *Chinese Youth* (*Zhongguo qingnian* 中國青年) vol. 3, no. 1 (1940), respectively. See YU Guangyuan 1983, 6-7; and GONG 1996, 10-11.

scientists in general, or on mathematicians and mathematics educators in the areas controlled by the Nationalist government. But this was all to change dramatically as soon as the People's Republic of China was founded in 1949 (see section three below).

In the 1950s, China witnessed the second influx of Marxist philosophy, to an extent that its influence on the natural sciences was on a scale much greater than the earlier experience of the 1930s. Two new translations of Engels' *Dialectics of Nature* were made one after the other.[28] Marxist interpretations of the history of science and mathematics— for example, those made by such influential Western Marxist scholars as the above mentioned John D. Bernal, John B. Haldane, and Dirk J. Struik— were also quickly introduced into China.[29] Quite naturally, there was a strong influence coming directly from the USSR. It was evident that the CCP, following the Bolsheviks, considered the social and natural sciences, as well as the arts, as part of an ideological superstructure. Accordingly, they had to confront the bourgeoisie. It is no surprise that the notorious Soviet criticism of "bourgeois" science during the Stalin era— particularly Mendelian genetics, quantum mechanics, contemporary Western theories of cosmology, and the American chemist Linus Pauling's resonance theory of chemical bonding, among others— were all introduced to China.[30] And similar criticisms of all these were immediately launched.[31] Although Chinese criticisms, both in scale and depth, were modest and

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[28] See Engels 1950; and 1955.

[29] See Bernal 1956; Haldane 1950; and Dirk J. Struik 1956. For Struik and the Chinese translation, see LIU Dun 2002.

[30] For a brief account of the historical background and development of dialectical materialism in the USSR, see Loren Graham 1987, 1-67. In addition, David Joravsky's *The Lysenko Affair* provides an excellent account of the Soviet's criticism of Mendelian genetic theory; see Joravsky 1970.

[31] For similar general criticisms in astronomy, biology, physiology, and physics by Chinese scholars, see GONG 1954. For Lysenkoism in China, see Schneider 1986; and 1989, 45-65. For a general account

restrained compared to those of the Soviets, they did have far-reaching negative consequences for the development of the natural sciences, as well as for the philosophy of science and the historiography of the history of science in China, especially during the Cultural Revolution. To criticize the idealism and metaphysics of "bourgeois" science has been a constant task, sometimes intense, sometimes not, for Chinese Marxist scientists and philosophers of natural science alike ever since.

Soviet Marxist philosophers also tried to bring mathematics under the umbrella of dialectical materialism not long after the October Revolution. Besides reiterating the practical origins of mathematics, and emphasizing that dialectics was a particularly valuable method for advancing mathematics, as Engels and Lenin had pointed out, Soviet Marxist philosophers based their criticism of set theory, mathematical logic, Western approaches to the foundations of mathematics, and probability theory, primarily on their so-called "mathematical idealism." Nevertheless, the criticisms raised in the 1920s represented no serious threat to Soviet mathematics, especially to the world-renowned Moscow school of mathematics, in part because of the abstract and advanced nature of these mathematical subjects, and in part because of the disunity among the Marxists themselves. Meanwhile, Soviet mathematicians, reluctantly, were called upon to interpret their own work in the light of materialist dialects. In fact, some well-known Soviet mathematicians, for instance, Sergei N. Bernshtein and A. V. Vasil'ev, openly expressed their doubts about the validity of finding any positive relation between mathematics and dialectical materialism, or, for example, interpretations of non-Euclidean geometry from

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criticisms of the theory of relativity in China, see Friedman 1983, 51-75. For Chinese translations and the overall influence of Soviet criticism in China, see GONG and LIU 1991.

the viewpoint of Henri Poincaré's conventionalism, which viewed mathematics as only a set of agreements among mathematicians.[32]

Soviet mathematicians, however, were not able to avoid interpreting mathematics according to Marxist principles for long. On March 15, 1931, the Bolshevik Central Committee sanctioned a proposal from the Communist Academy, which recommended ideological control of non-Communist scientific research establishments, and decreed the ideological status of the natural sciences. In the First All-Union Congress for the Planning of Science held the following month, the Bolsheviks imposed Marxist doctrine on all scientists and mathematicians.[33] As a result, the Bolsheviks launched an all-out attack on "bourgeois" science. Mathematics was no exception, and indeed became one of the battlefields.

According to Stalinist philosophers of mathematics, notably Ernst Kol'man and Sofia A. Yanovskaia (Ianovskaia), Marxist philosophy of mathematics makes the following four assumptions:

1. The axioms of mathematics are generated from the external world.
2. Formalism, logicism, intuitionism, and other Western approaches to the foundations of mathematics are all associated with idealistic philosophy. Accordingly, they all partake of mathematical idealism.
3. Mathematics is more than an organized collection of theorems, formulas, and methods. It has empirical value. Therefore, applied mathematics should and must be emphasized.

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[32] How Marxism entered into the field of mathematics, and the reaction of Soviet mathematicians in the pre-Stalin years, are covered in Vucinich 1999, 107-124.

[33] See Joravsky 1961, 233-249, esp. 244-245.

4. The progress of mathematics is driven mainly by practical problems and the needs of society.[34]

Soviet Marxist philosophers demanded that mathematicians reconstruct modern mathematics based on these assumptions. Although no renowned Soviet mathematician ever responded explicitly to this call, overall they did accept the above four assumptions in principle.[35] For instance, the leading mathematicians of the Moscow school, Andreĭ N. Kolmogorov and Aleksandr D. Aleksandrov, did offer Marxist interpretations of mathematics, just as they criticized Western "idealistic" philosophies of mathematics. They also acknowledged the importance of "applied mathematics" in their own writings. By doing so, they wanted to protect mathematical research from ideological and political meddling.[36] This strategy seems to have worked, and pure mathematical research in the Soviet Union during the Stalin era remained productive, although philosophy of mathematics suffered a severe blow due to its accommodation of Marxism.[37]

Soviet philosophy of mathematics was introduced to China mainly through Russian books by Soviet mathematicians and philosophers of mathematics, and through journals in which articles on Marxist philosophy of mathematics appeared. Some of these books and articles were translated directly into Chinese. For instance, Andreĭ N. Kolmogorov's article on mathematics for volume twenty-six of the *Great Soviet Encyclopedia* (1938), which is considered the first serious and detailed Marxist evaluation of mathematics by a

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[34] See Vucinich 2000, 54-76, esp. 62-63.

[35] *Ibid*.

[36] See Vucinich 2000, 61-62, 71-73.

[37] One example of the consequences of this compromise has been provided by Joseph W. Dauben in a study of the impact of the translation of Karl Marx's *Mathematical Manuscripts* into Chinese. See Dauben 2003,330.

mathematician, was translated into Chinese and published in 1956.[38] The article on geometry along the same lines in the same *Encyclopedia* by Aleksandr D. Aleksandrov was likewise translated into Chinese, as were his *Pridoda* articles, "Lenin's Dialectics and Mathematics," and "On Mathematical Idealism."[39] Other systemic treatments of mathematics from a Marxist point of view, including *Mathematics: Its Content, Methods and Meaning*, were also rendered almost immediately into Chinese, primarily mathematicians at Beijing University and at the newly founded (1952) Institute of Mathematics of the Chinese Academy of Sciences.[40] The first chapter of this book by Aleksandrov, "A General View of Mathematics," presents in considerable detail the essential Marxist view of mathematics based on Engels' account in the *Nature of Dialectics*, and includes the four assumptions about mathematics already mentioned. Aleksandrov asserts: "In reality, mathematics offers not the slightest support for idealism or metaphysics. Quite the contrary; when considered objectively in all its interrelations and developments, mathematics offers still another confirmation of dialectical materialism in contrast to idealism and metaphysics." [41]

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[38] See Vucinich 2000, 61. Kolomogorov 1938; and 1956.

[39] See Aleksandrov 1951a; 1951b; and 1955.

[40] *Matematika, ee soderzhanie metody i znachenie* (*Mathematics: Its Content, Methods and Meaning*, 3 vols.) was written by a group of leading Soviet mathematicians and edited by Aleksandr D. Aleksandrov, Andreĭ N. Kolmogorov, and M. A. Lavrent'ev. The book was also rendered into English and published in six parts by the American Mathematical Society as the first of the *Translations of Mathematical Monographs* series. Part I was published in 1962, and the rest in 1963. Nevertheless, sections eight and nine of chapter one entitled "The essential nature of mathematics" and "The laws of the development of mathematics," respectively, are omitted from the English translation because of their detailed descriptions of Marxist views. See Aleksandrov, Kolmogorov, and Lavrent'ev 1956, 1958-1962, and 1962-1963, esp. part I, 67.

[41] Quoted from Aleksandrov, Kolmogorov, and Lavrent'ev 1962-1963, 1: 7. The Chinese translation of this passage appears on p. 6.

Soviet philosophy of mathematics had a marked influence in new China. The Soviets' interpretations of the nature of mathematics and its historical development were echoed not only by Chinese philosophers of mathematics, but also by leading working mathematicians. Before we discuss this further, we should consider MAO Zedong's understanding of dialectical materialism in terns of philosophy of science.

7.2 MAO Zedong and Dialectical Materialism

MAO Zedong became the official leader of the Chinese Communist Party at the Zunyi Meeting 遵義會議 held during the Long March in January of 1935. Since then, he succeeded in maintaining power until his death in September of 1976. Over the span of four decades, he was the absolute authority and undisputed leader of the CCP. His writings and ideas were canonized by the CCP, and became an integral part of Party ideology, as well for the nation, along with Marxism and Leninism.[42] MAO exerted considerable influence on, and indeed determined, significant aspects of the development of recent science in China. During the Cultural Revolution, his words, like those in the Bible or Koran, were quoted to support or refute arguments made by different authors. Consequently, it is necessary to describe his understanding of dialectical materialism in general, as well as his views on the natural sciences in particular.

MAO's concern for dialectics is reflected in his notes for lectures on dialectical materialism written between 1936 and August of 1937, which he subsequently delivered at the Anti-Japanese Military and Political College (*kangri junzheng daxue* 抗日軍政大學) in Yanan. The main purpose of MAO's lectures was to criticize the dogmatists within the

[42] See Zhongguo Gongchandang 1981, 56-73.

CCP who, according to MAO, excerpted words and phrases from Marxist works and used the resulting fragments of Marxism to guide actions of the CCP without due consideration for either their meaning or the real situation in China. Such dogmatists were believed to be responsible for a series of setbacks for the CCP between 1931 and 1934. MAO, with this in mind, addressed two major issues: what was the essence of Marxist dialectical materialism and how could it be incorporated into revolutionary practices in China? Among MAO's lectures on *Dialectical Materialism*, three sections in particular are relevant to our current subject. These are section four of chapter two entitled "The Question of the Object of Dialectical Materialism – What is it that Dialectical Materialism Studies?"; section eleven of the same chapter, "On Practice"; and the first section of chapter three, "On Contradiction."[43]

In "The Question of the Object of Dialectical Materialism – What is it That Dialectical Materialism Studies?" MAO points out that dialectical materialism is not only a philosophical world outlook, but also provides the only "scientific" methodology and logic

[43] MAO's *Dialectical Materialism* (*Lecture Notes*) consists of three chapters. According to Stuart R. Schram, chapter one and the first six sections of chapter two were first published in the *Kangzhan Daxue* 抗戰大學 ([Journal of] the Anti-Japanese College), vol.1, no. 6 – no.8 (April to June, 1938). Sections seven through ten of Chapter two were also published soon thereafter, see Schram 1986, 837-838. Section eleven of chapter two, "On Practice," and chapter three and the last which only includes "On Contradiction," were also mimeo-graphed and circulated in the areas controlled by the CCP as mentioned in GONG 1996, 9. As early as the 1960s, some Western scholars, for example Karl A. Wittfogel 1963, 251-77; Arthur A. Cohen 1964; and Francis Y. K. Soo 1981, doubted for different reasons that "On Practice" and "On Contraction" were drafted in July and August of 1937, respectively. Some, including John E. Rue 1967, 464-468, have even gone so far as to argue that MAO was not the author of these two essays. Such arguments have been refuted by Stuart R. Schram 1967, 155-165; 1986, 837-860. See also Knight 1980, 641-668; 1990, 3-83; 1996, 84-116. Evidence revealed in recently published Chinese articles and books, for example, GONG 1986, 83-114; SHI 1988; and XU, CHEN, and SONG 1992, 283-313; also contradict Rue's claims.

"On Practice" 實踐論 was not published formally until December 29, 1950, and "On Contraction" 矛盾論 only appeared after some revision on April 1, 1952; both were published in *Renmin Ribao*. Later they were collected in *Mao Zedong xuanji* (*Selected Works of Mao Zedong*, vol. I) (Beijing: Renmin Chubanshe, 1951). The official English versions were first published in 1961 (Beijing: Foreign Languages Press). MAO's motivation for writing the two essays is clearly stated in the footnotes. See MAO 1975, 1: 295-296, 311.

for mathematics and the natural sciences. According to MAO, dialectical materialism provides a unification of worldview, logic, and methodology:

> [A]ll of the various sciences (mathematics, mechanics, chemistry and physics, biology, economics and other natural sciences and the social sciences) study the various aspects of the development of the material world and its knowledge. Because of this, the principles of the various sciences are restricted in a narrow and one-sided way by concrete realms of study. Dialectical materialism is however quite different; it is the universalisation, the totality, the conclusions and the general content of value from all of the concrete sciences and all of humankind's other scientific knowledge. In this way, the concepts, judgments, and principles of dialectical materialism constitute exceedingly extensive laws and formulations (incorporating the most general principles of all of the sciences, and consequently incorporating the essence of the material world). This is one side of the picture and from this perspective, dialectical materialism is a world view. From the other perspective, dialectical materialism is *the logical and epistemological foundation* for genuine scientific knowledge liberated from all idle speculation, fideism and metaphysics; hence it is at the same time the only true, objectively reliable methodology for the study of concrete science. This further adds to our comprehension of what we mean when we speck of dialectical materialism or dialectical materialism as a *unified system of world view and methodology*.[44]

The assertion that dialectical materialism is the *only* "true, objectively reliable methodology" for the sciences not only echoes Lenin's view of dialectics, but also reflects Marxist views with respect to the ongoing debate over formal logic and dialectical logic as mentioned above. Moreover, in the lecture *On Practice*, MAO acknowledged that dialectical materialism is concerned with class nature and the importance of practice, both as emphasized by Marx and Lenin.[45] The class nature of dialectics is a reflection of the service it renders to the proletariat, and likewise accounts for why it is negatively regarded by the bourgeoisie. MAO explains yet another characteristic of dialectics as follows:

> Discover the truth through practice, and again through practice verify and develop the truth. Start from perceptual knowledge and actively develop it into rational knowledge; then start from rational knowledge and actively guide revolutionary practice to change both the subjective and the objective world. Practice, knowledge, again practice, and again knowledge. This form repeats itself in

[44] Quoted from Knight 1990, 97, italics added.

[45] See MAO 1975, 295-309, esp. 298.

> endless cycles, and with each cycle the content of practice and knowledge rises to a higher level. Such is the whole of the dialectical-materialist theory of knowledge, and such is the dialectical-materialist theory of the unity of knowing and doing.[46]

In his essay "On Contradiction," MAO states that contradictions exist everywhere, and are presented in the process of development of any given thing from beginning to end. The contradictions in question, according to MAO, are of two sorts: major and minor. Major contradictions are what determine the nature and state of any given thing or object.

MAO accepted the three basic laws of dialectics as outlined by Engels. MAO nevertheless elevated, as suggested by Lenin, the stature of the law of interpenetration, or the unity of opposites, which he thought was the most fundamental and from which could be derived the law of the interchange of quantities and qualities and that of the negation of the negation.[47] The law of the unity of opposites, according to MAO, unifies the contradictions within each "thing." In the conclusion of "On Contradiction," he writes:

> The law of contradiction in things, that is, the law of the unity of opposites, is the fundamental law of nature and of society and therefore also the fundamental law of thought. It stands opposed to the metaphysical world outlook. It represents a great revolution in the history of human knowledge. According to dialectical materialism, contradiction is present in all processes of objectively existing things and of subjective thought and permeates all these processes from beginning to end; this is the universality and absoluteness of contradiction.[48]

The two essays "On Practice" and "On Contradiction" have been highly regarded by the CCP as MAO's original contributions to dialectical materialism ever since they

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[46] Quoted from MAO 1975, 308. Stalin also made a similar assertion about the relation between practice and theory. He says that theory and practice stand in a dialectical unity to one another: theory guides practice and practice is the test of theoretical truth.

[47] Lenin, in his *Philosophical Notes*, considers the law of the unity of opposites to be the most important of the three, along with the law of the interchange of quantity and quality to be derivable from the law of the unity of opposites. See Loren Graham 1987, 54, 448 note 80. MAO later rejected the negation of the negation as a principal law of dialectics during a talk given in Hangzhou 杭州 in 1965. Knight rightly points out that what MAO rejected was the title rather than the substance of the law; see Knight 1990, 19. As for the talk, see GAO Lu 1986, 115-147.

[48] Quoted from MAO 1975, 345.

were formally published in the early 1950s.[49] In particular, they had considerable influence on the polices of mathematical research and the interpretation of the history of mathematics, as well as on mathematical education during MAO's reign, which we shall discuss in the next section.

Another equally important result of MAO's thinking related to our current study is his dialectical view of an infinite universe and the infinite divisibility of matter. It may seem strange that MAO should have considered the infinite at all, especially infinite divisibility, but he was prompted to do so in part because of his interest in dialectics. In August of 1963, the journal *Correspondence on the Studies of Natural Dialectics* (*Ziran bianzhengfa yanjiu tongxun* 自然辯證法研究通訊), which was sponsored by the Institute of Philosophy of the Chinese Academy of Sciences, published an article translated from Russian entitled "New Views of Basic Particles (*Jiben lizi de xingainian* 基本粒子的新概念)" by the Japanese theoretical physicist SAKADA Shōichi 坂田昌一.[50] SAKADA was famous for his 1942 prediction of the existence of two different kinds of neutrons and mesotrons, which were later discovered by experiment, and for his 1955 particle model

[49] The *Resolution on Certain Questions in the History of Our Party Since the Founding of the People's Republic of China* states clearly that MAO illustrated and developed "the nucleus of Marxist dialectics," namely, the three laws of dialectical materialism as stated in Engels' *Dialectics of Nature*; see Zhongguo Gongchandang 1981, 68-9. A recent work on MAO Zedong also says that the two essays by MAO "provided an important theoretical foundation for forming the ideas and ways of thinking of the CCP"; see FENG Xianzhi 1993, 2: 6-7. Stuart R. Schram has pointed out MAO's debt in this respect to AI Siqi and to the Soviet Marxist theorist Mark B. Mitin; see Schram 1986, 838-39. Nick Knight has meticulously identified possible sources of MAO's *Materialism Dialectics* (*Lecture Notes*). Besides Ai and Mitin, Knight mentions the Chinese Marxist theorist LI Da; see Knight 1990, 83-86. In his latest publication, Knight argues that MAO made no original contribution to dialectical materialism, and that MAO's arguments are "derived from the interpretation of dialectical materialism deemed orthodox in the Soviet Union between 1931-36." See Knight 1996, 84-116, quoted from 107. Our purpose here is not to argue what statements are original contributions of MAO, but to focus on what MAO actually wrote or said.

[50] The article was originally published in Japanese in 1961in the *Journal of the Physical Society of Japan*, was translated into Russian. See SAKADA 1963.

later named after him.[51] In this paper, SAKADA wrote in a dialogue style as Galileo had in his well-known *Dialogues Concerning the Two Chief World Systems*. SAKADA, however, described his own thesis about the structure of particles while criticizing others from a dialectical point of view. In order to support his arguments that particles were not the ultimate elements of which matter is made, SAKADA cited Engels to the effect that molecules and atoms were just different stages of an infinite series through which matter was dissected, as described in the *Dialectics of Nature*. SAKADA also quoted Lenin's view that electrons are inexhaustible, as he suggested in *Materialism and Empirio-criticism*.

MAO happened to read SAKADA's article, and was fascinated by it. On August 18, 1964, he even invited several Marxist philosophers to discuss the article with him at the summer resort of Beidaihe 北戴河 in Hebei province.[52] In the course of their conversation, MAO echoed both Engels and Lenin on infinity and divisibility:

> Lenin once said that everything could be divided. For example, an atom, which was believed to be indivisible, cannot only be divided, but so, too, its electrons. The research subject of nuclear fusion is young, [but] over the last several decades scientists have subdivided the nucleus into protons, antiprotons, neutrons, anti-neutrons, mesotrons, antimesotrons, both heavy and light. As for the separability of electrons from the nucleus, it was known before.... The universe is infinite. Time and space are infinite. Space, both in terms of the macrocosm and micro-cosm, are infinite. Because of the infinite divisibility of matter, scientists will have work to do even one million years from now. I have heard some theories and read several articles, of which I enjoyed very much the one by SAKADA Shōichi published in *Correspondence on the Studies on Natural Dialectics*. I have never read such an article before. He is a dialectical materialist and quotes from Lenin.[53]

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[51] For the model, see QING and LIU 1965, 38-39.

[52] These philosophers are KANG Sheng 康生, CHEN Boda 陳伯達, AI Siqi, LU Ping 魯平, and GONG Yuzhi, among others. See ZOU, LIANG, and SHEN 1983, 109; Knight 1990, 18; Schram 1991, 49; and GONG 1996, 99-100.

[53] GONG 1996, 100. For Engels' account of mathematical infinity in the real world, see Engels 1966, 54-64; 1940, 313-319, Appendix I.

MAO had another similar conversation six days later with Yu Guanyuan and the theoretical physicist ZHOU Peiyuan 周培源, in the course of which he voiced views similar to those reflected in the above quotation.[54] Because of MAO's interest and some inaccuracy in the Russian translation (as pointed out by SAKADA himself, who was then in Beijing leading a Japanese delegation of scientists), the article was immediately retranslated directly from Japanese, and published under the new title: "Dialogues on the Various New Views of Particles (*Guanyu xin jiben liziguan de duihua* 關於新基本粒子觀的對話)," which appeared in *Red Banner* (*Hong Qi* 紅旗), the propaganda organ of the Central Committee of the CCP. Accompanying it was an editorial note based on the words of MAO. The editorial began as follows:

> The world is infinite, and full of contradictions. Everything in it is a unity of opposites, and none of them is without contradiction, nor is indivisible. That every unit can be halved is a universal phenomenon. This is dialectics. It is true in nature, as well as in society and the human understanding. Denying this is metaphysics.[55]

Because of the pivotal role of *Hong Qi* in the politics of new China, SAKADA's article together with the editorial were immediately reprinted in major newspapers such as *People's Daily* (*Renmin Ribao* 人民日報) and journals across the country. Seminars devoted to the subject were organized and attended by leading Marxist philosophers, physicists, and scientists in other areas. Numerous articles discussing the importance of dialectics in natural science and scientific research flooded newspapers and magazines

[54] MAO stated similar views on the infinite divisibility of matter as early as January 15, 1955, during a conversation with nuclear physicists including QIAN Sanqiang 錢三強, who was a student of both Frédéric and Irène Joliot Curie; MAO repeated these views during a conference in Moscow when he visited there in 1957; See ZHU, SONG, and ZHU 1983, 324; GONG 1996, 100-103, and GE 2002, 114-116.

[55] SAKADA 1965, 19.

throughout the country.[56] MAO's talk and SAKADA's article, together with these intensified and highly publicized discussions, in retrospect, had various negative consequences for development of the philosophy of science in China over the next decade or so. On the positive side, they may have inspired physicists and mathematicians at the Institute of Atomic Energy, the Institute of Mathematics of the Chinese Academy of Sciences, Beijing University, and the Chinese University of Science and Technology in Beijing to work out a thesis regarding elementary particles called the stratum model (*qiangzi jiegou de cengzi moxing* 強子結構的層子模型).[57] This model speculated that even elementary particles like the straton might have internal structures. In other words, the meson was probably made up of other smaller particles. Indeed, the straton model has been treated by most Chinese Marxist philosophers as a successful case of using dialectical materialism to guide scientific research.[58]

On the negative side, examples may be cited from modern cosmology. MAO's endorsement of infinite time and space prohibited Chinese scientists from upholding any contemporary Western cosmological theories that might hypothesize a finite universe or consider any starting point for time itself. Theories like the Big Bang, or a Steady State Universe, were criticized through translations of Russian accounts in the early 1950s. As soon as background radiation was observed in the mid 1960s, the Big Bang theory has

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[56] For instance, *Hong Qi* and *Ziran Bianzhengfa Yanjiu Tongxun* devoted special issues to the subject (no. 9 (1965), and no. 3 (1965), respectively), and included discussion of SAKADA's article and the application of dialects in natural science.

[57] For an account of the possible influence of dialectical materialism on Chinese particle physics, see ZHU, SONG, and ZHU 1980, 1983, 321-329.

[58] See GONG 1996, 104; ZOU, LIANG and SHEN 1983, 110-111; and ZHU, SONG, and ZHU 1980, 321. The guiding role of dialectical materialism over particle physics seems rather superficial if one considers the fact that American physicists Murray Gell-Mann and George Zweig in 1963 had already proposed that the baryon and meson were not the ultimate particles, and were in turn made of three hypothetical quarks, namely up, down and strange, and their anti-parts.

generally been accepted in the West. But this has not been the case in China. The theory, in fact, was constantly attacked by the "Mass Criticism in Natural Sciences (*like da pipan* 理科大批判)" movement during the Cultural Revolution on the grounds that it assumed the finitude of the cosmos, which in turn was taken to be connected to the creation of the world by God, which was obviously in conflict with dialectical materialism.[59] Because of such philosophical interpretations of the Big Bang theory, studies of cosmology in China were adversely affected, a situation that only began to change in the 1980s.

MAO's assertion of an infinite universe and the infinite divisibility of matter in the real world, for better or worse, but quite remarkably for the case of mathematics, canonized the concepts of both the actual infinite and infinitesimals. The relation between finite and infinite, accordingly, was interpreted as two sides of the same coin: they cannot be separated from each other. Finites are components of the infinite, and the infinite is a sum of countless finites. Furthermore, infinite and finite are united under the law of the unity of opposites.[60] This dialectical view of the infinite and finite was to be applied in nation-wide discussions on the foundations of the calculus, including differentials and the infinitesimals, in the 1970s.

7.3 Politics in Demand

Immediately after the founding of the PRC, dialectical materialism advanced itself from the ideology of the Party to that of the State. Chinese mathematicians suddenly realized that they had to be familiar with dialectics. In order to educate themselves, mathematicians

[59] For a brief account of Chinese criticism of contemporary Western theories of cosmology in general, and of the Big Bang theory in particular, see LI Qibin 1983, 398-403; and Buckley 1991, 5-6.

[60] This, basically, was a dialectical view of the relation between finite and infinite. It was repeated in numerous articles and books. See AI Siqi [1959] 1983, 694-97; and LU Daji 1965.

and scientists organized symposia to discuss possible links between dialectics and their own subjects. As early as December of 1949, the Shanghai branch of the All-China Alliance of Scientific Workers and the Chinese Science Society invited leading Marxist philosophers, together with leading mathematicians and scientists in Shanghai, to discuss relations between mathematics, natural sciences, and dialectics. Mathematicians in the discussion, however, voiced their skepticism about opposing formal logic to dialectics, while acknowledging that dialectics was one of the mathematical methodologies.[61]

In Beijing, a symposium was organized by the Beijing branch of the Chinese Mathematical Society in July of 1950, where mathematicians and philosophers together discussed Engels' accounts of the origins of mathematics, the objects of mathematical research, and the article written by Andreĭ N. Kolmogorov on mathematics for the first edition of *The Great Soviet Encyclopedia*.[62] The participants also discussed what the dialectical methods of mathematics were, and what might be the possible relations between mathematics and dialectics.[63]

In 1951, along with other intellectuals, mathematicians found themselves facing a political movement called "Reform of Thought (Sixiang Gaizao 思想改造)." This movement was based on the assumption that the outlooks of the great majority of intellectuals were bourgeois. In order to reform their worldviews from bourgeois to

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[61] See ZHANG Mengwen 1950, 3-15.

[62] The Chinese Mathematical Society was founded in 1935 in Shanghai, but soon ceased to function as a result of the Japanese occupation between 1937 and 1945. In order to differentiate itself from the Society that had failed under the Japanese, the New Society of the Chinese Mathematical Society was founded in 1940 in Kunming, Guizhou province. The original name of the Society was restored at the end of World War II. The Chinese Mathematical Society did not join the International Mathematical Union (IMU) until 1986, for very complicated political issues related to Taiwan. As for the history of the Society, its relations with the IMU and the International Congress of Mathematicians (ICM), see Stone 1961, 624-625; REN and ZHANG 1995; and Lehto 1998, 126-130, 242-250.

[63] See GUAN Zhaozhi 1959b, 2.

proletarian, intellectuals were required to attend political meetings regularly to study MAO's *On Practice* and *On Contradiction*, along with Engels' *Dialectics of Nature* and other works about dialectical materialism. The experience of the renowned mathematician HUA Luogeng 華羅庚 illustrates well exactly what this movement entailed.

HUA was a legend among Chinese mathematicians. Like the Indian mathematician Srinivara Ramanujan, he had received no formal higher education. But as a mathematical genius, HUA established himself in his twenties as a first-rate mathematician for his work largely on analytic number theory. Following World War II, he came to America, first as a member of the Institute for Advanced Study at Princeton, and then as a full professor at the University of Illinois at Urbana. But as soon as the PRC was founded, he wrote a famous letter urging overseas Chinese to return home to help build the new China, and HUA himself returned immediately to Beijing. HUA was soon appointed as the director of the newly founded Institute of Mathematics at the Chinese Academy of Sciences, and was elected president of the Chinese Mathematical Society.[64]

Like other mathematicians in China, HUA studied MAO's two treatises (as well as the *Dialectics of Nature* by Engels) as a result of the reform of thought movement. But unlike many of his colleagues, Hua's assessment of these works was published because of his prestigious status. In an article entitled "A Mathematician's Preliminary Understanding of *On Practice* and *On Contradiction*" (DATE), HUA used dialectical interaction of perceptual and rational knowledge to explain the development of elementary mathematics, emphasizing that mathematical knowledge is derived from practice. He echoed Engels' assertion that the main foundation of higher mathematics is contradiction,

[64] For a definitive biography of HUA, see WANG Yuan 1994, and its English translation by Peter Shiu Man-Kit in 1999.

and used the differential and integral calculus as an example to illustrate this point. He also applied, rather clumsily, the notions of universality and contradictions to criticize empiricism and "dogmatism," which he ambiguously associated with any trend that focused on getting more generalized results or propositions with increasingly less useful properties.[65]

Mathematicians like HUA became familiar with dialectical materialism as a result of the reform of thought movement. But when this movement faded from the political scene in the early 1950s, this did not mean an end to political pressure on mathematicians. In fact, at the first National Congress of the reorganized Chinese Mathematical Society, which was held at Beijing University August 15-20, 1951, the official government ideology was stressed, and the relation between mathematics and dialectical materialism specifically was taken up in a panel discussion. Western schools of mathematics such as formalism, logicism, and intuitionism were all condemned for treating mathematics either as a game of symbols, or as a free creation of pure human thought.[66] The guiding role Marxism and Leninism were expected to play in mathematical research and teaching was soon emphasized in a subsequent symposium organized by the Chinese Mathematical Society in September of 1953. At this meeting, HUA Luogeng and SU Buqing 蘇步青, the latter a renowned geometer and vice president of the Society, both expressed in their opening address and closing remarks of the symposium, respectively, the importance of Marxist ideology for Chinese mathematicians.[67]

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[65] See HUA Luogeng 1953, 482-88, esp. 485-86. Other well-known mathematicians, for instance XU Baolu 許寶祿, also wrote about their understanding of dialectical materialism. See XU Baolu 1953.

[66] See REN and ZHANG 1995, 195-196; and ZHANG 2000, 200.

[67] See REN and ZHANG 1995, 199-207.

Because of the stress they placed on ideology, the foundations of mathematics and mathematical logic suddenly became dangerous subjects for research, because foundations had to deal with logicism, formalism, or intuitionism in one way or another, and mathematical logic had direct links to logicism.[68] This situation further discouraged Chinese scholarship in these areas that were already relatively weak. Mathematicians who had studied mathematical logic either turned their interests to such practical subject as newly-developing computer science, or focused on such politically-free areas as algorithms and recursion theory.

Marxist interpretation of the history of mathematics likewise became orthodox as a consequence of the demands of politics. The position adopted by Engels and later detailed in *Mathematics: Its Content, Methods and Meaning*, written by a group of leading Russian mathematicians, was widely adopted. Chinese accounts interpreting the development of mathematics and the philosophical meaning of modern mathematics in the 1950s and 60s basically followed the orthodox line of Engels' *Dialectics of Nature*.[69] Articles by the well-known mathematician GUAN Zhaozhi 關肇直 deserve brief mention here not only because his influence among Chinese mathematicians was on a par with that of HUA Luogeng, but also because his approach to the subject was typical and quite systematic.[70]

GUAN surveyed what such leading western mathematicians as Joseph Fourier, Carl F. Gauss, Bernhard Riemann, Henri Poincaré, and David Hilbert had written on

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[68] See GUAN Zhaozhi 1955b, 36.

[69] See, for example, WU and HU 1956; ZHAO Zhongzhe 1956; HU Shihua 1956; XIA Hong 1956; MO Shaokui 1957; WU Chunfei 1960; and HUANG Yaoshu 1964.

[70] GUAN Zhaozhi became a member of the Party in 1947, and was then sent to the Institute Poincaré in Paris where he studied under the guidance of the French mathematician Maurice Fréchet. GUAN returned to Beijing when the new China was founded, without having completed the requirements for a Ph.D. degree. For his biography, see FENG and ZHU 1996, 370-77.

epistemology and ontology of mathematics. After analyzing their positions according to the principal criterion of Marxism, namely that matter exists before thought, GUAN concluded that these first-rate mathematicians, although acknowledging that geometry was derived from experience and relied on experiments to check its propositions, nevertheless adopted ontological views of mathematics that were primarily idealistic, influenced by Kant's *a priori*.[71] GUAN held that geometry should not be differentiated as pure or applied, *a priori* or *a posteriori*. Instead, he believe that the theoretical and practical, the abstract and concrete, were all simply to be understood as geometry.[72]

In the first half of the 1950s, the CCP treated mathematicians rather loosely, although the party did emphasize the guiding role dialectical materialism should play as a guiding principle over mathematics. But when the Anti-rightist Campaign 反右 was unveiled in 1957, mathematicians could no longer avoid being drawn into political whirlpools. Some were branded as rightists. Some lost their jobs, and were sent to the countryside as manual laborers.[73] Although the Anti-rightist Campaign did considerable harm largely to individual mathematicians, the Great Leap Forward launched the following year caused even greater damage to mathematical research and education.

During the Great Leap Forward, which lasted about three years, mathematics was called upon to serve the interests of national production, and mathematicians were

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[71] See GUAN 1958a. Felix Klein was taken to reflect a basic materialistic position because he once said that axioms were idealizations of empirical data, and advocated the integration of theory with practice. See GUAN 1958a, 13.

[72] See GUAN 1958b.

[73] See ZHANG Dianzhou 2000, 244-48. For the Anti-rightist Campaign, especially in the field of arts and literature, see Goldman 1987a, 253-58.

expected to integrate mathematical theories with practice.[74] Applied mathematics—especially operations research and linear programming—suddenly became the first priority for mathematicians. Research in pure mathematics consequently suffered, especially in such areas as number theory, topology, and the theory of functions, which were abandoned even in the most prestigious research institution, the Institute of Mathematics of the Chinese Academy of Sciences, since pure mathematics was held to have no practical value for socialist construction.[75] The resulting over-emphasis of the utilitarian side of mathematics resulted in a lack of attention of basic theories. It was even proposed that Euclidean geometry be eliminated from high schools, and Cauchy's approach to the calculus from colleges, because they failed the test of political correctness. Both of these proposals were later enacted during the Cultural Revolution. The need to reform calculus textbooks in China to better serve political orthodoxy also played a part in efforts to translate Marx's mathematical manuscripts into Chinese.

As for the Great Leap Forward, it was soon understood to have been instead a great leap backward. Hundreds of thousands of people died of hunger, and the state's economy was at the brink of bankruptcy. The CCP was forced to reevaluate its policies in all spheres of the life of the nation. In science, the Central Committee of the CCP decreed fourteen-points on July 19, 1961, requiring immediate attention. These were a series of proposals by the Chinese Academy of Sciences concerning the very serious state of research in the natural sciences, and were intended in part to lessen the Party's grip on science. The fourth point, for example, urged that scientists should have the right to choose

[74] See for instance, HU Shihua 1958; and LIU Yangqiao 1958, 1-2, 11.

[75] See WANG Yuan 1994, 227-247, 254-263; and ZHANG Dianzhou 2000, 248-259. For a brief account of the consequences of the Great Leap Forward, see Goldman 1987b, 432-34.

their own research projects. Similarly, the tenth called for reimplementation of the hundred-flowers policy, namely the encouragement of academic freedom, without the application of such labels such as "bourgeoisie, proletariat, capitalist, or socialist" to the natural sciences.[76] Another measure meant to provide similar relief for institutions of higher education was also instituted. Over the next four years, in fact, mathematicians began to enjoy relatively greater academic freedom.

Nevertheless, despite political constraints and ideological demands, the results of Chinese mathematical research in the first seventeen years of the new China, before the Cultural Revolution, all in all, was quite impressive. More than one thousand papers were published in Chinese journals alone in the first decade of the PRC, and reflected work primarily devoted to analytic functions, analytic number theory, differential geometry, and algebraic topology.[77] Above all, Chinese results in algebraic topology and the theory of analytic functions were highly regarded, even by so prominent and critical a mathematician as Marshall Stone.[78] The extent of this achievement may in part be due to the fact that mathematics, apart from foundations, has little to do with philosophy. Moreover, the majority of working mathematicians in China were as uninterested in dialectical materialism and politics as were most mathematicians elsewhere in the world.

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[76] See WANG Yuan 1994, 261.

[77] According to a survey by ZHANG Dianzhou, the number of published papers was 983; see ZHANG 2000, 236. TSAO Chia Kuei in another study lists 1271 titles, but some are repeated and several others were written by foreign mathematicians; see TSAO 1961. For the achievement of Chinese mathematicians in particular in the 1950s, see, for example, HUA 1959; SU and GU 1959; LU 1959; TSAO 1961; Stone 1961; Fitzgerald and MacLance, 1977, 3-5; and ZHANG 2000, 236-241.

[78] See Stone 1961, 619. For instance, WU Wenjun's 吳文俊 work on characteristic classes and imbedding classes was awarded the First Prize of the National Award for Natural Sciences in 1956. This has been widely taken to be one of the major reasons why WU was later awarded the State Supreme Science and Technology Award, the highest award in China, in 2001.

The importance of the dialogue between mathematics and dialectics, between theory and practice, as it occurred in the first decade of the PRC was significant for mathematicians not only for its immediate impact, but even more importantly, for the precedent it set for the much sharper ideological debates that were to occur during the Cultural Revolution. How ideology dramatically affected mathematicians during the Cultural Revolution, especially those whose research focused on the infinite (or infinitesimals), is the subject of what follows.

Chapter 8 Karl Marx's *Mathematical Manuscripts* and Their Significance for Mathematics in China

> The fundamental principle characteristic of the [mathematical] manuscripts of Marx has essential significance even in the present day.
>
> - Sonia Alexandrovna Yanovskaya[1]

8.1 Marx's *Mathematical Manuscripts*: Their Contents, History, and Introduction to China

Marx was awarded a doctoral degree in philosophy for his work at the University of Jena on Greek atomism in 1841. Thus, his education was primarily as a philosopher, rather than as a scientist or mathematician. His formal mathematical education did not go beyond the high school level. However, Marx had a special interest in mathematics, and spent a great deal of time studying it in the last two decades or so of his life, primarily because it was relevant for his writing of *Das Capital*. Marx realized he needed a good foundation in mathematics for the applications he had in mind for his dialectical materialism. As Marx noted in a letter to Friedrich Engels in 1858: "In elaborating the PRINCIPLES of economics I have been so damnably held up by errors in calculation that in DESPAIR I have applied myself to a rapid revision of algebra. I have never felt at home with arithmetic. But by making a detour via algebra, I shall quickly get back into the way of things."[2] As for higher mathematics, especially the differential and integral calculus,

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[1] Quoted from the English translation of Marx's *Mathematical Manuscripts* by Aronson and Meo 1983, xv.

[2] See Marx's letter of January 11, 1858, to Engels. Quoted from Marx and Engels 1983, 40: 244.

Marx regarded these as the "touchstone of the application of the method of dialectical materialism to mathematics."[3]

It was Engels who first brought Marx's mathematical studies to public attention when he said that Marx had made some "independent discoveries" in mathematics in a speech Engels delivered at Marx's graveside at the Highgate Cemetery, London. In his preface to the second edition of his *Anti-Dühring* (1885), Engels noted that Marx had left some "extremely important mathematical manuscripts," and also hoped to edit and publish them along with his own work on the dialectics of nature.[4] Nevertheless, this wish was never realized. Marx's manuscripts, together with those of Engels himself, were left in the possession of the German Social Democratic Party (SDP) after Engels' death in 1895. In order to assess the value of their manuscripts, Eduard Bernstein, who was Engels' close friend and leader of the SDP, consulted Albert Einstein, among others, for their opinions.[5] Discouraged by the negative assessments he received, Bernstein, a revisionist in the eyes of Lenin and his follows, did not place much value on Marx's mathematical manuscripts, and took no further steps to publish them. Nevertheless, The Karl Marx-Friedrich Engels Institute, founded in Moscow in 1920, secured photocopies of 865 quarter-sheets of Marx's manuscripts from the Archives of the SDP in 1923, thus carrying out in part Lenin's instructions to collect the manuscripts of both Marx and Engels. The manuscripts were later augmented by several hundred more sheets scattered elsewhere.

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[3] Quoted from Kennedy 1977, 316.

[4] Engles, 1966, 17.

[5] Engles, 1940, xiv.

In order to reassess the scientific value of the manuscripts with reasonable accuracy, the Institute in 1925 commissioned the German mathematician Emil J. Gumbel, along with two Russian mathematicians, R. Mateika and R. S. Bogdan, who worked as his assistants in helping to investigate Marx's manuscripts. The result of Gumbel's research, "On the Mathematical Manuscripts of K. Marx," was published in 1927 in the Russian journal, *The Chronicle of Marxism*. Having combed through all of the manuscripts available to him, Gumbel classified them into four categories: calculation sketches, reading notes, outlines of studies, and Marx's own completed work. In his published report, Gumbel also identified the primary sources of Marx's notes, and categorized the main contents of the manuscripts under arithmetic, geometry, algebra, differential calculus, and Taylor's theorem. Furthermore, he pointed out that Marx's independent work and notes also dealt with the foundations and history of the differential calculus. Gumbel briefly presented an overall picture of the manuscripts, but did not give a very high opinion of them.[6]

While Gumbel was conducting his investigation, Ernst Kol'man, then president of the Soviet Association of the Scientific Institute of Natural Science, head of the Marx Study Center at The Karl Marx-Friedrich Engels Institute, and a "spokesman for Stalinist Marxism," also showed considerable interest in the manuscripts.[7] It is no surprise that Kol'man was dissatisfied with Gumbel's article, and with respect to Marx's manuscripts, he criticized Gumbel for being "unable to appreciate completely either the importance of

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[6] Gumbel 1927, 56-60. For Gumbel and his study of the manuscripts, see Vogt 1995, 26-41.

[7] See Vucinich 2000, 54; Ford 2002, 348-349. Kol'man is also briefly introduced in an introduction by P. G. Werskey to the new edition of *Science at the Crossroads* (London: Frank Cass, 1971); see Kol'man, 1971, xvi.

their publication or their philosophical and historical-mathematical significance."[8] Not only did Kol'man advocate and publicize the importance and significance of the manuscripts, but at the Second International Congress of the History of Science and Technology held in London in 1931, he presented a paper entitled: "The Present Crisis in the Mathematical Sciences and General Outline for Their Reconstruction." In this article, Kol'man discussed the contemporary crisis of "bourgeois" mathematics from a Marxist point of view, and demanded that mathematicians both in the USSR and abroad use dialectics as Marx had set the example in his mathematical manuscripts to reinterpret mathematics and to fix the crisis. As Kol'man wrote, "The hitherto unpublished writings of Marx dealing with mathematics and its history…are of tremendous methodological importance."[9] During the Congress, Kol'man also delivered "Short Communications on the Unpublished Writings of Karl Marx Dealing with Mathematics, the Natural Sciences, Technology and the History of these Subjects," in which he gave brief summaries of the manuscripts.[10] In another conference paper delivered at the International Congress of Mathematicians held in Zürich, Switzerland, the following year, Kol'man again emphasized the importance of both Marx's account of the historical development of the calculus and Marx's dialectical approach to its foundations for the history and philosophy of mathematics.[11]

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[8] Kol'man 1968, 101-112. See the English translation, Aronson and Meo 1983, 217-234, esp. 226.

[9] See Kol'man 1971, 215-229, esp. 227.

[10] About the manuscripts, Kol'man writes: "Mathematical manuscripts consisting of 31 different calculations and excerpts from arithmetic, algebra, analysis and geometry, and of 19 drafts and studies for independent mathematical works. In addition, there are applications of mathematics to the problems of political economy, differential rent, the process of circulation, the rate of surplus value and the rate of profit, the problem of crises." Quoted from Kol'man 1971, 234.

[11] Kol'man 1932, 349-351. A revised and extended version of this paper was published later under the same title in *Archeion*: *Archivio di storia della scienza*; see Kol'man 1933, 379-384.

8.2 Russian Collations and Western Translations of Marx's *Mathematical Manuscripts*

At Kol'man's suggestion in 1931, the Karl Marx-Friedrich Engels Institute in Moscow established a study group to carry out further research on Marx's mathematical manuscripts and to translate and edit them for publication. He also recommended that the Institute enlist the mathematician Sofia A. Yanovskaya to lead this group, which included D. A. Raikov and A. I. Nakhimovskaya.[12] Within two years the group had completed its first phase of intensive research and prepared a Russian translation of Marx's original accounts, including three of the most important, although not yet published manuscripts: "The Derivative and the Symbolic Differential Coefficient," "The Differential and Differential Calculus," and "A Historical Sketch."[13] These translations were published first in *Under the Banner of Marxism*, the chief philosophical journal of Soviet Marxism, and later were reprinted simultaneously as a book entitled *Marxism and Science*, which appeared in the politically meaningful year of the fiftieth anniversary of Marx's death.[14] On both occasions, the translations were accompanied by a detailed introduction entitled: "On the Mathematical Manuscripts of Karl Marx" written by Yanovskaya, who was in full agreement with Kol'man about the great importance of the manuscripts for the history and philosophy of mathematics.[15]

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[12] See Kol'man 1983, 226.

[13] The original manuscripts do not bear these headings, which were added by Yanovskaya's group.

[14] *Pod znamenem Marksizma* no. 1 (1933), 15-73. *Marksizm i Estestvoznanie* (Moscow: Partizdat, 1933), 5-61.

[15] Yanovskaya 1933a, 74-115; and 1933b, 136-180.

Publication of the Russian translations from excerpts of Marx's manuscripts immediately stimulated considerable interest among Russian mathematicians. For instance, within a year Valère I. Glivenko argued that Marx treated the differential as an operative symbol, anticipating the French mathematician Jacques Hadamard who advanced a similar idea in connection with applications to functional analysis in 1911.[16]

Unfortunately, the outbreak of World War II temporarily brought a halt to work on Marx's manuscripts in the USSR, but interest revived immediately thereafter. In 1947, the mathematician L. P. Gokieli published a pamphlet entitled *Karl Marx's Mathematical Manuscripts and Questions on the Foundations of Mathematics*, wherein Gokieli discussed the manuscripts in considerable detail.[17] In the meantime, Konstantin A. Rybnikov, one of Yanovskaya's graduate students, joined the study group investigating the manuscripts. Rybnikov conducted a painstaking investigation of the sources of Marx's manuscripts, leaving no stone unturned, as his doctorate thesis. At the second All-Soviet Symposium on the History of Natural Sciences held in Moscow in May of 1953, Rybnikov delivered a lecture on the manuscripts. He also delivered a similar lecture the following year at a national conference sponsored by the Moscow Mathematical Society. Meanwhile, Rybnikov wrote an article on the manuscripts for the second edition of the *Bolshevik Soviet Encyclopedia*.[18] Thanks to the interest and support of the prominent mathematician and historian of mathematics G. F. Rybkin, study of Marx's mathematical manuscripts in the USSR received renewed momentum in the early

[16] Glivenko 1934, 79-85.

[17] Gokieli 1947.

[18] Rybnikov 1955, 197-199; and Rybnikow 1954, 496-498. The encyclopedia article, however, has been deleted from the third and subsequent editions.

1960s. As a result, the manuscripts were published in a dual German-Russian edition in 1968, along with a lengthy introduction, extensive notes, and an inventory of the manuscripts and their sources, written by Yanovskaya and the other members of her group.[19]

Following publication of this definitive volume, interest in the manuscripts soon became worldwide.[20] This is evident from the fact that the first part of the Yanovskaya's edition was reedited or translated, and then published in major languages, one after another. First, a new German edition was reedited and published in 1974, followed by an Italian translation. An English translation was completed in 1983, followed by a French translation the following year. Most recently, a new English translation was produced by an Indian scholar and published in 1994.[21] Moreover, articles and even pamphlets devoted to Marx's manuscripts began to appear in America and Europe.[22]

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[19] See Marx 1968.

[20] Before 1968, Dirk J. Struik was one of the few scholars outside the USSR who showed an interest in and appreciation of the manuscripts. See Struik 1948, 181-196.

[21] *Mathematische Manuskripte*, ed. with an introduction and commentary by Wolfgang Endemann (Kronberg Taunus, BRD: Scriptor Taschenbücher, 1974); *Manoscritti Matematici*, trans. and ed. by Francesco Matarrese and Augusto Ponzio (Bari: Dedalo Libri, 1975); Marx's "On the Concept of the Derived Function" was translated into Italian three years earlier by L. Lombardo Radice under the title "Sul concetto di funzione derivata," *Critica Marxista – Quaderni* 6 (1972), 278-286; *Mathematical Manuscripts of Karl Marx*, trans. by C. Aronson and M. Meo (Clapham, London: New Park, 1983); *Les Manuscrits Mathématiques de Marx*, trans. A. Alcouffe (Paris: Éditions Économica, 1985); and *Karl Marx, Mathematical Manuscripts*, trans. and ed. by Pradip Baksi (Calcutta: Viswakos Parisad, 1994). For a review of both the German edition and the Italian translation, see Kennedy (1976), 490-494. For that of Baksi, see Mobasher, 1996.

22 For example, Miller 1969, 649-659; L. Lombardo Radice 1972, 273-277; Kennedy 1977, 303-318; Moretto 1978, 201-218; Guerraggio and Vidoni 1982; Drago 1982, 107-116; Magnani 1983, 55-89; Vogt 1983, 50-61; Gerdes 1985; and Dauben 1999, 799-810; 2003; 2004.

8.3 The *Mathematical Manuscripts* of Karl Marx in China

The first introduction of Marx's manuscripts to China was not made via the USSR, but came rather from Japan.[23] When excerpts from Marx's manuscripts were published in Moscow in 1933, among the first to take an interest was the Japanese Marxist TAMAKI Hidehiko, who quickly translated them into Japanese. Meanwhile, the Japanese scholar KONNO Takeo, using the translated Russian excerpts and the article by Yanovskaya, wrote an article of his own: "An Historical Approach to Natural Sciences: the Differential Calculus of Karl Marx," wherein he discussed Marx's contributions to the differential calculus and foundations of the subject in its historical context.[24] This article later was included with Tamaki's translation, along with his introduction, in a pamphlet entitled: *Marx's Mathematical Manuscripts*.[25] Nearly twenty years later, this Japanese translation served as the basis for the translation of Marx's mathematical manuscripts into Chinese.

The first study of Marx's mathematical manuscripts to be translated into Chinese was most probably an article by KONNO Takeo. In a note to the translation dated February 10, 1935, the translator HUA Rong 華蓉, who had returned as a student from Japan to China, wrote:

> The Russian translation of the portion of Marx's mathematical manuscripts edited and interpreted by Yanovskaya was published last year, and now has been translated into Japanese this year. The Russian translation has received the attention of some Japanese scientists and scholars. I am wondering what our dignified philosophers, scientists, and mathematicians think of that. Recently I thought about translating Marx's "The Foundation of the Differential Calculus

[23] For a brief account of the Chinese introduction of Marx's manuscripts, see WU Yanfu, LIU Dachun, and XUE Yuchuang 1983, 344-352.

[24] KONNO 1934, 27-50.

[25] See TAMAKI and KONNO 1949.

> and Dialectical materialism" into Chinese, but it is too long and premature to publish it as a pamphlet. Accordingly, I translated this article by KONNO instead, and introduce it to our countrymen because I think it is necessary [for our people] to acquire a brief idea [of Marx's mathematical manuscripts].[26]

Due to the fact that this translation was published in a journal that only appeared irregularly, *Kexue Luncong* (*Science Series*), whose mission was to promote Marxism in China (but ironically was edited chiefly by YE Qing, one of the protagonists mentioned above in the debate over formal versus dialectic logic in the 1930s), HUA's translation did not receive as much attention as the translator had hoped. In the 1930s and 40s, CHEN Xiaoshi, in her book of 1948 with the same title as Engels' *Dialectics of Nature*, also commented briefly on Marx's mathematical manuscripts. In particular, she pointed out that Marx inherited the critical Hegelian concept of the differential, applied dialectical materialism to the differential calculus, and made important contributions to the history of mathematics. Nevertheless, Marx's mathematical manuscripts remained known only to a very few in China.[27]

The mathematical manuscripts themselves were again brought to the attention of Chinese readers more systematically in the early 1950s. This was chiefly due to a personal interest in the subject taken by XU Mofu 許默夫, who worked in the research section of the Bureau of Chemical Industry of the Northeast Industry Department. In 1948, XU wrote an article commemorating the 131st anniversary of Marx's birthday for *Dongbei Ribao* (Northeast Daily), a major newspaper in the liberated Northeast of China. In his article, XU mentioned Marx's mathematical work and its importance from a

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[26] The original date of this note was February 10, 1934. However, this must be wrong because KONNO's article was not published until June of 1934. The quotation made here may be found in KONNO 1935, 48.

[27] See, WU, LIU, and XUE 1983, 345.

dialectical point of view.[28] Based on Yanovskaya's article and the Russian translation published in 1933, XU outlined Marx's dialectical interpretation of the foundations of the differential calculus in an article entitled "Marx's great contributions to Mathematics," which appeared in the journal *Ziran kexue* (Natural Science).[29] He also stressed the significance of dialectical materialism for its influence on pure mathematics, as well as for the history of philosophy of mathematics. XU went on to write two more articles for *Shuxue Tongbao*, one of two journals then sponsored by the Chinese Mathematical Society, in a role similar to that of the *American Mathematical Monthly* in the U.S. In these articles XU addressed the nature and importance of Marx's manuscripts for Chinese mathematicians and mathematics educators.[30] In his first article, XU described why and how Marx had studied mathematics, especially the differential calculus, and outlined the main sources of Marx's manuscripts on the differential calculus. In the second article, he presented a synopsis of Marx's account of the three stages of the development of the differential calculus, concluding as follows:

> Marx's analysis of the foundation of the differential calculus profoundly reveals the metaphysical nature of [the works of those mathematicians] from Newton and Leibniz to Lagrange and their successors, and leaves no ground of higher mathematics for any idealistic and scholastic attempts to stand on. Although mathematics has been advanced a great deal since Marx's death seventy years ago, his manuscripts are still a model of applying dialectical materialism to solve specific scientific problems. Especially, his manuscripts on historical criticism [of the differential calculus] set an excellent example for studying the history of natural sciences.[31]

XU also discussed the ways in which Marx used dialectics to bridge the gap between elementary and higher mathematics, and tried to create a dialectical unification, in

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[28] XU Mofu 1949.

[29] XU Mofu 1951, 354-358.

[30] XU Mofu 1953a, 1-5; and 1953b, 31-36.

addition to presenting Marx's dialectical explanation of the algebraic differential symbols.[32]

By the mid 1950s, Chinese mathematicians, mathematics educators, and philosophers of mathematics were all acquainted with both the nature of Marx's mathematical manuscripts, and their significance as described by Kol'man, Yanovskaya, and their Russian colleagues through the above-mentioned articles by XU, and Chinese translations of articles by Kolomogorov, Glivenko, and Rybnikov, respectively.[33] At the time, however, Chinese mathematicians and mathematics educators did not pay much attention, if any, to the manuscripts. It is safe to say that they did not use Marx's dialectical views of the differential calculus to interpret the foundations of differentials, nor did they incorporate them into their teaching of the calculus. But as China entered into one of the most catastrophic periods of its history (1966-1976), things began to change dramatically. Marx's mathematical manuscripts were just beginning to receive significant and serious attention.

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[31] XU Mofu 1953b, 36.

[32] XU Mofu 1955, 37-41.

[33] Rybnikov's "Matematicheskie rukopisi Marksa" (Marx's Mathematical Manuscripts) [Rybnikov (1954), 197-199] was translated into Chinese by Zhao Mengyang and published under the title "Makesi de shuxue shougao" (Marx's *Mathematical Manuscripts*), in *Shuxue Tongxun*, see Rybnikov 1956, 1-5. Meanwhile, Alexander N. Kolomogorov, in his article entitled "Mathematics" for the second edition of the *Bolshivic Soviet Encyclopedia*, also described the nature and importance of Marx's mathematical manuscripts. See *Bol'shaia Soveskaia Entsiklopediia 26* (1954), 478. As noted in the previous chapter, this article was translated into Chinese and published as a pamphlet under the title: *Jiaxue: Shuxue* (*Teaching: Mathematics*) (Beijing: Gaodeng Jiaoyu Chubanshe, 1956). The translation of Kolomogorov's article also played a role in disseminating Marx's manuscripts in China.

8.4 Contexts of the Chinese Translations of the *Mathematical Manuscripts* in the 1970s

The Cultural Revolution, according to the official view of the CCP, began in May of 1965 and ended slightly more than a decade later in October of 1976. Scholars, however, are generally agreed that the Revolution actually began in November of 1965 with the publication of a review article entitled: "Ping Xinbian Lishiju Hairui Baguan" (On the New Historical Peking Opera *Hairui Dismissed from Office*)" by YAO Wenyuan 姚文元, then a propagandist at the *Liberation Daily* of Shanghai.[34] Following this notorious article, criticism in other cultural areas beyond the arts soon spread to political circles with the immediate result that, on May 16, the Cultural Revolution Group was established directly under the standing committee of the Politburo of the Central Committee of the CCP.[35] YAO was soon made one of the five members of this Group, chiefly because of his review article. When the Eleventh Plenary Meeting of the Eighth Congress of the Central Committee of the CCP was held on August 8, 1966, a "Decision of the Central Committee of the Chinese Communist Party Concerning the Great Proletarian Cultural Revolution" was passed which outlined the guiding principles of the Revolution. This decision, often referred to as the "Sixteen Articles," was to change dramatically the Party's scientific and educational policies not only with regard to changing realities within these fields, but also with respect to the changing requirements of politics. While a full examination of these relationships is beyond the task of the present work, some peculiarities of the decision must be considered, for they were to

[34] See Zhongguo Gongchandang 1981, 32. Most recently, HAO Ping argues that the Revolution began even earlier with the Socialist Education Campaign at Beijing University in July of 1964. He also provides a survey of other views on the starting point of the Cultural Revolution. See Hao 1996, 66-86. YAO's article was first published in *Wenhui Daily* on November 10, 1965; see YAO 1965.

have a profound impact on the motivation for Chinese translations of Marx's mathematical manuscripts, and served to set the tone for the subsequent nationwide discussion of the manuscripts in greater detail. The first of the "Sixteen Articles" outlined the general goals of the Revolution:

> Our current goals are to struggle against and overthrow those persons in authority who are taking the capitalist road; to criticize those *reactionary bourgeois academic authorities*, and *the ideologies of the bourgeoisie* and all other exploiting classes; and to *reform education*, literature and arts as well as all other parts of the superstructure unfit in the socialist economic bases, in order to consolidate and develop the socialist system.[36]

Of the three italicized objectives, the first two are related and pertinent to problems of the infinite in Chinese mathematics. As for the third objective, educational reform, Article 10 of the Decision further called for the reform of education, transformation of the principles and methods of teaching, and an end to the domination of the schools by bourgeois intellectuals, especially at colleges and universities. Moreover, in addition to shortening the period of schooling, the Decision mandated fewer but better courses, and a thorough revision of all teaching materials, in some cases beginning with the simplification of anything that seem too complicated.

In an attempt to follow the Revolution's guidelines for educational reform, colleges and universities immediately ceased to function properly.[37] Beginning in the fall of 1966, colleges and universities actually discontinued recruitment of new students. When students were again admitted to universities in 1970, the new recruits were not

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[35] See, *Renmin Ribao*, May 17, 1966.

[36] For the Chinese text of the Decision, see *Hongqi* 10 (August 10, 1966), 1-9. Emphasis added.

[37] For a general account of educational reform in this period, see, for example Theodore His-en CHEN 1974; Julia KWONG 1988; and Hayhoe 1996, in particular chapter 3: "The Socialist Story, 1949-1978," 73-113.

fresh high school graduates, but previous workers in farms and factories, or members of the People's Liberation Army (PLA). The criteria for selecting students were no longer based on merit as determined by regional or nationwide college-entrance examinations, as had been the case between 1950 and 1965, but now depended upon political attitude, family background, and personal connections.[38] Accordingly, the new peasant-worker-solider students required new simplified textbooks to accommodate their usually insufficient preparation for university study.

Textbook reform is a constant concern almost everywhere, not only to up-date what is taught, but to reflect political, cultural, and social changes as well. The new China, of course, took up this crusade with a specific political agenda in mind. In the 1950s, as politics demanded, China was influenced pedagogically by Russian textbooks. But as Sino-Soviet relations deteriorated in the early 1960s, Soviet textbooks were abruptly abandoned. As a result, the Chinese government put textbook reform at the top of its agenda for education. As the Cultural Revolution got underway, Chinese leaders saw textbooks as an important battlefield of the ideological struggle against both Western idealism and Soviet revisionism. They called for new textbooks that adopted the point of view of dialectical materialism, uniting theory and practice to produce teaching materials that would be revolutionary, practical, and advanced.

Mathematics textbooks were no exception. Here we consider, to take but one example, college textbooks on the differential and integral calculus. Since they were first introduced to China in the mid-19th century, calculus texts had been primary Western, especially American. In the 1950s, this changed abruptly, and calculus textbooks in

[38] The importance of personal connections in this respect has also been observed by Susan L. Shirk and Jonathan Unger, respectively. See Shirk 1982 and Unger 1982.

China were predominantly translations from Russian models. These differed considerably from their American counterparts, and gave special emphasis to theories and axiomatic systems.[39] Russian textbooks, however, at least in the eyes of the workers and members of the PLA propaganda team for Mao's thought stationed at Qinghua University (one of the leading and most prestigious universities in China), represented a "system of idealist and metaphysical 'axioms'." They specifically criticized the concepts of the differential and integral calculus in standard textbooks for being "derived from piles of axioms and theorems that were very mysterious and unfathomable." All this was noted in a featured article in a special issue of *Hongqi* devoted to educational reform: "Striving to Build a Socialist University of Science and Engineering."[40] Of particular interest is an example the team offered as to how the concepts of the differential and integral calculus should be derived from the experience of workers:

> [W]hen a bench worker uses a file to round off a piece of metal, every stroke of the file makes short straight lines, which finally combine to form curves. This process of turning the whole into parts and parts into the whole vividly illustrates the concepts of differential and integral calculus. After studying this the worker-students said: "There is nothing really mysterious about calculus; a mere file pierces the paper obscuring the window. Higher mathematics has again returned to the hands of we laboring people.[41]

This way of introducing the two most fundamental concepts of the calculus came to be known as "The Method of Piercing Window-Paper," and was to become the guiding principle of a new calculus textbook used by some members of the faculty who taught the subject at Qinghua University. The textbook, entitled *Weijifen* (The Differential and

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[39] For an account of mathematics education in China from 1949 to 1966 in general, and in teachers colleges in particular, see Swetz 1970, 1097-1103.

[40] See Anon. (IV) 1970, 5-19, esp. 16-17. An English translation of this article has been made by Peter J. Seybolt; see Seybolt 1973, 272-300. The quotation is from Seybolt 1973, 295.

[41] Seybolt 1973, 295. For the original Chinese text, see Anon. (IV) 1970, 17.

Integral Calculus), was published in April 1971, and immediately became popular across the country.[42] In the preface to the textbook, the authors wrote:

> [While] compiling the *Weijifen*…We used Chairman Mao's two splendid philosophical works, *On Practice* and *On Contradiction*, as weapons to expose and analyze contradictions [in the calculus], and to criticize idealism and metaphysics in the old textbooks. [We] have been careful to derive mathematical concepts and rules from specific production experience, and to highlight both the dialectical relationship between the fundamental contradiction of the differential and the integral, and the idea of mathematics serving practice. [We also] have tried to make this new calculus textbook a tool of understanding and means of reforming the world for laboring people.[43]

The political objectives and guiding principles of this textbook are clearly stated. As for the technical parts, for instance the concepts of the differential and integral, the book avoids using axiomatic systems and the conventional epsilon-delta approach. Instead, fragmentary quotations from both Mao and Engels are used to analyze some specific examples through which the book stresses that the basic ideas of the differential and the integral may be derived from the practical experience of workers.[44]

Following this example set by Qinghua University, in November of 1972 a writing group at Shanghai Normal University designed another calculus text known as the "3-in-1 textbook." This was specifically aimed at mathematics students in teachers colleges.[45] Concerning the reform of mathematics textbooks in China at this time,

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[42] See Qinghua Daxue Jichuke "Weijifen" Bianxiezu 1971, 27-77, esp. 28.

[43] *Ibid*., i.

[44] *Ibid*., 28-32.

[45] In April of 1972 Shanghai Huadong Shida (East China Normal University), Shanghai Shiyuan (Shanghai Teachers College), Shanghai Bangong Bandu Shiyuan (Shanghai Half-Work Half-Study Teachers College), Shanghai Jiaoyu Xueyuan (Shanghai Education College), and Shanghai Tiyu Xueyuan (Shanghai Physical Education College) merged to become: Shanghai Shida (Shanghai Normal University). In April of 1978, the merged one split. The so-called 3-in-1 group was meant to combine peasants, workers and teachers in one group. For details of the textbook on the calculus, see Shanghai Shida 1973.

Chandler Davis, who visited China in 1973 as a member of an American delegation of mathematicians almost immediately after President Nixon's historical trip, observed:

> The curriculum [of mathematics] includes problems arising from current industrial practi[c]e, which teacher[s] and students are to attack jointly. (At least in principle it does; no case was reported to me of an under-graduate class solving a new practical problem.) Organized expositions of standard topics are also included, but they are being completely re-thought to bring them closer to practi[c]e. Epsilon and delta are banished from [the curriculum].[46]

Another American mathematician, George D. Mostow, who was a member of a group that also visited China in 1973 from Yale University, captured succinctly the political spirit of Chinese mathematical research and mathematics education that prevailed at the time in China as follows:

> The centrally-directed ideology in China determines the guiding principles in mathematics as well as the other academic disciplines. But the post-Cultural Revolution reforms pose more of a pedagogical challenge in mathematics than elsewhere. The central goal is to design a program that serves the people—workers, peasants and soldiers.[47]

Given the political realities of educational reform in the wake of the Cultural Revolution, Chinese mathematics educators did their best to meet the needs of largely unprepared college students, while applying Marxist philosophy of mathematics as well as Mao's thoughts in the textbooks they wrote. The calculus textbooks produced at Qinghua University and Shanghai Normal University are but two examples in this trend. It was also natural that concomitantly, some mathematics educators also turned their attention to Marx's mathematical manuscripts, which they believed might offer guidance

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[46] Chandler Davis, "A Mathematical Visit to China" pp. 7-8, in box four, the folder for Albert Feuerwerker (1973), National Archive on Sino-American Relations Records, 1971-1984, The Bentley Historical Library of the University of Michigan at Ann Arbor, Michigan. Material quoted here is on p. 7b. The article also can be found in Davis 1972.

[47] See G. D. Mostow, "Impressions of China," a report of the Yale University Delegation to China, pp. 28-29, in box four, the folder for Albert Feuerwerker (1973), National Archive on Sino-American Relations Records, 1971-1984, The Bentley Historical Library of the University of Michigan at Ann Arbor, Michigan. See p. 28c.

in writing revolutionary calculus textbooks. Indeed, the need to revise calculus textbooks served to generate an interest in Marx's manuscripts, which in turn influenced this reform, including teaching of the calculus and a better understanding of the history of the calculus as well.

In addition to textbook reform, mass criticism of the "bourgeois" sciences as called for in the above-mentioned guiding principles of the Cultural Revolution was also launched.[48] On February 1, 1968, the *Wenhui Daily*, the most popular newspaper among intellectuals in China, published an article entitled "Swift Change for the Current Situation in the Fields of Science and Technology" by HE Gechang 何革昌. HE's basic point was both simple and obvious: since the theoretical systems of the modern natural sciences were fabricated by bourgeois scientists in capitalist society, they should be firmly rejected.[49] This article signaled a movement in China against Western science similar to what had already happened in the Soviet Union during its own Cultural Revolution in the 1920s.

Among Western sciences and prominent scientists, the theory of relativity and its creator Albert Einstein were first singled out for criticism by CHEN Boda 陳伯達, a leading Marxist theorist and director of the CCP's Central Group of the Cultural Revolution from May of 1966 to August of 1970. CHEN doubtless singled-out relativity for several reasons: 1) the theory was the most "infamous" one, exercising a profoundly "vicious" impact on all of the sciences, natural and social, in the twentieth century; and 2)

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[48] Before the Cultural Revolution, there were certainly earlier criticisms of the natural sciences. But the scale was usually small and criticism was often targeted on a specific scientist's research orientation and philosophical outlook. For an overview of criticisms against the natural sciences in China, including mathematics, in the late 1950s, see HU and GOU 2004.

[49] See *Wenhui Daily* February 1, 1968. The given name of the author of this article means "prosperous revolution." It is obviously a fabricated pseudonym for a powerful organization or person.

Einstein was considered a Machist by Soviet critics, and a substantial amount of criticism against Einstein and his theories had already been published by Soviet Marxists in the 1920s. Following CHEN's lead, the Chinese Academy of Sciences selected leading physicists from the Institute of Physics, the Institute of Atomic Energy, the Institute of High-Energy Physics, as well as mathematicians from the Institute of Mathematics to comprise a group charged with drawing up a global criticism of Einstein and his theories.

Meanwhile, ZHANG Chunqiao 張春橋 and YAO Wenyuan 姚文元, both of whom were also members of the Central Group of the Cultural Revolution, and later together with MAO's widow JIANG Qing 江青 and WANG Hongwen 王洪文 were called the "Gang of Four," also selected scientists—primarily from Fudan University in Shanghai—to form "The Shanghai Mass Criticism Group on Natural Sciences," primarily to compete with CHEN over who had more influence in the field of ideology.

The "collaboration" of the two groups in Beijing and Shanghai soon brought "fruitful" results which were published on June 15, 1970, as a book: *Xiangduilun Wenti Taolun* 相對論問題討論 (*Discussion over Problems on Relativity Theory*) vol. I.[50] Less than two months later, CHEN was ousted as director of the Central Group of the Cultural Revolution by MAO because CHEN had insisted upon re-establishing the position of President of the People's Republic of China for which LIN Biao 林彪 was the only possible candidate. CHEN had done so during the Party Plenum which was held in summer of 1970 at Lushan, Jiujiang, Jiangxi Province. Immediately after the Lushan meeting, a new wave of studying Marxism swept across China as MAO called for all citizens to study the works of Marx, Engels and Lenin directly, *not* through their

[50] For contexts of the book as well as a detailed account of criticism of the theory of relativity and Albert Einstein during the Cultural Revolution, see QU and XU 1984/85; and HU Danian 2005, 152-167.

interpreters like CHEN. As a result, the major works of these revolutionary mentors were reprinted for all to read, and a new Chinese translation of Engels' *Dialectics of Nature* was also made available by the Central Translation Bureau for the works of Marx, Engels, Lenin and Stalin.[51] Marx's *Mathematical Manuscripts* had just appeared on the scene, waiting to be translated.

Meanwhile, the group criticizing Einstein and his theory of relativity at the Chinese Academy of Sciences collapsed with the fall of its sponsor, CHEN Boda. The entire movement, however, regained strength when YAO Wenyuan became the new czar of ideology. The Shanghai Mass Criticism Group on Natural Sciences, YAO's most important propaganda tool, launched a full assault against relativity theory and Einstein. Now, in addition to attacking physics, the Group was ready to move on to criticize a broad spectrum of western science and theories of cosmology such as the Big Bang, Norbert Weiner's cybernetics, Mendelian genetics, Linus Pauling's resonance theory of chemical bonding, and of course, a major target in mathematics: axiomatics and logic.

The power struggle between MAO and his successor as guaranteed by the Constitution, LIN Biao, ended when the latter fled China and died in an airplane crash in Mongolia on September 13, 1971. Subsequently, Premier ZHOU Enlai 周恩來 became the political archenemy of the "Gang of Four." Meanwhile, among many issues to be faced was the integration of theory and practice, which seemed to contend with each other. When the Cultural Revolution was launched in 1966, colleges were immediately closed. Research scientists and college professors were ordered to work in factories or farms to put their knowledge in practice. Scientific journals ceased publication. Basic

[51] See ZHU Lan 1983, 129-130.

theoretical studies and research were condemned. Within a few years, this intolerable situation prompted responsible scientists to voice their worries about the future development of science and technology, and the future of China as a whole. Through different channels, they reported to Premier ZHOU Enlai that scientific theories should not be ignored, and that basic research was just as important as practice. In responding to such requests from the Chinese Academy of Sciences, in January of 1970 ZHOU issued instructions to the following effect: "It is necessary to bring scientific research to a higher level on the basis of going deep into reality on a wide scale. Do some basic research and elevate practice to the theoretical level it ought to enjoy." [52] Despite these directives from ZHOU Enlai, basic researches continued to be ignored.

In the summer of 1972, YANG Chen-Ning 楊振寧, a 1957 Nobel laureate in physics who spent most of his academic career in the United States, at the Institute for Advanced Study in Princeton, visited Beijing University and the Institute of Physics of the Chinese Academy of Sciences. He noticed the serious lack of studies and research on basic sciences even in China's top research institutions, and he consequently wrote a letter to Premier ZHOU, voicing his concerns. On July 14, 1972, when Premier ZHOU received YANG and other members of a delegation of visiting Chinese American scientists, he praised YANG in particular for his expert advice. He also immediately instructed ZHOU Peiyuan 周培源, a prominent physicist, Vice-Chairman of the Chinese Association for Science and Technology, and Vice Chairman of the Revolutionary Committee of Beijing University, to write an article promoting the importance of basic scientific research. Following the Premier's directive, ZHOU Peiyuan wrote a paper:

[52] Quoted from ZHOU Peiyuan 1977, 3.

"Some Personal Opinions about the Revolution of Education in Science Departments in Universities," in which he used examples from history of science, including mathematics, to argue that basic research is essential for the development of science, and that theory and practice should be integrated and well balanced. The article was intended for publication in the *People's Daily*, but was rejected there and eventually appeared in the *Guangming Daily* on October 6, 1972.[53]

The "Gang of Four" believed the purpose of the article was to oppose any revolution in Chinese education, and regarded it as aimed directly at them. They also suspected, rightly, that the paper by ZHOU Peiyuan was supported by their political rival, Premier ZHOU Enlai. As a result, vehement attacks on the paper and the value of research on basic science which it supported were immediately launched by the "Gang of Four" and their associates. To provide a "theoretical" basis for their assault, YAO Wenyuan advanced the following "theory," according to which: "The foundation of basic theories of the natural sciences is Marxist philosophy. Marxism is the most fundamental theory, without which the natural sciences would be out of the question."[54]

In order to provide new demonstrations that Marxism is "really" the most fundamental theory of "proletarian" sciences, and above all to further combat relativity theory and other "bourgeois" sciences, a new journal, *Ziran Bianzhengfa Zazhi* 自然辯證法雜誌 (*Journal for the Dialectics of Nature*), was launched on October 26, 1972, in Shanghai

[53] See ZHOU Peiyuan 1977, 4. For the article see the *Guangming Daily*, October 6, 1972, pp. 1-2.

[54] Quoted from Theoretical Study Group of the Chinese Academy of Sciences 1977b, p. 141. For the roles of YAO Wenyuan and others who had Shanghai connections and played roles on the national scene in the 1970s, see Goodman 1981.

as a propaganda organ of the "Gang of Four."[55] From June, 1973, when the first issue appeared, until the downfall of the "Gang of Four" in October of 1976, the journal has published thirteen issues in all. Among the articles to be found in the *Journal for the Dialectics of Nature* is a Chinese translation of Marx's *Mathematical Manuscripts*.

8.5 Translations of the *Mathematical Manuscripts*

Translating Marx's *Mathematical Manuscripts* had multiple objectives. Firstly, it was to provide a piece by Marx for Chinese readers to study, in response to a call by MAO. It was considered politically correct to establish the image of Marx as a mathematician. Secondly, the manuscripts provided a model for applying materialist dialectics to study the foundations of the differential calculus. That in turn was expected to help develop foundations for other branches of mathematics, or for other natural sciences. Thirdly, the manuscripts might provide insights for calculus curriculum reform, and for providing exampled for criticizing idealism and metaphysics in pure mathematics.

In the fall of 1970, with these objectives in mind, a number of mathematicians at Fudan University, including the renowned SU Buqing and GU Chaohao 谷超豪, formed a group to study and translate Marx's mathematical manuscripts.[56] The source they used was the Japanese pamphlet entitled "*Marx's Mathematical Manuscripts*" published in 1948. The pamphlet, in two parts, provided first a Japanese translation of Marx's

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[55] Another journal entitled "*Xuexi yu Pipan* 學習與批判 (*Study and Criticism*)" was created around the same time. But it focused on non-science areas. For the political nature of this journal as well as the great influence it exerted from 1973 to 1976, see John Gardner (1981), 326-342; 409-412. For *Ziran Bianzhengfa Zazhi*, see ZHU Lang 1983, 125-126; LIU Bing 1997.

[56] The Group was certainly formed under the direction of the Revolutionary Committee of Fudan University. Evidence is lacking, however, to prove the link between the formation of the Group and the Shanghai Mass Criticism Group on Natural Sciences.

manuscripts by TAMAKI Hidehiko, based upon the Russian version produced by Sonia A. Yanovskaya and her collaborators in 1933. The second part was an essay on the importance of Marx's manuscripts written by IMANO Takeo.[57] Of these, the Fudan Group only translated Marx's manuscripts as they appeared in the first part of the Japanese pamphlet. The resulting Chinese translation consisted of four parts. The first three followed the same rubrics as added by the Russian editors: I) "The Derivative and the Symbolic Differential Coefficient"; II) "The Differential and Differential Calculus"; and III) "A Historical Sketch." The forth part comprised an appendix of excerpts from correspondence between Marx and Engels related to mathematics. This part was translated directly from the German. The Chinese translation of all four parts was printed by Fudan University as study materials for internal use in April of 1971.[58]

Because of its author (Marx) and a catchy title, this translation was soon read widely across the Fudan campus, and was quickly reprinted at other universities, for instance at Nanjing Teachers College in August, 1972. The translation served to make intellectuals and politicians alike aware of the mathematical manuscripts left by Marx, and generated serious interest in their contents.

For YAO Wenyuan and his associates in Shanghai, Marx's manuscripts were especially important because they could be used to support YAO's "theory" that Marxism was the most appropriate foundation for the natural sciences. Not long after the publication of the Fudan translation of Marx's *Mathematical Manuscripts*, the translators were asked by ZHU Yongjia 朱永嘉, leader of the powerful Writing Group of the

[57] See the notes by the Chinese translators in Marx 1971 [1972], 69.

[58] *Ibid*.

Shanghai Municipal Revolutionary Committee and Party Secretary of Fudan University, to re-translate Marx's manuscripts, this time directly from the original German manuscripts preserved in the Marx archives of the International Institute of Social History in Amsterdam, The Netherlands. In order to assure the accuracy of the new translation, WANG Fushan 王福山, professor of physics at Fudan, and CHENG Qixiang 程其襄, professor of mathematics at Shanghai Normal University, both of whom had been educated in Germany and received their doctorate degrees there in the 1930s, were asked to join the translation group. CHENG had just been released from the "57 Cadre School" where he had been sent for political rehabilitation, and was grateful for the opportunity to return to intellectual pursuits.[59]

Parts of the new translation were published in the recently-created *Journal for the Dialectics of Nature*. Marx's essays on the concepts of the derivative and the differential appear in the fourth issue of the journal.[60] Marx's comments on the historical development of the differential calculus, his notes on the history of the calculus, and D'Alembert's method of the differential, were all published in the next issue.[61] The new

[59] See Anon. (II) 1974, 17-18. *Wuqi Ganxiao* 五七干校 (57 Cadre School) was a special place where selected intellectuals and officials suspected of minor political offenses were sent for rehabilitation or brainwashing. CHENG Qixiang told the author in an interview at his home in East China Normal University, on August 13, 1999, that at the time he was invited by his friend, WANG Fushan, to join the Fudan translation group, he had just been released from the "57 Cadre School." Sadly, Professor CHENG died in the year of 2000. The author is grateful to Professor ZHANG Dianzhou for arranging the interview, and for taking time to join him in the interview and later correcting the notes on behalf of Professor CHENG Qixiang in a letter dated December 23, 1999.

[60] Marx 1974a.

[61] Marx 1974b.

translations were subsequently published together as a special issue of the *Journal of Fudan University* (natural sciences edition) at the order of ZHU Yongjia.[62]

When the translation of Marx's *Mathematical Manuscripts* from the Japanese by mathematicians at Fudan University was printed for internal use in 1971, a group of mathematicians and philosophers of mathematics at Beijing University was also thinking of renewing a similar translation project, which had been conceived in 1958 but was abandoned soon after because of the deteriorating political environment in China.[63] When the Vice-President of Beijing University, ZHOU Peiyuan, published the above mentioned article in the *Guangming Daily* emphasizing the importance of study and research on basic scientific theories, the journal *Hong Qi* 紅旗 (*Red Banner*), as an organ of the Central Propaganda Department under the control of the "Gang of Four," began to solicit articles to rebut ZHOU's arguments. Since ZHOU used examples from the history of science, the *Red Banner* created a special column devoted to the subject. Among the articles the *Red Banner* solicited was one on history of the calculus, because as ZHOU explained:

> From history of mathematics we do not see the practical usefulness of some mathematical branches in the early stages of their development. The calculus was invented about three hundred years ago after a long period of time trying to solve the problems of planetary motion and tangents to curves. But it is now an indispensable mathematical tool to solve practical problems.[64]

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[62] Marx 1975a. Comparing the texts published in the *Journal of the Dialectics of Nature* and the *Fudan Journal*, the latter includes the following additional contents: 1) About the symbol: $\frac{dy}{dx}$ (pp. 11-12); 2) Three drafts on the differential (pp. 26-51); 3) One facsimile of a list of mathematicians and their works and translation thereof (pp. 73-74); 4) Another example using D'Alembert's method (pp. 81-86); 5) On the second degree of the derivative (pp. 87-88); 6) Analysis of Lagrange's methods (pp. 89-123); and 7) Several manuscripts written in the 1870s (pp. 124-165).

[63] See "Notes on the Translation," in Marx 1975b, ii.

[64] See the *Guangming Daily*, October 6, 1972, p. 2.

The editors of the *Red Banner* asked the Department of Mathematics and Mechanics at Beijing University to contribute an article on the calculus. More than twenty faculty members in the Department, including DENG Donggao 鄧東皐, SUN Xiaoli 孫小禮, WU Guanglei 吳光磊, DING Shisun 丁石孫, and LIU Xiyuan 劉西垣, soon agreed to devote themselves to this assignment. The article they co-authored, "*weijifen de lilun shi zengmo laide*? 微積分的理論是怎麼來的? (How Did the Theory of the Calculus Originate?)," was soon published under the name "*Shuli* 舒立" in the first issue for 1973 of the *Red Banner*. It was also the inaugural article for a new column on the history of science.[65] The article, like Boris M. Hessen's "The Social and Economic Roots of Newton's '*Principia*'," emphasized the needs of production, which together with theoretical concerns in astronomy and mathematics, prompted discovery of the theories of the calculus by Newton and Leibniz.

While writing the article, the authors consulted a great number of historical accounts of the history of the calculus by Western scholars. They soon found that the accounts by Western historians were in marked contrast to what Marx had said about the subject in the newly translated *Mathematical Manuscripts* by their colleagues at Fudan University. This too helped to revive their interest in Marx's manuscripts. In the spring of 1973, when the Shanghai group was retranslating Marx's manuscripts directly from the German, WANG Delu 王德祿, who was in charge of the Central Translation Bureau of the Works by Marx, Engels, Lenin and Stalin, proposed that Beijing University should

[65] In an interview at her home in Beijing on August 26, 1999, Professor SUN Xiaoli told the author that she and her colleagues did not know the article they wrote was intend to criticize ZHOU Peiyuan. She also confided that the presumed name of the author, "*Shuli,*" was a pseudonym based upon "*shu li xi*," namely, the Department of Mathematics and Mechanics. For the article, see SHU Li 1973; and its reprint SHU Li 1994.

prepare a new translation based upon the 1968 Russian-German version edited by Sonia A. Yanovskaya, a copy of which he immediately provided. In the meantime, the Foreign Ministry also secured photocopies of Marx's *Mathematical Manuscripts* from the International Institute of Social History in Amsterdam.[66] With the help of both the Central Translation Bureau and the Foreign Ministry, Beijing University began to prepare yet another and in fact the third Chinese translation of Marx's *Mathematical Manuscripts*.

The translation group at Beijing University consisted of mathematicians, along with experts on German and Russian. Among mathematicians were the well-known JIANG Zehan 江澤涵, DENG Donggao 鄧東皋, and LENG Shengming 冷生明. The Beijing group's translation was conducted with the utmost seriousness. Based on the original German manuscripts, the Russian translation, and above all the notes provided by Yanovskaya and her associates, the group at Beijing University made rapid progress. When its draft translation was ready, the group also sent it to mathematicians at the Institute of Mathematics of the Chinese Academy of Sciences. Additional copies were sent to mathematicians at Beijing Normal University such as HU Shihua 胡世華, ZHANG Herui 張禾瑞, JIANG Shuomin 蔣碩民, and the philosopher of science, YU Guangyuan 于光遠, for their comments in particular.[67] The translation completed by the Beijing Group was first published in a special issue of the *Journal of Beijing University* (natural science edition) in 1974. Several months later it also appeared independently as a separate

[66] In the notes to the second translation of Marx's manuscripts, the Shanghai Group writes that it was based on a German source, but does not specify the actual source used. However, from facsimiles of the manuscripts printed in the *Journal of the Dialectics of Nature,* as well as the *Journal of Fudan University*, the stamp of the International Institute for Social History in Amsterdam is clearly visible. This suggests that the Shanghai and Beijing groups did not share the original sources, but indeed were rivals.

[67] See "Epilogue" by SUN Xiaoli and DENG Donggao, in *Shuxue yu Wenhua* 數學與文化 (*Mathematics and Culture*), eds., DENG Donggao 鄧東皋, SUN Xiaoli 孫小禮, and ZHANG Zugui 張祖貴, Beijing: Beijing University Press, 1994, 346.

pamphlet printed by the most prestigious press in China: Renmin Chubanshe (The People's Press).[68]

Although the two groups in Shanghai and Beijing worked from the same original German manuscripts, the differences in their styles, structure, and the wordings of separate headings can be easily recognized. In their "Note on The Translation," although the translators of the Beijing Group did not mention the Russian-German dual edition of Marx's *Mathematical Manuscripts* published in Moscow in 1968, let alone the detailed notes provided by Yanovskaya and her collaborators, the influence of this edition on the Chinese translation is obvious. However, considering the fact that at the time China and the Soviet Union considered each other as enemies, it is perhaps understandable that the helpfulness of the Soviet edition was not acknowledged explicitly.[69] What all this translation activity by Chinese mathematicians devoted to the mathematical manuscripts of Karl Marx would mean for the future of mathematics in China is the subject of the next and concluding chapter of this dissertation.

[68] See Marx 1974c; and Marx 1975b.

[69] In her latest article on Marx's mathematical manuscripts, SUN Xiaoli does acknowledge the Russian-German dual edition edited by Sonia Alexandrovna Yanovskaya and her colleagues; see SUN Xiaoli 2003.

Conclusion

> I can well imagine that a serious mathematical philosophy based on the dialectical approach will make its appearance…. As far as the detailed analysis of mathematics or of a mathematical theory (e.g. the calculus) by the dialectical method is concerned, my reading, beginning with Hegel's work in this area, has not led me to find anything that can stand up to serious criticism.
>
> -Abraham Robinson[1]

When Karl Marx's notes on the concept of derived function and on the differential as well as his comments on the historical development of the differential calculus were published in the *Ziran Bianzhengfa Zazhi* (*Journal for the Dialectics of Nature*), readers, who were college professors, factory workers, and even high school teachers and students, wrote letters or sent articles from all parts of the county to the editorial office of the *Journal* to express their new understanding of the differential and limits after studying Marx's mathematical manuscripts.[2] In the meantime, a symposium on Marx's manuscripts was organized by the editorial office of *Fudan Xuebao* 復旦學報 (*Fudan Journal, Natural Sciences*). A number of prominent professors of mathematics, including those who were translators of the manuscripts at Fudan University and Shanghai Teachers College, were invited.[3] Their politically-correct praise for the value of Marx's manuscripts in applying dialectics to study the foundation of the differential calculus and

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[1] Quoted from Robinson 1979, 566.

[2] In the fourth issue of *Ziran Bianzhengfa Zazhi* (*Journal for the Dialectics of Nature*) for 1974, there is a special column devoted to "Discussion of Problems on the Calculus and Limits" (pp. 145-187). The column features five articles by HE Fang, FU Xitao, ZHENG Lixing, XUE Shouguo, and LI Guozhen, along with excerpts from several more letters, including one by SHEN Tianji; see HE Fang 1974; FU Xitiao 1974; ZHENG Lixing 1974; XUE Shouguo 1974; LI Guozhen 1974; and SHEN Tianji 1974. Some of these letters and articles are discussed in Dauben 2003, 342-345; 2004.

[3] See Anon. (II) 1974.

in reforming the calculus curriculum set the tone for further studies and discussion of the manuscripts. Subsequently, papers on Marx's manuscripts inundated the *Fudan Journal*.[4]

When the translation of Marx's manuscripts prepared by a group of scholars at Beijing University was published in 1975 as a pamphlet by the most prestigious press in China, the People's Press, the manuscripts were immediately discussed at length in universities and research institutes not only in Beijing, but throughout the country. The study groups were formed even at such universities as Beijing University, Qinghua University and Beijing Normal University. Symposia were organized, including one facilitated by the Editorial Office of the *Shuxue de shijian yu renshi* 數學的實踐與認識 (*Practice and Understanding of Mathematics*), one of the key journals sponsored by the Chinese Mathematical Society in the 1970s. Mathematicians and philosophers of mathematics from Beijing University, Qinghua University, Beijing Normal University, Beijing Teachers College, Nankai University, as well as from the Institute of Mathematics and the Institute of Computing Technology of the Chinese Academy of Sciences, were invited by the Editorial Office to exchange their understanding of Marx's manuscripts in order to stimulate further studies.[5] Between 1975 and 1977, Marx's manuscripts were the most widely-discussed topic among Chinese mathematicians and philosophers of mathematics. According to Professor LIN Xiashui 林夏水, philosopher of

[4] Between 1975 and 1977, numerous articles on Marx's manuscripts were published in the *Fudan Journal*. For instance, LI Ke 1975; DAI Qiaoming 1975; OUYANG and ZHU 1975a, 1975b; XIE 1975; WU Wenjing 1975; YAN Shaozong 1975; Anon. (II) 1975, 1977; SHEN Tianji 1976; NI and RONG 1976; and ZHOU and WANG 1977. Some of these articles are discussed in Dauben 2003, 345-347; 2004.

[5] See Anon. (III) 1976.

mathematics at the Chinese Academy of Social Sciences in Beijing, more than a hundred articles were published during this short period.[6]

A thorough discussion of these papers would require a dissertation of its own. Here we limit ourselves to an outline of the major views presented in the articles by mathematicians and philosophers of mathematics. Virtually all of the papers published during this period claim, whether their authors really believed it or not, that Marx using dialectics had provided a solid foundation at least for the differential, if not for the whole of calculus. The specific arguments, however, differ from paper to paper. The three laws of materialist dialectics, namely the law of the transformation of quantity into quality and *vice versa*, the law of the negation of the negation, and the law of the unity of opposites, are applied in these papers, and some of them appeal to all three laws. For instance, the article, "Weiwu Bianzhengfa de Guanghui Shengli 唯物辯證法的光輝勝利 (A Splendid Victory of Materialist Dialectics)," published in the *Acta Mathematica Sinica* (18 (3)(1975): 149-156) by the Study Group of Marx's *Mathematical Manuscripts* in the Department of Mathematics at Zhejiang Normal College, uses the law of the negation of the negation in explaining Marx's notes on the concept of derived function as follows:

> At the beginning of his account of the differential calculus, Marx writes very clearly: "Let variable x increase to x_1, accordingly, the dependent variable y increases to y_1."… By using the view point of change in motion, Marx first negates "instantaneous" and "one point," and lets the variable really change and observes [these changes] in the change of motion. It is well known that when x_1 returns to x, and y_1 goes back to y during the second negation, [they] are not simply restored to their original places, but

[6] An e-mail note dated April 27, 2001, from LIN Xiashui to the author. The author is grateful to Professor LIN for the insightful information he provided about the discussions of Marx's *Mathematical Manuscripts* and nonstandard analysis in China in the 1970s in the aforementioned e-mail and in another one dated May 6, 2001 as well. For lists of some major Chinese articles on Marx's manuscripts, see WU, LIU and XUE 1983, 348-352; and LIN Xiashui 1983, 359-360.

> move up spirally at the new position of the ratio: $\frac{\Delta y}{\Delta x} = f^1(x)$. Once $\Delta x = x_1 - x = 0$, the increment of the function Δy and the increment of the variable Δx disappear. But the relation they have to each other is retained. The result of the negation of the negation makes $\frac{\Delta y}{\Delta x}$ change to $\frac{0}{0}$, and the derivative $f'(x)$ deduced from the preliminary derivative $f^1(x)$ leads to the ironclad fact: $\frac{0}{0} = f'(x)$.
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> 馬克思在其微分學論述一開始，就旗幟鮮明的提出：“**設自變量 x 增長到 x_1，隨之因變量 y 增長到 y_1**。”…… 馬克思用運動變化的觀點，先否定“瞬時”、“一點”，讓自變量變化起來，在運動變化中加以考察。如所周知，當 x_1 回復到 x，y_1 回復到 y 的第二次否定過程中，已不是簡單的回復，而是在比式 $\frac{\Delta y}{\Delta x} = f^1(x)$ 这一新環節上進行螺旋式上升。一旦 $\Delta x = x_1 - x = 0$ 時，函數增量 Δy 和自變量增量 Δx 消失了，但是它們的依賴關係卻真確地保留下來。否定之否定的結果，使 $\frac{\Delta y}{\Delta x}$ 變爲 $\frac{0}{0}$，而導數 $f'(x)$ 又從**預備導數** $f^1(x)$ 中結晶了出來，從而帶來了不依人們意志為轉移的鉄的事實：$\frac{0}{0} = f'(x)$。[7]

The law of the transformation of quantity into quality and *vice versa* is also used in the same article in explaining the differential:

> Dialectical logic advocates applying the view point of the transformation from change of quantity to change of quality to analyze questions. We deduce logically from $\frac{\Delta y}{\Delta x} = f^1(x)$ to $\Delta y = f^1(x)\Delta x$. Let Δx, Δy vanish, if we use dx to denote the discarded or vanished difference Δx, then the discarded or vanished Δy should be denoted as dy. When $\frac{\Delta y}{\Delta x} = f^1(x)$ is transformed or qualitatively changed to $\frac{dy}{dx} = f'(x)$, why cannot we hold that $\Delta y = f^1(x)\Delta x$ has transformed or qualitatively changed to $dy = f'(x)dx$? The change of dx in places does not affect at all its relation with dy. Accordingly we can always treat $dy = f'(x)dx$ as the equivalent or another form of $\frac{dy}{dx} = f'(x)$.

[7] Quoted from Zhejiang 1975, 151.

辯證邏輯主張用量變到質變的觀點分析問題。我們從 $\frac{\Delta y}{\Delta x} = f^{1}(x)$ 邏輯地推出 $\Delta y = f^{1}(x)\Delta x$。令 Δx，Δy 消失，如果我們用 dx 表示被揚棄了的或消失了的差 Δx，隨之被揚棄了的或消失了的 Δy 就表示為 dy，那麼當 $\frac{\Delta y}{\Delta x} = f^{1}(x)$ 轉化、質變為 $\frac{dy}{dx} = f^{'}(x)$ 時，我們爲什麼不可以認爲 $\Delta y = f^{1}(x)\Delta x$ 轉化、質變為 $dy = f^{'}(x)dx$ 呢？dx 在位置上的這種改變絲毫不影響它和 dy 之間的關係，因此我們可以始終把 $dy = f^{'}(x)dx$ 看作 $\frac{dy}{dx} = f^{'}(x)$ 的等價物或另一形式。[8]

In the same article, we can also find arguments which are based on the law of the unity of opposites:

> The differential as a vanished difference has two sorts of meanings: On the one hand, the absolute value of the differential as the ultimate development of algebra vanished and becomes 0. [It] has some properties of zero. On the other hand, the differential as the starting point of the differential calculus—is a mathematical magnitude in a broad sense representing the properties of the differential calculus having non-zero operational properties for "difference," which thus guarantees the differentials can operate freely like any non-zero number, in addition to its special methods of operation in the new domain of the differential calculus. Of course, this discussion assumes that the functional relation is provided, and is also based on $\frac{dy}{dx} = f^{'}(x)$. It would be fruitless if [one] considers the vanishing of the differences Δx and Δy in isolation.
> The differential has the property of zero in its numerical aspect, and has the property of non-zero in its operational aspect. Accordingly, a general mathematical magnitude such as the differential is the unity of opposites of zero and non-zero.

> 微分作爲消失了的差有兩層意思：一是講微分作爲代數發展的終點在絕對值上已經消失，變爲 0，具有零的屬性；另一是講微分作爲微分學的出發點——標誌微分學特性的廣義數學量而具有"差值"的非零運算屬性，從而保證微分在新的微分學領域理象非零的數那樣可以自由運算，並且還有特殊的計算方法。當然，這一切是在函數關係給定的條件下進行討論的，是以 $\frac{dy}{dx} = f^{'}(x)$ 爲基礎的，孤立地考察差值 Δx 或 Δy 的消失，必將一無所得。
> 微分在絕對量上具有零的屬性，而在運算上又具有非零屬性，因此，微分這種廣義數學量包含了是零和非零矛盾的對立統一。[9]

[8] Quoted from Zhejiang 1975, 153.

[9] Quoted from Zhejiang 1975, 153-154.

The dialectical arguments supporting Marx's notes on the differential calculus presented in the article by the Study Group of Marx's *Manuscripts* in the Department of Mathematics at Zhejiang Normal College may be difficult to follow. But they represent typical ways of reasoning in the articles published during the Cultural Revolution. In order to establish squarely the thesis that Marx had provided a solid foundation for the differential calculus, Chinese mathematicians and philosophers alike criticized Cauchy whose approach had generally been adopted in the calculus textbooks in mainland China before the Cultural Revolution, and who was commonly regarded as the "founder of the exact differential calculus in the modern sense."[10]

It is well-known that Cauchy defined the differential based on the concept of the derivative, which in turn was based on his notion of limit, infinitesimal and infinity. Considering the function $y = f(x)$, let the increment of the variable x be h or Δx, and the corresponding increment of the function be Δy. Then the derivative of the function, represented by y' or $f'(x)$ is the limit of the ratio of Δy to Δx. This may be written in symbols as follows:

$$y' = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+h) - f(x)}{h}.$$

If the differential of the variable is denoted by dx, then the differential of the function, dy, can be defined as $f'(x)dx$.

Cauchy's approach was acknowledged by Chinese philosophers and mathematicians as rigorous from the view point of formal logic, but it was not regarded as having provided a solid foundation for the differential calculus.[11] Among the arguments raised against Cauchy and in favor of Marx, those by the philosophers HUANG Shunji 黄順基 and WU Yanfu 吳延涪 of the People's University of China are

[10] Quoted from Boyer 1959, 282.

[11] See LI Guozhen 1974; OUYANG 1975; OUYANG and ZHU 1975a; 1975b.

representative.[12] HUANG and WU are perhaps the most prolific authors on Marx's mathematical manuscripts in China. They have written dozens of articles on the manuscripts, some of which are the result of a series of lectures they gave for the Study Group of Marx's Mathematical Manuscript at Beijing Normal University. Their lectures, published in eight installments in the *Journal of Beijing Normal University*, offer a systematic discussion of Marx's manuscripts.[13]

The fourth installment of the lecture series particularly interesting, and is entitled "Daohanshu Xia 導函數 (下) (Derived Function (III))." In this article, HUANG and WU criticize Cauchy' theory of the differential by using both dialectics and Marx's notes on the derived function. Their specific arguments run as follows:

> Cauchy's limit method in no way treats the whole process of the differential as a process of dialectical development. It starts with a form of summation [namely $x + \Delta x$] (that is a totally-wrong starting point), and it absolutely eliminates the contradictory motion of the variable x. Accordingly, it denies the duality of the infinitesimal Δx. As a result, the whole differential process [in Cauchy's approach] becomes a meaningless mechanical four-step procedure, and a purely formal calculation. This kind of approach leads to the formalism of mathematical idealism.
>
> 哥西的極限方法根本不是把整個微分過程看成是一個辯證發展的過程，它從和的形式出發（這是根本錯誤的出發點），完全抽去了變量 x 的矛盾運動，從而也就取消了無窮小 Δx 的兩重性，這樣一來，整個微分過程就變成為仿佛是毫無內容的機械的四步規則，變成為單純的形式的演算。這種觀點導致數學中的唯心主義的形式主義。[14]

[12] Between July 1973 and July 1978, the Department of Philosophy of the People's University of China merged with that of Beijing Normal University. During this time, both HUANG Shunji and WU Yanfu were affiliated with Beijing Normal University.

[13] The *Journal of Beijing Normal University* originally planned to publish their lecture notes in ten installments. But at the end of the eighth installment, the *Journal* announced it would not publish the remaining two parts because of the changed editorial policies; see HAUNG and WU 1977d, 81. The first four installments were published under the pseudonym XUE Shu 薛舒, but the lecture notes numbered 5 through 8 were published under their real names. See XUE Shu 1976a; 1976b; 1977a; and 1977b; HUANG and WU 1977a; 1977b; 1977c; and 1977d.

[14] XUE Shu 1977b, 29.

> Cauchy's limit method treats infinitesimals as non-zero. Therefore the infinitesimal, though a variable with 0 as its limit, can approach infinitely close to 0, but cannot be exactly 0 at any time. Thus, Cauchy has taken away the duality of the infinitesimal, and has created an impassable chasm between 0 and non-0.
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> 哥西的極限方法把無窮小看成只是非 0，因此無窮小雖然是以 0 為極限的變量，但是它僅僅無限趨近于 0，任何時候都不能和 0 重合，這樣哥西就取消了無窮小的兩重性，在 0 和非 0 之間設置了一道不可逾越的鴻溝。[15]
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> In Cauchy's view, the infinitesimal is either 0 or non-0, and must be one or the other. He obstinately insists his metaphysical view of the infinitesimal is non-0. But under certain circumstances, [he] let the infinitesimal be directly equal to zero [as Marx did]. This sort of concept did not exist in Cauchy's mind. Therefore, whether or not the infinitesimal has the duality of 0 and non-0, and whether or not the opposites of 0 and non-0 can be mutually transformed, reflect the opposite views of Cauchy's limit method and Marx's algebraic differential method. The contrast between the two methods is really a contrast between metaphysics and dialectics.
>
> 在哥西看來，無窮小要么是 0，要么是非 0，二者必居其一，他頑固地堅持無窮小是非 0 的形而上學觀點，至於在一定條件下肯定地、直接了當地令無窮小等於 0，這個概念在他的頭腦中是沒有的。所以無窮小有沒有 0 和非 0 的兩重性，0 和非 0 這一對對立面是否可以互相轉化，在這個問題上，哥西極限方法和馬克思代數微分法的對立是形而上學和辯證法的對立。[16]

From the above lengthy quotation, one can see that Cauchy's limit theory was criticized for failing both to reflect the dialectical process of the differential and to treat infinitesimals as being as small as one pleased but never equal to 0. Its "weaknesses" were alleged because Cauchy's theory was in conflict with Marx's so-called algebraic differential method. Marx's viewpoint may be sketched as follows. Consider the function $y = f(x)$, and define the ratio of Δy to Δx as the preliminary derived function, $f^1(x)$.

Since $$f^1(x) = \frac{\Delta y}{\Delta x},$$

[15] XUE Shu 1977b, 29.

[16] XUE Shu 1977b, 30.

then $\Delta y = f^{1}(x)\Delta x$.

Let $x_1 - x = \Delta x$; $y_1 - y = \Delta y$,

then $\frac{\Delta y}{\Delta x} = \frac{y_1 - y}{x_1 - x}$;

let $x_1 - x = 0$, then $y_1 - y = 0$ and

$$\frac{\Delta y}{\Delta x} = \frac{0}{0} = \frac{dy}{dx};$$

$$f^{'}(x) = \frac{dy}{dx};$$

$$dy = f^{'}(x)dx .$$

The above outlined derivation reflects Marx's algebraic differential method.[17] The method was thought to be "permeated by the dialectics of contradiction from the beginning to the end."[18] Taking the difference $x_1 - x = \Delta x$, this was interpreted as the first negation of *x*, and to having brought motion into mathematics by letting the variable *x* really change. Similarly, $x_1 - x = 0$ was thought to be the negation of the negation. These procedures as a whole were explained to have reflected "contradiction of the change of motion from the quantitative aspect."[19] The equalities $\frac{\Delta y}{\Delta x} = \frac{0}{0} = \frac{dy}{dx}$ were interpreted to mean that Marx held that the infinitesimal Δx was both zero and non-zero. When Δx is close to zero, or in Marx's own words "is vanishing,"[20] it should be regarded as a non-zero number. But unlike Cauchy's potential infinitesimal, Marx's infinitesimal

[17] See Marx 1975b, 1-8, 34-40, 103-104.

[18] XUE Shu 1977b, 29.

[19] XUE Shu 1977b, 29.

[20] Marx 1975b, 2, 78-79, 96-97.

is an actual one, and can also become exactly zero. The process of the infinitesimal going from a small quantity to actually zero was thought to have fulfilled the transformation from quantitative change to qualitative change. Once the infinitesimal Δx is zero, it can be replaced by the symbol dx, as can Δy be replaced by dy. Marx's view of the infinitesimal as both zero and non-zero was hailed as having "completely unveiled the dialectical nature of infinitesimals."[21]

Discussions of Marx's views on infinitesimals link naturally to the subject of nonstandard analysis, the creation of Abraham Robinson in the 1960s, because Robinson's theory supplies a rigorous theory of infinitesimals.[22] As early as 1972, Chinese mathematicians were aware of his newly-created nonstandard analysis. Moreover, the year 1972 was a turning point for Chinese mathematics during the Cultural Revolution. Studying pure mathematics was finally allowed again after having been prohibited from 1966 to 1971. In that year, the logician WANG Shiqiang 王世強 returned to his teaching position at Beijing Normal University after laboring three years in the countryside. Accidentally, he found a copy of the first edition of *Nonstandard Analysis* (1966) by Abraham Robinson in a second-hand bookstore in Beijing. He immediately purchased it, studied it passionately, and highly recommended the book to his colleagues in Beijing.[23] Also in 1972, Professor SHEN Youcheng 申又棖 of Beijing University, a leading expert on functions of a complex variable and differential equations, read an

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[21] See XUE Shu 1977b, 29-30.

[22] For the creation of nonstandard analysis and its founder Abraham Robinson, see Dauben 1988; 1990; 1995 and 2005.

[23] WANG Shiqiang 1998, 395. In an e-mail note dated November 19, 2001, from LI Banghe to the author, Professor LI writes: "Prof. WANG Shiqiang of Beijing Normal University played an important role in introducing nonstandard analysis into China. He found the book by Robinson and started to organize studies and a translation of the book in 1972."

introductory article on nonstandard analysis by Martin Davis and Hersh Reuben in *Scientific American*, and was fascinated.[24] He too was enthusiastic about introducing nonstandard analysis particularly from the view point of the history of the calculus. He translated chapter ten of Robinson's *Nonstandard Analysis*, "Concerning the History of the Calculus," into Chinese, and then circulated his translation among his colleagues in order to show the significance of nonstandard analysis from a historical point of view.[25] Later, SHEN Youcheng and WANG Shiqiang, in collaboration with others including the logician ZHANG Jinwen 張錦文 of the Institute of Computing Technology of the Chinese Academy of Sciences, translated the complete revised edition of *Nonstandard Analysis* (1974), which appeared in Chinese in 1980.[26]

Thanks to the early efforts of SHEN Youcheng and WANG Shiqiang, nonstandard analysis was quickly embraced by Chinese mathematicians and philosophers of mathematics, especially the younger generation. Among these who were influential in promoting nonstandard analysis was the aforementioned logician ZHANG Jinwen. After the publication of Marx's *Mathematical Manuscripts* by two independent groups of scholars at Fudan University and at Beijing University, interest directed to foundations of the calculus was immediately aroused. ZHANG organized an informal seminar devoted to study of the foundations of both the calculus and nonstandard analysis. The seminar met once a month beginning in 1975 and ran through the end of 1977. Participants in the seminar included mathematicians and philosophers of mathematics from the Institute of

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[24] See Davis and Hersh 1972; also Dauben 1995, 282.

[25] DING and LI 2002, 11. For a detailed and interesting discussion of Robinson's accounts on the history of the calculus, see Dauben 1995, 349-355; and 2005, 266-271.

[26] Robinson 1980.

Computing Technology and the Institute of Mathematics of the Chinese Academy of Sciences, Beijing University, Beijing Normal University, the Chinese Academy of Social Sciences, Beijing Institute of Technology, and the Chinese Electrical Power Research Institute, among others. At the seminars, ZHANG Jinwen himself was often the key speaker lecturing on nonstandard analysis using the book by Abraham Robinson.[27]

ZHANG Jinwen's seminar was supported by GUAN Zhaozhi, then the most influential mathematician in Beijing. In an article on "Shuxue tuili de yangexiang he renshilun zhong de shijian biaozhun 數學推理的嚴格性和認識論中的實踐標准 (Rigor of Mathematical Reason and Practical Standards of Epistemology)," GUAN wrote: "Nonstandard Analysis, which appeared in 1966 and is based on current mathematical rigor and modern mathematical logic, has created a new kind of mathematical analysis which differs from traditional (or standard) analysis. [It] treats the infinitesimal as actual, and mathematical analysis built upon it can perform all of the functions of the old analysis. It has foundations that meet current standards of rigor, and yet preserves the vivid and heuristic infinitesimals of Newton and Leibniz."[28] Moreover, GUAN also encouraged young mathematicians, for instance, LI Banghe 李邦河, to study nonstandard analysis. With GUAN's support and the collative efforts of ZHANG Jinwen, LI Banghe and other scholars, in October of 1976 the Institute of Mathematics of the Chinese

[27] Information about the seminar on foundations of the calculus and nonstandard analysis organized by ZHANG Jinwen provided here is based on an e-mail note dated April 27, 2001, from LIN Xiashui to the author. In their article entitled "Makesi de shuxue shougao zai zhongguo de chuanbo 馬克思的數學手稿在中國的傳播 (The Dissemination of Marx's Mathematical Manuscript in China)," WU Yanfu, LIU Dachun, and XUE Yuchuan also mention this seminar. But they say the seminars were held from 1976 to 1977. See WU, LIU, and XUE 1983, 346. Meanwhile, in his e-mail note dated November 20, 2001, LI Banghe told the author that among participants was the renowned mathematician Professor CHENG Minde of Beijing University.

[28] See GUAN 1976, 5.

Academy of Sciences mimeographed articles and translations on nonstandard analysis as a book with the title: "Shuxue de lishi luoji yu jichu zhilaio xuanji: feibiaozhun fenxi (I) 數學的歷史、邏輯與基礎資料選集：非標準分析（一）(*Selected Papers on History, Logic and Foundations of Mathematics: Nonstandard Analysis* (I)). "[29]

In the book there are five articles by Chinese scholars and one translation of a famous paper by Martin Davis and Reuben Hersh, "Nonstandard Analysis," published in *Scientific American* (translated by YANG Dongping 楊東屏 on pages 58-74). Of the Chinese articles, the first by ZHANG Jinwen is entitled "Wuqiongxiao liang fangfa yu fei biaozhun fenxi 無窮小量方法与非標準分析 (The Methods of Infinitesimal and Nonstandard Analysis)." This paper outlines different methods for dealing with infinitesimals before the twentieth century, including approaches taken by Newton, Leibniz, Cauchy, and Weierstraß, and then goes on to introduce Abraham Robinson's proof of the existence of infinitesimals in a nonstandard field, R*, an extension of the real numbers that contains both infinitely large and infinitely small elements. ZHANG also talks about how Robinson's new theory of infinitesimals answers the well-known criticisms of the calculus made by Bishop Berkeley against Newton's fluxions. The most interesting part of ZHANG's analysis is his use of nonstandard analysis to justify Marx's algebraic differential method.

Marx, in order to find the derivative of the function $y = f(x)$, first takes the difference $x_1 - x$, and then sets the ratio $f(x_1) - f(x)$ to $x_1 - x$, as the preliminary derived function $f^1(x)$. He then lets $x_1 - x = 0$, accordingly, $f(x_1) - f(x) = 0$. Thus the

[29] The author is very grateful to Professor LI Banghe for securing a copy of this book in a two-volume version reproduced by Guiyang Normal College in August, 1977. References to the *Selected Papers* in this thesis are based on the two-volume reproduced version. The author acknowledges his gratitude to Professor GUO Shirong for photocopying and mailing him the copies.

derivative of the function is $\frac{\Delta y}{\Delta x} = \frac{0}{0} = \frac{dy}{dx}$. ZHANG quotes Marx as follows: "Though x returns to itself through x_1, it is not the same thing at all." He then argues that what $x_1 - x = 0$ really means is that x_1 is a point within the monad x, and the difference of their standard parts is zero.[30] Since $x_1 - x$ is an infinitesimal, so is $f(x_1) - f(x)$. Therefore, operations with nonstandard infinitesimals can be handled like any real numbers. Consequently, letting $\frac{dy}{dx}$ directly equal $\frac{0}{0}$ can be justified from the view point of nonstandard analysis.[31]

ZHANG was convinced by Robinson's sophisticated technical derivation of nonstandard analysis, and praised it for having opened "a new field for mathematical analysis."[32] But in order to make his paper politically correct, he criticized Robinson's philosophy of mathematics for maintaining that the absolute infinite does not exist in any form, physically or ideally, and for separating mathematical reasoning from the real world in opposition to the point of view of materialist dialectics.[33]

Following ZHANG's article in the *Selected Papers on History, Logic and Foundations of Mathematics: Nonstandard Analysis*, is a paper entitled "Xiandeng de bianzhengfa 相等的辯證法 (Dialectics of Equal)" by LI Banghe.[34] In this paper, LI uses

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[30] The word "monad" is a basic terminology in nonstandard analysis. It consists of all those numbers infinitesimally close to a given finite number. Considering a given number *a*, there are infinitely many infinitesimals around *a*. The number *a* together with its infinitesimals are called a monad.

[31] ZHANG Jinwen 1976, 24.

[32] ZHANG Jinwen 1976, 26.

[33] ZHANG Jinwen 1976, 26-29.

[34] This paper is the second LI wrote on Marx's *Mathematical Manuscripts*; see LI Banghe 1976b. The first one is entitled "Weijifen zhong de duili tongyi 微積分中的對立統一 (The Unity of Opposites in the Calculus)," which LI submitted to *Acta Mathematica Sinica,* where it was accepted for publication. But

different levels of quantities to discuss the meaning of "equal" under certain circumstances, arguing the correctness of Marx's algebraic differential method. Say *a* and *b* are two finite numbers in R*, if they are equal, their difference *a* – *b* can be treated as an infinitesimal. If two infinitesimals σ and ρ are equal, then they are said to be of the same level, and also for another infinitesimal *t*, the ratio σ to *t*, and the ratio ρ to *t*, are finite numbers. If the difference between σ and ρ is a higher infinitesimal than *t*, then σ = ρ. This establishes equivalence for infinitesimals of the same level as *t*, but is a different one in nature from *a* = *b*. Having explained this, LI continues:

> Let *dx* be a positive or negative infinitesimal *t*, which can be taken randomly as long as it is within an infinitesimal boundary. As for *dx*, whatever kind of infinitesimal *t* it may be, form the point of view of finite numbers, it is always as Marx said, *dx* = 0. Suppose the function $y = f(x)$ has a finite derivative $f'(x)$ at the point *x*; let *dx* = *t*, then
>
> $$dy = f(x+t) - f(x) = f'(x)t + o(t),$$
>
> where $o(t)$ is a higher infinitesimal of *t* in R*.
> For the infinitesimal *dx* and others at the same level, we have
>
> $$dy = f'(x)dx.$$
>
> According to the previous explanation of "equal," the above is a rigorous equality. Meanwhile, from
>
> $$\frac{dy}{dx} = \frac{f(x+t) - f(x)}{t} = f'(x) + o(1),$$
>
> where $o(1)$ is an infinitesimal in R*, we get the following rigorous equality among finite numbers:
>
> $$\frac{dy}{dx} = f'(x).$$
>
> Accordingly, whether we derive $\frac{dy}{dx} = f'(x)$ from $dy = f'(x)dx$, or $dy = f'(x)dx$ from $\frac{dy}{dx} = f'(x)$, mathematically speaking, they are totally consistent and are rigorous.[35]

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since the journal soon stopped publishing any articles on philosophy of mathematics, the paper remained unpublished. The author is grateful to Professor LI Banghe for providing a copy of his manuscript and telling the story of the manuscript to Professor GUO Shirong. The author is equally grateful to Professor GUO for photocopying the manuscript and for relaying the story in written form. See LI Banghe 1976a.

[35] LI Banghe 1976b, 34.

Based on the above technical arguments in R*, LI Banghe justifies separating the ratio *dy/dx* and operating with it like a fraction. Moreover, in the same article he also discusses the equality between curves and straight lines, limit and equality by using the language of nonstandard analysis in order to justify what Marx had said about his algebraic differential method.

The third article, "Tantan shuxue jiegou **R* de xianshi yuanxing 談談數學結構**R* 的現實原型 (Discussing the Physical Origins of the Mathematical Structure of *R)," is by SHU Ji 舒基 of Northwest University in Xi'an.[36] This article argues that both infinitesimals and infinity are ontologically real. SHU based his arguments on quotations from Marx's *Mathematical Manuscripts*, MAO Zedong's essay "On the Correct Handing of Contradictions among the People," and Engels' *Dialectics of Nature*, especially his account of mathematical infinity in the real world.[37]

Compared with the immediately previous article, the fourth one is quite technical. This one is entitled "Tantan duiyu lianxutong de yixie renshi 談談對于連續統的一些認識 (Some Understanding of Physical Origins of Numbers),"[38] and was authored by YUAN Meng 袁萌. YUAN worked at the Chinese Electrical Power Research Institute, and was one of the earliest ardent students of nonstandard analysis in China. The paper first briefly outlines how the standard number field *R* was extended from the rational field *Q*, and holds that this extension was a result of man's deep understanding of the real world.

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[36] The article also appeared in the *Journal of Northwest University* in Xi'an and in *Ziran Kexue Zhengming* 自然科學爭鳴 (*Debate on Natural Sciences)*. For detailed references, see SHU Ji 1976. According to LI Banghe, SHU Ji is a pseudonym which means "foundation of mathematics."

[37] For Engels' account of mathematical infinity in the real world, see Engels 1966, 54-64; 1940, 313-319, Appendix I. For a discussion of this article by SHU Ji, see also Dauben 2003, 355-356.

[38] The title of the article is misleading. The words "連續統" in the paper actually refer to physical origins of numbers, rather than to the continuum or field of real numbers.

It then goes on to discuss whether the extension of a nonstandard filed *R** has its origin in the physical universe. Based on zero-sequences constructed by an ultrafilter in *R**, the paper argues that it has just an origin. The paper concludes that "The discovery of *R** and the creation of nonstandard analysis are leaps in the human understanding of the physical origins of numbers." [39]

The last article in the *Selected Papers*, "Wuxianxiaoliang de keguan beijing 無限小量的客觀背景 (Physical Models of Infinitesimal)," was by LIAO Zuwei 廖祖偉. This paper uses a set of small measurements and the golden ratio in one dimension constructing two different zero-sequences and argues they provide an objective background for the concept of monad.[40]

Soon after volume one of the *Selected Papers on History, Logic and Foundations of Mathematics: Nonstandard Analysis* circulated among scholars, the Institute of Mathematics of the Chinese Academy of Sciences compiled a second volume. In this volume, there are two translations, one of Robinson's article, "Non-Standard Analysis," which was originally published in 1961 in the *Proceedings of the Koninklijke Nederlandse Akademie Van Wetenschappen,* and in *Indagationes Mathematicae*.[41] The other, "Truth with Respect to an Ultrafilter or How to Make Intuition Rigorous," by Donovan H. van Osdol, was originally published in 1972 in the *American Mathematical*

[39] YUAN Meng 1976. Besides the article discussed here, YUAN also wrote an article introducing the basics of nonstandard analysis, see YUAN Meng 1977.

[40] See LIAO Zuwei 1976.

[41] See Robinson 1961. The translation (on pages 97-109) was done by the mentioned SHU Ji of Northwest University in Xi'an. For the background of the paper and detailed discussion of its contents, see Dauben 1995, 283-287.

Monthly.[42] The former is the first article in which Robinson revealed his revolutionary idea of nonstandard analysis. The latter by van Osdol introduced some basics of nonstandard analysis by using a similar approach to the one W. A. J. Luxemburg took in his *Lecture Notes on Nonstandard Analysis*, which used an ultrafilter construction of the nonstandard reals rather then the approach using model theory and symbolic logic that Robinson had taken.

Three articles in the second volume are by Chinese scholars. One is an introductory presentation of nonstandard analysis by DAI Zhizhong 戴執中 of Jiangxi University, whereas the other two are research papers by LI Banghe.[43] The first one, "Zai yige fei Ajimide yu shang de fenxi 在一個非阿基米德域上的分析 (Differential and Integral Calculus on a Non-Archimedean Field," was published in *Acta Mathematica Sinica* in 1979. In this paper, LI investigated differential and integral calculus and generalized power series with real coefficients and real exponents by using the method of nonstandard analysis. Specifically, LI provided necessary and sufficient conditions for the intermediate value theorem and the mean value theorem of the differential calculus for some functions in a field *L* of generalized power series with real coefficients and real exponents. Moreover, similar necessary and sufficient conditions are also given for such theorems as assuming the maximum value on a closed interval, Rolle's theorem,

[42] See van Osdol 1972. This translation was done by WU Wangming 吳望名, pp. 110-123.

[43] In an e-mail message dated November 19, 2001, to the author, Professor LI Banghe writes: "I was the first one who wrote creative papers on nonstandard analysis in China in 1975. I advocated the benefits of nonstandard analysis during a meeting held at the Institute of Mathematics of the Chinese Academy of Sciences in 1975 or so. I also participated in the seminars organized by ZHANG Jinwen."

Lagrange's form of the remainder of Taylor's expansion, Cauchy's theorem, and the mean value theorem of integral calculus.[44]

LI's second article, "Fei biaozhun fenxi yu guangyi hanshu chengfa (I) 非標準分析與廣義函數的乘法（I）(Nonstandard Analysis and Multiplication of Distributions)," was published in 1978 in the most prestigious journal in China, *Scientia Sinica*. In this article, LI discusses how to use a nonstandard analytic representation of distributions to define products of distributions. The product may or may not be a distribution. The paper provides some explicit calculations for various products occurring in applications.[45]

The two volumes of the *Selected Papers on History, Logic and Foundations of Mathematics: Nonstandard Analysis* reflect the serious regard Chinese mathematicians had for nonstandard analysis, the connection with Marx's mathematical manuscripts, and how quickly Chinese mathematicians were able to make their own original contributions to this nascent field. How Chinese mathematicians used nonstandard analysis to justify Marx's views on the differential has already been discussed. In order to show how discussions of Marx's mathematical manuscripts served to promote nonstandard analysis in China, one more example is illuminating, namely an article, "Dian de kefenxing yu fei biaozhun fenxi 點的可分性與非標準分析 (The Divisibility of Point and Non-standard Analysis)," again by ZHANG Jinwen.[46]

In his *Mathematical Manuscripts*, Marx commented on tangent problems as discussed in Jean-Louis Boucharlat's *Élémens de calcul différential et de calcul intégral*

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[44] See LI Banghe 1976c; 1979.

[45] LI Banghe 1976d; 1978.

[46] ZHANG Jinwen 1977a.

(*Elements of the Differential Calculus and the Integral Calculus*, Paris, 1838). Among his comments, Marx wrote: "The two sides of the auxiliary triangle are made by *dx* and *dy*. They are smaller than points. Therefore, under this circumstance one can dare to take the chord as equal to the arc or *vice versa*."[47] ZHANG Jinwen links what Marx has said about the divisibility of points with infinitesimals.[48] Using nonstandard analysis, he "proved" that the divisibility of points and infinitesimals is a necessary and sufficient condition as follows:

> If a point can be divided, and has some parts, then there exist infinitesimals. This is because taking a small point x_1 from one part of the point *x*, and taking another small point x_2 from another part of the point *x*, $x_1 - x_2$ must be an infinitesimal. Otherwise, $|x_1 - x_2|$ is a positive real number, and there is a real distance between x_1 and x_2. Thus, x_1 and x_2 are not small points within different parts of the point x, and that contradicts the premise. Therefore, $x_1 - x_2$ must be an infinitesimal. Consequently, that points can be divided means that infinitesimals exist.
> One the other hand, if infinitesimals exist then points can be divided. This is based on the following reasoning: If an infinitesimal η exists, then for any point *x* of a real number, $x - \eta$ and $x + \eta$ are points in different parts of *x*, which is because the distance between them is an infinitesimal, otherwise the two points would be two points of real numbers.
> Standard analysis is based on a traditional view of the indivisibility of points, and naturally excludes infinitesimals. In the calculus, the term "infinitesimal" is used, but it is a potential one (namely, a variable or a function with 0 as its limit). Standard analysis uses the method of limits, but nonstandard analysis employs the method of infinitesimals. Thus, points can be divided.
>
> 如果點是可分的，有部分的，則存在無限小量。因爲任取點 *x* 的一部分中的一小點記作 x_1，再另取一個不同的部分中的一小點 x_2，那麼 $x_1 - x_2$ 必然是一個無限小量，否則 $|x_1 - x_2|$，為一正實數，x_1 與 x_2 之間有一個實數距離，這樣 x_1 與 x_2 就是不同的點了，而不再是在點 *x* 的兩個不同的部分中了，與前提相衝突。因此，$x_1 - x_2$ 必須是一無限小量，也就是說，點可分就存在着無限小量。
>
> 反之，如果存在無限小量，則點就是可分的了。因爲如果存在一個無限下量 η，那麼對於任意一個實數點 *x*，$x - \eta$ 和 $x + \eta$ 是屬於點的不同部分，這是因爲

[47] See Marx 1968, 330-331; 1975b, 22-24.

[48] There are other scholars who have made similar links. For instance, XUE Shu 1977a, 26-27.

> $x - \eta$ 與 $x + \eta$ 的距離是一個無限小量，而任意兩個實數點之間的距離都只能是一個實數，而不能是一个無限小量。
>
> 標準分析由於依據點的傳統的不可分的概念，自然要排除無限小量，排除它作爲論域的對象，作爲微分學領土上的成員，雖然那裏也用無限小量這個名稱，但那只是一個虛名了，人們稱爲潛在無限小或潛無限小（即以 0 為極限的變量或函數）。標準分析使用的是極限方法，而非標準分析運用的是無限小量方法，這樣，點也是可分的了。[49]

Once the link between Marx's views and infinitesimals had been "firmly established," ZHANG proceeds to discuss infinitesimal methods in nonstandard analysis, and concludes that "Nonstandard Analysis by using the methods of mathematical logic has correctly solved the logical dilemma [of Newton's and Leibniz's infinitesimals], and has made great progress for the methods of infinitesimals. The new methods and concepts of nonstandard analysis will have a deep and far reaching impact on the development of mathematics and applications of mathematics as well. It is a very important research field to which one should pay attention."[50]

The reception of nonstandard analysis in China, however, was not unanimously warm and welcoming. Immediately after publication of ZHANG Jinwen's article, two mathematicians at Fudan University, LI Jizhong 李繼宗 and YAN Shaozong 嚴紹宗, wrote an article entitled "Dui fei biaozhun fenxi de jidian kangfa 對非標準分析的幾點看法 (Some Points of View on Nonstandard Analysis," in which they criticized ZHANG's rosy view. From the standpoint of materialist dialectics, LI and YAN argued that nonstandard analysis did not conform with what Marx and Engels had said about infinity, and nor did it provide a solid foundation for the calculus. They cautioned: "Do not rely upon dialects and only use deductive methods of formal logic (including mathematical logic) to find

[49] ZHANG Jinwen 1977a, 48.

[50] ZHANG Jinwen 1977a, 53.

foundations for the calculus, or forcefully cut the feet of the calculus to fit the shoes of the deductive system of mathematical logic; all of these are doomed to failure."[51]

Unlike LI's and YAN's total rejection or ZHANG's whole-hearted embrace of nonstandard analysis, some Chinese mathematicians took a middle ground between these two opposite views. On the one hand, they acknowledged that nonstandard analysis rigorously proved the existence of infinitesimals and also provided a powerful method for dealing with infinitesimals. They also believed that nonstandard analysis did put the calculus on a solid foundation. But on the other hand, they held that the philosophical assumptions of the founder of nonstandard analysis, Abraham Robinson, must be criticized, and pointed out that he was not only a bourgeois mathematician and a logical positivist, but his philosophy of mathematics was a mixture of logical positivism and pragmatism.[52]

Using Marx's mathematical manuscripts and Mao's thoughts to evaluate nonstandard analysis was something Abraham Robinson had never considered. But he did foresee the dialectical approach and once wrote: "I can well imagine that a serious mathematical philosophy based on the dialectical approach will make its appearance…. As far as the detailed analysis of mathematics or of a mathematical theory (e.g. the calculus) by the dialectical method is concerned, my reading, beginning with Hegel's

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[51] LI and YAN 1977, 53. ZHANG Jinwen responded to the criticisms of LI and YAN almost immediately. For details, see ZHANH Jinwen 1977b. Other scholars also criticized the article by LI and YAN; see for example CHENG 1977, and LIN Xiashui 1979.

[52] Among these who took middle position was ZHOU Guanxiong 周冠雄 of Central China Industrial College. ZHOU wrote two articles evaluating nonstandard analysis, one entitled "Using the Philosophy of Marxism to Evaluate Nonstandard Analysis" (ZHOU Guanxiong, 1977a), the other "Judging Non-standard Analysis Should Take Two Different Views" (ZHOU Guanxiong, 1977b). For a detailed discussion of the first of these, see Dauben 2003, 357-359.

work in this area, has not led me to find anything that can stand up to serious criticism."[53] Robinson was right. The dialectical approach to examining the foundations of the calculus taken by Chinese scholars in the 1970s not only did not reach as serious a level as Robinson had expected but it could not "stand up to serious criticism." Within a decade, Chinese mathematicians were no longer obsessed with finding appropriate Marxist critiques or foundations for mathematicians, but were instead back to concentrating on pure mathematics.

In retrospect, discussions of the foundations of the calculus in terms of Marx's mathematical manuscripts and of nonstandard analysis focused Chinese scholars' attention on both infinitesimals and infinity. From a purely mathematical point of view, these discussions may have added few valuable insights to foundational issues, but it provided an opportunity for Chinese mathematicians and philosophers of mathematics to ponder questions related to infinity. This also stimulated their interest in the historical development of the concept. The translation of two influential books, Carl Boyer's *The History of the Calculus and Its Conceptual Development*, and Morris Kline's *Mathematical Thought from Ancient to Modern Times*, added basic historical details to what Chinese scholars could read about Western mathematics and especially the development of the calculus.[54] In turn their historical and foundational concerns aided

[53] Quoted from Robinson 1979, 566.

[54] Carl Boyer's *The History of the Calculus and Its Conceptual Development* was translated by a group of mathematicians at Shanghai Teachers College (East China Normal University). The main translators were LI Ruife 李銳夫, ZHOU Kexi 周克希, WEI Zongshu 魏宗舒, LING Kangyuan 凌康源, and ZHOU Pengnian 周彭年. The Chinese title of the book is: 微積分概念史. The Chinese translation was published by Shanghai Renmin Chubanshe in 1977. Morris Kline's *Mathematical Thought from Ancient to Modern Times* was translated by a group of mathematicians at Beijing University. The Chinese version, entitled 古今數學思想, was published by Shanghai Kexue Jishu Chubanshe in 1979.

the introduction of nonstandard analysis to Chinese mathematicians during the tumultuous period of the Cultural Revolution.

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CHINESE and JAPANESE

ABBREVIATION
Books and Collections:

BZFLSXZ *Zhongguo ziran bianzhengfa yanjiu lishi yu xianzhuang* 中國自然辯證法研究歷史與現狀 (*Research in Dialectics of Nature in China: Its History and Present Status*), eds. Zhongguo ziran bianzhengfa yanjiuhui 中國自然辯證法研究會, and Ziran bianzhengfa yanjiu ziliao bianjizu 自然辯證法研究資料編輯組 (Beijing: Zhishi chubanshe, 1983).

FBZFX *Fei biaozhun fenxi* 非標準分析（I, II）(*Nonstandard Analysis*), ed. Zhongguo kexueyuan shuxue yanjiusuo ziliaoshi 中國科學院數學研究所資料室. Mimeograph in October of 1976. Remimeographied by Guiyang shifan xueyang shuxuexi 貴陽師範學院數學係 in August of 1977.

HNZ *Huainan Zi* 淮南子, 2 vols., 1897 tushu jicheng ben.

LQ *LI Yan, QIAN Baocong kexueshi quanji* 李儼錢寶琮科學史全集 (*The Completed Works by LI Yan and by QIAN Baocong*), 10 vols., ed. DU Shiran 杜石然 (Shengyang: Liaonin jiaoyu chubanshe, 1998).

JASSYLU *Jiuzhang suanshu yu LIU Hui*《九章算術》與劉徽 (*The Nine Chapters on Mathematical Procedures and LIU Hui*), ed. WU Wenjun 吳文俊 (Beijing: Beijing shifan daxue chubanshe, 1982).

KXCBS *Zhongguo xiandai kexuejia zhuanji* 中國現代科學家傳記 (*Biographies of Contemporary Chinese Scientists*), 6 vols., ed. Kexue chubanshe zhuanji bianjibu 科學出版社傳記編輯部 (Beijing: Kexue chubanshe, 1990-1994).

MQSXSLWJ *Ming Qing shuxueshi lunwenji* 明清數學史論文集 (*Collected Papers on the History of Mathematics during the Ming and Qing Dynasties*), ed. MEI Rongzhao 梅榮照 (Nanjing: Jiangsu jiaoyu chubanshe, 1990).

SBBY *Sibu Beiyao* 四部備要 (*Essentials of the Four Libraries*), Shanghai zhonghua shuju 1936 edition.

TH Zhongguo kexue jishu dianji tonghui: Shuxue juan 中國科學技術典籍通匯: 數學卷 (A Compendium of Chinese Classics of Science and Technology: Mathematics Section), 5 vols., eds. GUO Shuchun 郭書春 et al. (Zhengzhou: Henan jiaoyu chubanshe, 1993).

ZGSXJS *Zhongguo Shuxue Jianshi* 中國數學簡史(*A Brief History of Chinese Mathematics*), Bianxiezu 編寫組 (Jinan: Shangdong jiaoyu chubanshe, 1986).

ZGSXSLWJ *Zhongguo shuxueshi lunwenji* 中國數學史論文集 (*Collected Papers on the History of Chinese Mathematics*), 6 issues, first five are edited by WU Wenjun 吳文俊, issue 6 by BAI Shangshu 白尚恕, LI Di 李迪 and SHEN Kangshen 沈康身 (Jinan: Shangdong jiaoyu chubanshe, 1985-1996).

ZJSHMZJ *Zhangjiashan Han mu zujian* (*ersiqi hao mu*) 張家山漢墓竹簡(二四七號墓) (*Bamboo Strips from Han Tombs at Zhangjiashan*), edited by Zhangjiashan ersiqi hao Han zhujian zhengli xiaozu (Beijing: Wenwu chubanshe, 2001, pp. 83-96; 249-272.).

Journals:

BD *Beijing daxue xuebao* 北京大學學報 (*Journal of Beijing University, Natural Sciences*), Beijing,

BSD *Beijing shifan daxue xuebao* 北京師範大學學報 (*Journal of Beijing Normal University, Natural Sciences*), Beijing,

FD *Fudan xuebao* 復旦學報 (*Fudan Journal, Natural Sciences*), Shanghai,

HZGXY *Huazhong gongxueyuan xuebao* 華中工學院學報 (*Journal of Central China Industrial College*), Wuhan,

KX *Kexue* 科學 (*Science*), Ithaca, New York; Shanghai, 1915-1950.

KXTB *Kexue tongbao* 科學通報 (*Science Bulletin*), Beijing, 1950-

KXSJK *Kexueshi jikan* 科學史集刊 (Collected Papers on the History of Science), Beijing, 1958-1984, 11 issues.

SXDSJYRS *Shuxue de shijian yu renshi* 數學的實踐與認識 (*Practice and Understanding of Mathematics*), Beijing, 1971-1994, 95 issues.

SXJZ *Shuxue jinzhan* 數學進展 (*Advances in Mathematics*), Beijing, 1955-

SXTB *Shuxue tongbao* 數學通報 (*Mathematics Bulletin*), Beijing, 1951-

SXXB *Shuxue xuebao* 數學學報 (*Acta Mathematica Sinica*), Beijing, 1951-

ZGKJSL *Zhongguo keji shiliao* 中國科技史料 (*China Historical Materials of Science and Technology*), Beijing, 1980-

ZGKX *Zhongguo kexue* 中國科學 (*Scientia Sinica*), Beijing, 1952-

ZRKXZM *Ziran kexue zhengming* 自然科學爭鳴 (内部) (*Debate on Natural Sciences, internal use only*), Beijing, 1975-1977, 10 issues.

ZRBZFTX *Ziran bianzhenfa tongxun* 自然辯證法通迅 (*Journal of Dialectics of Nature*), Beijing, 1978-

ZRBZFYJ *Ziran bianzhenfa yanjiu* 自然辯證法研究 (*Studied in the Dialectics of Nature*), Beijing, 1985-

ZRBZFYJTX *Ziran bianzhenfa yanjiu tongxun* 自然辯證法研究通迅 (*Correspondence of Research on the Dialectics of Nature*), Beijing, 1956-1960; 1963-1965.

ZRBZFZZ *Ziran bianzhenfa zazhi* 自然辯證法雜誌 (*Magazine of the Dialectics of Nature*), Shanghai, 1972-1976, 13 issues.

ZRKXSYJ *Ziran kexueshi yanjiu* 自然科學史研究 (*Studies in the History of Natural Sciences*), Beijing, 1982-

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Vitae

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1. "The First Chinese Translation of the Last Nine Books of Euclid's *Elements* and its Source," *Historia Mathematica* **32**, no. 1(2005):4-32.

2. "*Mojing* yu *Zhuangzi* youmeiyou shuxue wuqiong gainian?《墨經》與《莊子》有沒有數學無窮概念？(Do the *Moist Canon* and *Zhuangzi* Really Have Concepts of Mathematical Infinity?)," in *Lixing, xueshu he diode de zhishi chuantong* 理性、學術和道德的知識傳統 (*Proceedings of the Symposium on New Meanings of Traditional Chinese Civilization in the 21st Century (III), held in Taipei, Taiwan, December 27-29, 2001*), ed. Thomas H. C. Lee. Taipei: The Himalaya Foundation, 2004, pp. 131-160 (in Chinese).

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5. "Chinese-U.S. Mathematical Relations, 1859-1949," in *Mathematics Unbound: The Evolution of an International Mathematical Community, 1800-1945*, edited by Karen H. Parshall and Adrian C. Rice. Providence, Rhode Island: American Mathematical Society & London: The London Mathematical Society, 2002, pp. 287-309.

6. "Diyiben zhongyi X shexian zhuzuo: *Tongwu dianguang*" 第一本中譯 X 射綫著作—《通物電光》*Tongwu dianguan*: The First Chinese Translation of a Book on X-rays), *Kexue jishu yu bianzhengfa* 科學技術與辯證法 (*Science, Technology and Dialectics*) **19**, no. 3 (2002):76-80 (with LI Di, in Chinese).

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10. "Qingdai dao minguo chunina ge minzu de ziran kexue chengjiu: Wulixue yu huaxue" 清代到民國初年各民族的自然科學成就：物理學與化學 (Chinese Minority Scholars' Contributions to Physics and Chemistry in the Late Qing Dynasty and the Early Republic), in LI Di, ed., *A History of Chinese Minorities' Science and Technology* (introductory volume), Nanning: Guanxi kexue yu jishu chubanshe, 1996, pp. 452-460 (in Chinese).

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