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Extreme value analysis of return time series: Stock market volatility and its possible causes revisited

Pericli, Andreas Neophytou, Ph.D.

City University of New York, 1993

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

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Extreme Value Analysis of Return Time Series:

Stock Market Volatility and its Possible Causes Revisited

by

Andreas N. Pericli

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Chair of Examining Committee

August 24, 1993
Date

Michael Grossman
Executive Officer

Ronald W. Anderson

Michael Grossman

Salih N. Neftci
Supervisory Committee

THE CITY UNIVERSITY OF NEW YORK

ABSTRACT

EXTREME VALUE ANALYSIS OF RETURN TIME SERIES: STOCK MARKET VOLATILITY AND ITS POSSIBLE CAUSES

by

Andreas N. Pericli

Asvisor: Salih Nefchi

At the moment there is a plethora of research trying to examine market uncertainty or market risk through time. Most of the outstanding research links market risk to the standard deviation of the conditional or unconditional distribution of market returns. This study approaches market risk by considering the tails of the distribution of returns only. This new approach, in examining market risk through time, relies on two important observations: i) market participants are highly concerned with the extreme returns of their portfolio, and ii) extreme negative returns coincide with severe economic problems such as financial failures, business failures, and recessions. To this end, we consider the extreme returns of a well diversified portfolio to follow a stochastic process and we estimate crossing rates and quantile values on these extremes to see if they are statisically different through time. We also examine whether changes in the crossing rates or quantile values are correlated with important economic events.

Aknowledgements

I am very indebted to Professors Michael Grossman and Salih Neftci for their academic support and guidance. For the completion of this work, I am fully grateful to professor Salih Neftci. To Peter Theodossiou a deeply felt aknowledgement for his academic help.

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1. Measures of Uncertainty in Asset Prices: A Brief Review of the Literature

After the pioneering papers by Markowitz (1952) and Tobin (1958), it was well understood that risk averse investors will require higher return for higher volatility in returns (uncertainty). Both Markowitz and Tobin, in their theories, quantify volatility (risk) of an investement choice by computing the standard deviation of the distribution of returns (in Tobin's analysis, return is the interest rate on government securities).

Quantifying volatility, in a sensible way, is central to much modern finance theory. The valuation of risky securities requires computation of risk premium that is determined by the covariance between the future return on the security and the market portfolio. Measuring volatility associated with the future price of the underlying asset is the most important task on the valuation of derivative securities, including options, warrants and convertibles. [1]

In this section, we intend to review the most cited approaches, in the finance literature that have been employed to measure volatility in speculative prices. We proceed by deviding all the proposed measures of volatility in asset prices into two general categories: (a) The Autoregressive Conditionally Heteroskedastic (ARCH) volatility measures and (b) the alternative volatility measures.

1.1 ARCH Measures of Volatility:

Prior to 1982, econometric modeling of financial data was based on the assumption that uncertainty, as measured by variances and covariances, was constant through time. This unrealistic assumption [2]was necessary because, by that time, no model was flexible enough to allow time variation in the second or higher

[1] See Black and Scholes (1973)

[2] Among others, Fama (1965) indicated that variances and covariances of stock returns are changing through time.

moments. The pioneering paper of Engle (1982) and its various extentions provide the tools that allow the second or higher moments of financial series to be conditioned on time.

1.1.1 The Linear ARCH(p) Model:

Engle (1982) suggests a stochastic series of the form,

$$\epsilon_t = z_t(\sigma_t^2)^{\frac{1}{2}} \tag{1.1.1.1}$$

where z_t is an iid process, $E(z_t) = 0$, $Var(z_t) = 1$, and σ_t^2 a linear function of past square values of the process,

$$\sigma_t = a_0 + \Sigma_{i=1}^{q} a_i \epsilon_{t-i}^2 \tag{1.1.1.2}$$

where $a_0 > 0$ and $a_i \geq 0$, and assuming,

$$\epsilon_t | \psi_{t-1} \sim N(0, \sigma_t^2) \tag{1.1.1.3}$$

where ψ_{t-1} the information set availiable at period t-1, we have what Engle called as autoregressive conditional heteroskedastic process which could be used to model financial series whose conditional variances are changing through time, while their unconditional variance remains constant. In particular (1.1.1.2) captures the tendency for volatility clustering, i.e. large (small) price changes to be followed by other large (small) price changes of unpredictable sign.

In most cases, ϵ_t corresponds to the innovation to the mean of some other stochastic process, y_t, where,

$$y_t = f(x_{t-1}; b) + \epsilon_t \tag{1.1.1.4}$$

and $f(x_{t-1}; b)$ is a function of x_{t-1} and the parameter vector b, x_{t-1} belongs in the ψ_{t-1} information set.

1.1.2 The Linear Generalized Autoregressive Conditional Heteroskedastic (GARCH) model.

Bolleslev (1986) generalized the ARCH process by allowing lagged conditional variances to be part of the process. This alternative is called the GARCH (Generalized Autoregressive Conditional Heteroskedastic) process, and given (1.1.1.1),

$$\sigma_t^2 = a_0 + \Sigma_{1=1}^{q} a_i \epsilon_{t-i} + \Sigma_{i=1}^{p} \beta_i \sigma_{t-i}^2 \qquad (1.1.2.1)$$

where

$$p \geq 0, q > 0$$

$$a_0 > 0, a_i \geq 0, i = 1, ..., q$$

$$\beta_i \geq 0, i = 1, ..., p$$

For $p = 0$ the process reduces to the ARCH(q) process, and for $p = q = 0$ ϵ_t is a white noise. In the ARCH(q) process the conditional variance is defined as a linear function of past sample variances only, whereas the GARCH(p,q) process allows conditional variances to be part of the process as well.

1.1.3 Non-Linear and Non-Parametric ARCH:

While the GARCH(p,q) model has been applied extensively, in modelling the conditional variances on speculative prices, its structure seems to be in contrast with the empirical behavior of stock prices where leverage effect is present. The leverage effect theory is based on the idea that as equity values fall the debt to equity ratio rises, increase in the leverage increases the likelihood that the firm will go bankrupt and consequently raises the risk of the firm. This theory was based on empirical evidence [3] and implies that stock returns are negatively correlated with changes in stock return volatility. The exponential GARCH(p,q), or

[3] See Black (1976).

EGARCH(p,q), model introduced by Nelson(1990) allows σ_t to respond asymmetrically to negative and positive residuals. Given (1.1.1.1), Nelson suggested

$$\log \sigma_t^2 = a_0 + \Sigma_{i=1}^{q} a_i(\phi z_{t-i} + \gamma[|z_{t-i}| - E|z_{t-i}|]) + \Sigma_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 \qquad (1.1.3.1)$$

In addition, the EGARCH(p,q) model, in comparison with the GARCH(p,q), model imposes no restrictions on the parameters a_i and β_i. Given $a_i\phi < 0$, the variance tends to rise (fall) when ϵ_{t-i} is negative (positive). This asymmetric behavior of the variance is compatible to empirical evidence.

Alternative parametric ARCH formulations have been considered in the literature, including power transformations of ϵ_t^2 as in the nonlinear ARCH model of Higgins and Bera (1989) and a threshold ARCH model as in Zakoian (1990). In the threshold approach, the σ_t^2 is a linear piecewise function which allows different reactions of volatility to different signs and magnitudes of the shocks.

Several non-parametric techniques have also been proposed in order to approximate the conditional variance. Such approaches include Kernel methods and Fourier Forms. In the Kernel methods σ_t^2 is estimated as a weighted average of ϵ_t^2. Several weighting schemes are possible but the most frequent in the estimation of ARCH models have been Gaussian Kernels. The flexible fourier transformation introduced by Gallant (1981) approximates σ_t^2 by a function of polynomial and trigonometric terms in lagged values of ϵ_t.

1.1.4 ARCH in Mean Models:

The ARCH in mean models, or ARCH-M, model was introduced by Engle, Lilien and Robins (1987). This model assumes that the conditional mean of some process y_t is an explicit function of the conditional variance of the process. In short, given (1.1.1.1),

$$y_t = f(x_{t-1}, \sigma_t^2; b) + \epsilon_t \qquad (1.1.4.1)$$

In this specification, the conditional mean of y_t will depend on the value of the conditional variance σ_t^2. Increase in the conditional variance, σ_t^2, will have a positive effect on the conditional mean of y_t depending on the sign of the partial derivative of $f(x_{t-1}, \sigma_t^2; b)$ with respect to σ_t^2. The significance of this specification, in finance, comes from the positive relationship that exist between risk, as measured by variance, and the expected return.

1.2 Alternative Measures of Volatility:

Several alternative measures to the ARCH model defined above have been employed in describing uncertainty in stock prices. One such alternative measure was proposed by Officer (1973). Officer uses monthly returns of the NYSE index and estimates rolling 12-month standard deviations as indicators of annual return variability. His approach was criticized, by French, Schwert and Stabaugh (1987), for two reasons: First, the use of monthly returns, rather than daily, eliminates the precision and accuracy of the variability indicator. Second, the rolling 12-month variability estimator uses overlapping samples of returns, consequently annual variability changes will be smoother.

A second alternative approach to the ARCH models discussed above was proposed, originally, by Merton (1980). Merton constructed the monthly stock return variance by adding the squared daily returns in the month plus twice the products of the adjacent returns:

$$\sigma_{mt}^2 = \Sigma_{i=1}^{N_t} r_{it}^2 + 2\Sigma_{i=1}^{N_t-1} r_{it} r_{i+1,t} \qquad (1.2.1)$$

where there are N_t daily returns, r_{it}, in month t [4]. This formulation accounts for first order serial correlation in the returns of portfolios induced by non-synchronous trading[5]. French, Schwert, and Stambaugh (1987) fitted an ARIMA

[4] French, Schwert, and Stambaugh (1987) tried several modifications of (1.2.1), including: (i) substracting the within month mean return from each observation and (ii) ignoring the ross-products. These modifications had little effect on the variance.

[5] For evidence on this issue see Fisher (1966) and Scholes and Williams (1977).

process on the estimates of (1.2.1) in order to construct monthly conditional variances. This method does not make efficient use of the data. In addition, Pagan and Ullah (1988) indicated that if the information matrix for the full model is not block diagonal between the parameters in the mean variance, the actual parameters may be inconsistent.

Schwert (1989,1990b) proposed the estimation of conditional standard deviations, as measures of variability, in a two step procedure. In the first step, an ARMA model is fitted on the returns. In the second step, an ARMA model is fitted on the residuals derived from the first step. The absolute conditional mean estimates of the second fit are the conditional standard deviations.

Another popular approach for assessing the volatility in the financial data is based on the implied volatility from option theory. Under specific assumptions Black and Scholes (1973) derived a valuation formula for options. The implied volatility may be calculated from the Black and Scholes formula. Even this method may yield results superior to ARCH alternatives, not all financial assets have actively traded options.

A further way for assessing the volatility in financial data is to make use of daily high and low prices. Parkinson (1980) showed that if the logarithm of asset prices follows a continuous random walk with constant instantaneous variance then the use of high/low prices provides 5 times more efficient variance than closing prices.

2. Extreme Value Theory: Stock Market Volatility Revisited

Several severe stock market drops have occured, during the last few years, within a short period of time. The October, 1987 and October, 1989 are prominent examples. This huge losses in private and social welfare, within short period of time, have alarmed investors, regulators, and the public in general. Moreover, the recent interest in stock market volatility has been spurred by these severe stock market drops. In this study, we employ extreme value theory in order to analyze the volatility of the extreme stock market returns, their magnitudes, and their possible causes, through time. Towards this end, we plan to do one or more of the following:

i) Estimate the frequency with which specified extreme return levels are crossed, and especially examine whether there is any statistical evidence of the frequency of the extremes changing through time. We consider a statistical increase in the frequency of the extremes (in particular the frequency of the negative return extremes) through time as increase in market risk. This is because increase in the frequency of the negative return extremes through time increase the likelihood of a severe stock market drop.

ii) Estimate extreme quantile values on the extreme returns and examine whether there is any statistical evidence the quantile values changing over the period under study. We consider the quantiles of extreme stock market returns as risk indicators since they provide measures for maqnidute comparison of the extremes through time.

iii) Estimate probabilities by which specific extrema occur, and examine whether there is any statistical evidence of the probabilities changing through time. The estimation of probabilities with which specific extreme returns occur may be used to compare whether the likelihood for having such extremes changes through time.

In this study, we assess the risk for extreme return occurances by using the first two approaches only. The third approach can be derived from our results very easily. We also examine whether any changes in the negative extreme return occurances are correlated with important economic events such as: i) recessions. ii) the creation of the security exchange commission (or the establishment of margin requirements). iii) the start of options trading.

While there are several methods of analyzing extremes, we have found the threshold method to fit better to our case. The threshold method is based on the distribution of exceedances over a high threshold u (say). Under the assumption that these extremes (exceedances over the high threshold (u)) are independent, and identically distributed, their limiting form converges to the Generalized Pareto Distribution[6] . For references on this approach see Smith (1984,1989), Davison (1984), Hosking and Wallis (1987), and Davison and Smith (1990).

Alternative methods for analyzing extremes include the traditional approach and the point process approach. The traditional approach relies on the asymptotic distribution of the maximum and minimum of independent, and identically distributed random variables. The classical reference on this approach is Gumbel (1958), while more recent references include Prescott and Walden (1980,1983), and Hosking, Wallis, and Wood (1985).

The most recent approach of analyzing extremes is based on viewing the high level exceedances over a fixed threshold as points of a poisson process. This method is more general and includes both the traditional and threshold approaches as special cases. For references on this approach see Leadbetter, Lindgren and Rootzen (1983), Resnick (1987) and Smith (1989). [7]

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[6] The mathematical form of the Generalized Pareto Distribution is given in section (3.2)

[7] This approach has been tried, originally, but the regularity conditions for maximum likelihood extimation were not satisfied. The estimation procedure of

Section 3 briefly reviews the extreme value methods we discussed above, and presents the statistical methodology for fitting these methods on samples of extreme stock market returns. Section 4 contains the preliminary analysis, while section 5 presents the statistical results of this research, and section 6 provides a summary.

In this study, we make use of three different indices: (i) The value weight index, (ii) The Smith and Cole's index and (iii) The Dow Jones Industrial Average index. The value weighted NYSE index is composed from daily returns which have been taken from CRSP data bases, and begin July 1962 and end December 1990. The Smith and Cole's index involves monthly returns from July, 1948 through December, 1884[8]. The Dow Jones Industrial Average index involves monthly returns from January, 1985 through December, 1991. These returns have been constructed from monthly average prices of the index.

this method is presented in section 3.

[8] Schwert (1990) explain the formation of this index.

3. Extreme Value Theory: A Brief Review

3.1. The Classical Approach:

Suppose X_1,X_2,...denote an i.i.d sequence of observations with common distribution function F and $M_n = max(X_1, ..., X_n)$. Classical Extreme Value Theory models look for normalizing sequence $\alpha_n > 0$, β_n such that $(\frac{M_n - \beta_n}{\alpha_n})$ converges in distribution so that

$$\lim_{n\to\infty} Pr[(\frac{M_n - \beta_n}{\alpha_n}) \leq \chi] = \lim_{n\to\infty} F^n(\alpha_n\chi + \beta_n) = H(\chi) \qquad (2.1.1)$$

where $H(\chi)$ is a nondegenerate distribution function. Given $H(\chi)$, the distribution of the random variable X_j does not matter any more, provided α_n and β_n can somehow be calculated. Furthermore, it is not necessary to know the detailed nature of F to know which limiting form (if any) it gives rise to, i.e. which "domain of attraction" it belongs to[9] . In fact, this is determined just by the behavior of the tail of $F(\chi)$ for large χ. Consequently a good deal may be said about the asymptotic properties based on a rather limited knowledge of F.

The convergence in equation (3.1.1) occurs if and only if:

$$\lim_{n\to\infty}[n(1 - F(\alpha_T\chi + \beta_T)] = -logH(\chi) \qquad (3.1.2)$$

Given (3.1.2) is satisfied, $H(\chi)$ takes one of the following three limiting distributions:

$$H(\chi) = e^{-e^{\chi}}, -\infty \leq \chi \leq \infty.$$

$$H(\chi) = \begin{cases} 0, & \chi \leq 0; \\ e^{-\chi^{-\alpha}}, & \text{for some } \alpha > 0, \chi > 0; \end{cases} \qquad (3.1.3)$$

$$H(\chi) = \begin{cases} e^{-(-\chi)^{-\alpha}} & \text{for some } \alpha > 0, \chi \leq 0; \\ 1, & \chi > 0; \end{cases}.$$

[9] The limiting form of $H(\chi)$ is provided in the next page.

known as the three types of extreme value limiting distributions [10] . These three types of distributions can be combined into a single generalized extreme value distribution which takes the following form:

$$H(\chi;\mu,\sigma,\kappa) = e^{-(1-\kappa(\frac{\chi-\mu}{\sigma}))^{\frac{1}{\kappa}}} \tag{3.1.4}$$

known as the generalized extreme value distribution function.

The above is valid only in the range of χ that makes $1-\kappa(\frac{\chi-\mu}{\sigma}) > 0$ where σ, μ, and κ are the parameters of the distribution with $\sigma > 0$, μ, and κ being any real numbers.

The case $k = 0$ being interpreted as the $\lim_{\kappa\to 0}$,

$$H(\chi,\mu\,\sigma,0) = e^{e^{-(\frac{\chi-\mu}{\sigma})}} \tag{3.1.5}$$

is widely called the Gumbel distribution.

The parameter κ is called the shape parameter and can be used to model a wide range of tail behavior. The case $\kappa < 0$ is that of a polynomially decreasing tail function and therefore corresponds to a long tail parent distribution. The case $\kappa = 0$ is that of an exponentially decreasing tail function, while the case of $\kappa > 0$ is the case of a finite upper endpoint and therefore a short tailed function.

3.2 The Threshold Approach:

The threshold approach is based on the distribution of exceedances over a prespecified high threshold. Given that an observation exceeds a prespecified threshold, say u, the probability that it exceeds it by at least y is (1-F(u+y)/(1-F(u)). Assuming that conditions (3.1.1) and (3.1.2) hold, the extremes can be approximated for a large u by the family

$$G(y;\sigma,\kappa) = 1-(1-\frac{\kappa y}{\sigma})^{\frac{1}{\kappa}} \tag{3.2.1}$$

[10] These three limiting forms have been developed form the works of Fisher and Tippett (1928) and Gnedenko (1943).

known as the Generalized Pareto Distribution, GDP. This distribution is valid when $0 < y < \infty$ for $k \leq 0$, or when $0 < y < \frac{\sigma}{\kappa}$ for $\kappa \geq 0$. When $\kappa = 0$ the limiting form converges to the Exponential Distribution. The distribution function of the exponential distribution is:

$$G(y;\sigma) = 1 - e^{-\frac{y}{\sigma}} \tag{3.2.2}$$

The generalized extreme value distribution, and the Generalized Pareto Distribution, may be fitted to data using maximum likelihood and, thus, the parameters can be estimated. The applicability of the Generalized Distribution forms relies on the fact that, when there is a need to estimate the parameters, the Generalized Distribution forms will avoid to making an a priori choice among the possible distribution forms.

3.3 The Point Process Approach:

Assuming (3.1.2) holds for some normalizing sequences α_n and β_n; $X_1, ..., X_n$ denotes a random sample of size n from F and $Y_{n,i} = (\frac{x_i - \beta_n}{\alpha_n})$, let P_n denote the point process on R^2 with points at $(t_i = \frac{i}{n}, Y_{n,i})$, where $i = 1, .., n$. The ordinates of P_n will tend to cluster near the lower end points of the (rescaled) distribution, but away from the boundary the process will look like a nonhomogeneous poisson process. Weak convergence of P_n to poisson process may be established excluding a set bordering of the lower boundary.

The intensity measure of the limiting process derived from equations (3.1.2), and (3.1.3) is:

$$\Lambda(t_1, t_2) \times (x, \infty) = (t_2 - t_1)[1 - \kappa(\frac{x - \mu}{\sigma})]^{\frac{1}{\kappa}} \tag{3.3.1}$$

whenever $0 \leq t_1 \leq t_2 \leq 1$ and $1 - \kappa(\frac{x-\mu}{\sigma}) > 0$. The intensity function depends on the parameters μ,σ,κ, and may be fitted to the data by maximum likelihood estimation as we mentioned above.

The point process, extreme value, theory may be viewed as the general case while the clasical and threshold, extreme value, theories are special cases of the point process theory. For instance, in the classical extreme value theory, the probability that $(\frac{M_n-\beta_n}{\alpha_n})$ has no points above χ is identical to the probability that P_n has no points in $(0,1)\times(\chi,\infty)$. Under the poisson process with intensity (3.3.1), this precisely (3.1.4). The Generalized Pareto Distribution may be derived from the Poisson representation as well: the limiting conditional probability that $Y_{n,i} > u + y$ given $Y_{n,i} > u$ is given by:

$$\frac{\Lambda(0,1)\times(u+y,\infty)}{\Lambda(0,1)\times(u,\infty)} = [1 - \frac{ky}{\sigma - ku + k\mu}]^{\frac{1}{k}} \tag{3.3.2}$$

This equation is the GPD when replacing σ on (3.2.1) with $\sigma - ku + k\mu$.

Assuming, after the above generalization, that the excesses over a fixed threshold level, u, follows a Poisson process with an annual crossing rate, $\lambda(u)$, then an unbiased estimate of $\lambda(u)$ is $\frac{n}{m}$, where m the number of years, with estimated variance $\frac{n}{m^2}$. The annual crossing rate over a specific extreme return level, let's say, χ is:

$$\lambda(u)[1 - \frac{k(\chi - u)}{\sigma}]^{\frac{1}{k}} \tag{3.3.3}$$

4. The Extreme Value Theory Applied to Stock Market Returns:

4.1. Parameters Estimation:

Assuming that our sample of extreme stock market returns is made from annual maxima of a series of daily returns. Let n denote the sample size [11]in the examined time period and y_m (for m between 1 and n) denote the individual maxima for the examined period. Provided that equation (3.1.1) is satisfied, the limiting distribution of the annual maxima is given by equation (3.1.4). The parameters of this distribution are μ, σ and κ and may be estimated through maximum likelihood.[12] The likelihood function of these extreme return occurances is:

$$L = \Pi_{m=1}^{n} e^{-(1-\kappa(\frac{y_m-\mu}{\sigma}))^{\frac{1}{\kappa}}} (\frac{(1-\kappa(\frac{y_m-\mu}{\sigma}))^{\frac{1}{\kappa}-1}}{\sigma}) \quad (4.1.1)$$

The log likelihood function of the above is:

$$\log L = \Theta = -n \log \sigma - (1-k)\Sigma_{i=1}^{n} \phi_i - \Sigma_{i=1}^{n} e^{\phi_i} \quad (4.1.2)$$

with $\phi_i = \frac{1}{\kappa} log(-\kappa(\frac{y_i-\mu}{\sigma}))$.

Now, fix a threshold u, and let u denote the number of returns that exceed u, for the period under study. Let n be the sample of returns that exceed the fix threshold u, and y_m (m take values from 1 to n) denote the differences between the observations that exceed u and the threshold level u. Pickand (1975) showed that the same assumptions lead to equation (3.1.1), the excesses over u will be approximated for a large u by the GPD which is given by equation (3.2.1). In short, as Pickand has shown the limiting distribution of the excesses over u will follow the GPD if and only if the parent distribution is in the domain of attraction of one of the extreme value distributions given by equation (3.1.3). The

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[11] n may be the sample of maxima over a different time period such as one month. By the same analogy n may apply to the number of annual minima or monthly minima.

[12] A more sophisticate model can be constucted if it is assumed that there is seasonal effect as the January effect (this case is not assumed here).

parameters of the GPD are σ and κ and may be estimated through maximum likelihood. The likelihood function of the excesses over the specified threshold u is:

$$L = \Pi_{m=1}^{n} \frac{1}{\sigma}[1 - (\frac{\kappa y_m}{\sigma})]^{\frac{1}{\kappa - 1}} \tag{4.1.3}$$

The log likelihood function of the above is:

$$\log L = \Theta = -n \log \sigma - \frac{1-k}{k} \Sigma_{m=1}^{n} \log(1 - \frac{\kappa y_m}{\sigma}) \tag{4.1.4}$$

Maximum likelihood (for both models) proceeds iteratively by using either Newton-Rampson or a quasi-Newton iteration.

The Newton-Rampson method solves the log likelihood equations $[\frac{\vartheta \Theta}{\vartheta \theta} = 0]$ by the iteration

$$\theta_{j+1} = \theta_j + \delta\theta \tag{4.1.5}$$

where $\theta = (\sigma, \mu, \kappa)'$, θ_0 is an initial estimate of the parameters and $[\delta\theta = M^{-1}$ grad $\Theta]$ and M^{-1} are the inverse information matrix.[13]

Crossing rates, extreme quantile values and probabilities of certain extrema may be directly obtained from the above models and clear quantified conclusions can be drawn in regard to the changes in stock market volatility as well as the magnitude of return extremes over time.

4.2. Quantile Estimation:

In this thesis, as it has been demonstrated in section 2, we intend to use extreme quantile values in order to study the magnitude of the extreme stock market returns through time.

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[13] The elements of the information matrix of the generalized extreme value distribution are provided by Prescott, P. and Walden, A. T. (1990), while, the elements of the information matrix of generalized Pareto distribution are provided by Hosking, J.R.M., and Wallis, J. R.(1987).

The quantiles for the Generalized Pareto Distribution are calculated as follows:

$$y(G) = \frac{\sigma}{k}(1-(1-G)^k) \tag{4.2.1}$$

A quantile estimator of y(G), $\hat{y}(G)$, is given by substituting the parameter estimates from the fitted models in (4.2.1). The variance of $\hat{y}(G)$ is given asymptotically by[14] :

$$(var\hat{y}(G)) \sim s(k)^2 var(\hat{\sigma}) + 2\sigma s(k)s'(k)cov(\hat{\sigma},\hat{k}) + (\sigma)^2 s'(k)^2 var(\hat{k}) \tag{4.2.2}$$

where

$$s(k) = \frac{(1-(1-G)^k)}{k}$$

and

$$s'(k) = \frac{-s(k) + (1-G)^k log(1-G)}{k}$$

By the same token, we may derive quantiles and confidence intervals for quantiles for the Generalized Extreme Value Distribution of the annual maxima. But, based on the generalization in section (3.3), we may show that the quantiles, z(G), of the distribution of annual maxima are related with the quantiles $y_u(G)$ of the distribution of crossing returns over a specified threshold level, u, as follows:

$$z(G) = \begin{cases} u + y_u[1+\lambda(u)^{-1} logG] & \text{if } e^{-\lambda(u)} < G < 1; \\ \leq u & \text{if } 0 < G \leq e^{-\lambda(u)}. \end{cases} \tag{4.2.3}$$

Provided that $k = 0$, the sample of excesses over a high threshold converges to the exponential distribution. Quantiles for the exponential distribution are computed by the following formula:

$$z(G) = -\sigma log(1-G) \tag{4.2.4}$$

The variance of these quantiles is given asymptotically by:

$$var\hat{z}(G) \sim (log(1-G))^2 var(\hat{\sigma}) \tag{4.2.5}$$

[14] This formula has been found in Hosking and Wallis(1987) paper.

5. Preliminary Analysis Using the Threshold Approach:

Tables 1 through 7 are part of the preliminary analysis. Tables 1 and 2 test, empirically, whether there is any upward or downward trend in the extreme market returns [15]in the 80's as compared to those of the 70's and 60's. In table 1, the threshold levels correspond to very high daily returns of the value weighted index for the 1962-1990 period. For example, the .0131 threshold is the daily return, of the value weighted index, which is 1.5 standard deviation above the mean return of this index for the 1962-1990 period. The interpretation of other thresholds is very similar[16] . The empirical results indicate, on the average, an upward trend in the frequency of the positive extremes in the 80's as opposed to those of the 70's and 60's. The results are independent of threshold level.

In other words, table 1 shows empirical evidence that high daily return levels (say the ones higher than 1.7%) were , on the average, in the 80's approximately 200% more volatile (frequent) than those of the 60's and at least 20% more volatile than those of the 70's. Extreme value theory will help us verify the statistical significance of these results and in addition may help us identify statistical changes in the magnitude of extreme stock market returns through time.

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[15] In this study, we analyze extremes using the threshold approach, thus from now and on the term extremes will refer to the returns that cross the specified threshold.

[16] Several threshold levels are tried in order to examine whether the results are sensitive to the threshold level.

Table 1

Empirical Crossing Rates over Several Subperiods for the 1962-1990 Period, of the Daily Value Weighted Index.					
Threshold level	.0131 [1.5]*	.0170 [1.96]	.0182 [2.1]	.0207 [.4]	.0232 [2.7]
Crossings 1962,July-1970,Dec.	73 (8.4)**	26 (3)	21 (2.4)	12 (1.4)	8 (.9)
Crossings 1971,Jan.-1980,Dec.	151 (15.1)	69 (6.9)	54 (5.4)	33 (3.3)	23 (2.3)
Crossings 1981,Jan-1990,Dec.	163 (16.3)	93 (9.3)	71 (7.1)	39 (3.9)	28 (2.8)

* A number in a bracel (for all tables presented in this paper) will indicate the number in standard deviations that the specified threshold level (the one above it) deviates from the mean return of the index. For example, .0131 in table 1, is the threshold level which is 1.5 standard deviations above the mean return value of the value weighted index for the 1962-1990 period. Other numbers in bracels are interpreted similarly.

** Numbers in parentheses, for tables 1 through 7, indicate empirical annual crossing rates.

Table 2 presents the same analysis as table 1, but now, for several low negative threshold levels. For example the -.0122 threshold corresponds to the daily return which is 1.5 standard deviations below the mean return of the index for the 1962-1990 period. Although there is no clear trend in the frequency of the negative extremes, for the threshold levels examined here, there is indication of an upward trend for the very low negative returns. For example, it seems justifiable to say that for the -1.98% threshold level annual crossing rates were at least 100% more volatile in the 80's as opposed to those of the 60's and at least 16% more volatile than those of the 70's. Again extreme value theory will help us examine whether these results are statistically significant and in addition provide us statistical evidence about changes in the magnitudes of the extreme stock market drops.

Table 2

Empirical Crossing Rates over Several Subperiods for the 1962-1990 Period of the Daily Value Weighted Index.					
Threshold level	-.0122 [-1.5]	-.0161 [-1.96]	-.0173 [-2.1]	-.0198 [-2.4]	-.0223 [-2.7]
Crossings 1962,July-1970,Dec.	46 (5.3)	25 (2.9)	22 (2.5)	17 (2)	9 (1)
Crossings 1971,Jan.-1980,Dec.	164 (16 .4)	80 (8.0)	61 (6.1)	36 (3.6)	21 (2.1)
Crossings 1981,Jan.-1990,Dec.	154 (15.4)	71 (7.1)	57 (5.7)	42 (4.2)	31 (3.1)

Tables 3 and 4 have been constructed to test whether the start of options trading has been associated with any increase in the volatility of extreme stock market drops. Following the October, 1987 crash many researchers have examined whether the start of trade in stock index futures and index options have increased speculative activity that in turn destabilized stock markets, causing higher volatility. We discuss the findings of past research on this issue and contrast them to the findings of this paper in section 6. Options began to trade on April of 1973. Table 3 shows empirical evidence of a positive association between options trading and negative return extremes. For example, prior to options trading the frequency of negative extremes was lower in comparison to that after the options began to trade on April of 1973. All results are independent of the threshold level.

Table 4 tests the same hypothesis by using the monthly returns of the Dow Jones Industrial Index for the 1940-1991 period. Again, there is empirical evidence that very low negative returns were less frequent prior to options trading as compared to after the options begun to trade. Again, all the results are independent of the threshold level.

Table 3

Empirical Crossing Rates over Several Subperiods for the 1962-1990 Period of the Daily Value Weighted Index.					
Threshold level	-.0122 [1.5]	-.0161 [-1.96]	-.0173 [-2.1]	-.0198 [-2.4]	-.0223 [-2.7]
Crossings 1962,July-1973,Mar.	91 (8.4)	28 (2.58)	21 (1.93)	12 (1.11)	8 (.74)
Crossings 1973,Aip.-1990,Dec.	308 (16.05)	157 (8.41)	126 (6.75)	86 (4.6)	60 (3.21)

Table 4

Empirical Crossing Rates of the Dow Jones Industrial Average for two subperiods of the 1940-1991 period and Three Low threshold Levels				
Threshold Level u	-.0197 [-.5]	-.0437 [-1.0]	-.0677 [-1.5]	-0898 [-1.96]
Crossings 1940,Dec.-1973,Aip.	76 (2.28)	30 (.9)	9 (.27)	5 (.15)
Crossings 1973,May-1991,Dec.	51 (2.88)	18 (1.018)	11 (.62)	5 (.28)

Table 5 examines, empirically, whether there is any association between the creation of the Security Exchange Commission (S.E.C.) and the frequency of negative return extremes. We test this hypothesis by employing the monthly returns of the Cole and Smith's index from July, 1948 through December, 1884 and the monthly returns of the Dow Jones Industrial Average index from January, 1885 through December, 1991. The S.E.C. was created in 1934 in order to observe trading practices and coincides with the establishment of margin requirements. Several authors have tried to determine whether the creation of the S.E.C. and the establishment of margin requirements caused a reduction in stock market volatility. In our analysis, we have assumed the 1929-1933 period as anomalous. The empirical findings indicate that the establishment of the S.E.C. associates with the reduction in the volatility of negative return extremes. In short, the frequency of the negative return extremes was higher prior to 1929 as compared to after 1933. The results are independent of the threshold level.

Table 5

Empirical Crossings of the Smith and Cole's and Dow Jones Industrial Average indices over Several Subperiods for the 1962-1990 Period and Several Threhold Levels.				
Threshold Level	-.0197 [-.5]	-.0437 [-1.0]	-.0677 [-1.5]	-.0898 [-1.96]
Crossings 1848,July.-1928,Dec.	238 (2.95)	110 (1.37)	48 (.6)	25 (.31)
Crossings 1929,Jan.-1933,Dec.	26 (5.2)	20 (4)	16 (3.2)	14 (2.8)
Crossings 1934,Jan-1939,Dec.	14 (2.33)	7 (1.17)	4 (.67)	3 (.5)
Crossings 1940,Jan.-1991,Dec.	128 (2.46)	49 (.94)	21 (.40)	11 (.21)
Crossings 1934,Jan.-1991,Dec.	142 (2.45)	56 (.96)	25 (.43)	14 (.24)

Tables 6 and 7 have been consructed in order to test, empirically, whether there is any association between recessions and voaltility of negative extreme return occurances. Table 6 contains a list of recessions as determined by the National Bureau of Economic Research (N.B.E.R.) from 1962 through 1990. Table 7 uses the daily returns of the value weighted index from 1962 through 1990. The cutoff points of each subperiod have been constructed in order to reflect the assumption that the stock market predicts recessions six months before they actually occur.[17]The empirical findings indicate a positive association between recessions and the volatility of negative return extremes. The effects of recessions are asymmetric. Again, in section 6 we revisit this concern using extreme value theory and in addition we examine whether recessions are associated with statistical changes in the magnitude of negative extreme return occurances.

[17] This assumption is based on the knowledge that the NBER announces that a recession has begun at least six months after it starts.

Table 6

Dates of N.B.E.R. Recessions from 1962-1990.
January, 1970 - November, 1970
December, 1973 - March, 1975
February, 1980 - July, 1980
August, 1981 - November, 1982

Table 7

Empirical Crossing Rates of the daily Value Weighted Index for Several Subperiods of the 1962-1990 Period and Four Low Threhold Levels.

Threshold Level	-.0080 [-1.0]	-.0122 [-1.5]	-.0161 [-1.96]	-.0173 [-2.1]
Crossings 1962,July-1969,June	100 (14.28)	55 (7.85)	14 (2)	11 (1.57)
Crossings 1969,July-1971,May	70 (37.04)	27 (14.09)	11 (5.73)	9 (4.69)
Crossings 1971,June-1972,May	22 (22)	9 (9)	1 (1)	0 (0)
Crossings 1972,June-1975,Sept.	180 (56.84)	93 (29.37)	56 (17.68)	36 (11.37)
Crossings 1975,Oct.-1979,July	109 (28.43)	40 (10.44)	25 (6.52)	19 (4.96)
Crossings 1980,Aug.-1983,May	167 (60.73)	87 (31.63)	48 (17.45)	44 (16)
Crossings 1984,June-1990,Dec.	258 (30.06)	138 (16.08)	71 (8.27)	69 (8.03)

In the remaining part of this paper we will use the estimation procedure of the Generalized Pareto Distribution which has been discussed in section 3 in order to examine statistically the several objectives of section 2.

6. Results of the Fitted Models:

This section of the paper fits the Generalized Pareto Distribution, or its special case the exponential distribution, to several samples of stock market return extremes. Our objectives, as stated in section 2, are: i) to study the volatility and the magnitudes of the stock market return extremes through time, and ii) to examine whether there is an association between several economic events and the volatility as well as the magnitude of severe stock market drops (negative return extremes). To this end, we have proposed i) Estimating the frequency with which specified stock market return extremes are crossed[18]ii) Estimating extreme quantile values on the stock market return extremes, and iii) Examining whether there is statistical change in the frequency or extreme quantile values on the extreme returns through time.

The stock market crash in October, 1987 raised the concern whether stock market volatility was unusually high in the 80's. Schwert (1990a), among others, showed that, apart from October, 1987 and October, 1988 stock market volatility was not particularly high in the 80's. We have revisited the same concern by analyzing the stock market return extremes only. In short, based on the definition of volatility, which is provided in section 2, we have examined whether stock market return extremes (positive and negative) were unusually frequent or extreme (high/low) in the 80's.

The daily returns of the value weighted index from January 1971 through December 1990 have been used. Samples of positive extremes have been constructed by dividing the 1971-1990 period into two ten year periods. The Generalized Pareto Distribution has been fitted to data based on three high positive threshold levels. [19]In tables 8 and 9 we show the estimated parameters, with their

[18] The terms: frequency and crossing rates, which we use interchangebly, refer to the average annual crossings over the specified threshold

[19] Through out the paper we fit the Generalized Pareto Distribution, or its

standard errors, plus point estimates and confidence intervals for the .9, .99, .999 quantile values for the fitted extremes. The estimated parameters of the fitted model were statistically significant in all cases. The location parameter, k, has satisfied the regularity conditions for asymptotically efficient estimators in 4 fits. The quantile estimates show a significant increase in the quantile values in 80's compared to those of the 70's which is independent of the threshold level.

To assess the significance of these results we propose at least one of the following: i) Estimate percentage changes in quantile values. For example, using this approach, we may conclude that for the .00887 (or .887%) threshold level, the 90th percentile values of the positive return extremes has been 1.2% higher in the 80's in comparison to its counterpart of the 70's. ii) Compare what quantiles, in one period, will correspond to specific quantile values from another period. This may be done by computing the cumulative probability, G, for a specific quantile value through equation (3.5.1) for G.P.D fits and through formula (3.5.1) for exponential distribution fits. For instance, based on this approach we have concluded that for the .887% threshold level the .02821 extreme return (which is the estimated 90th quantile value in the 80's) will correspond to the 86.5 [20]percentile value in the 70's. In conclusion, based on these results, we may say that there is an upward trend in the magnitude of the positive extremes which is statistically, but not economically, significant.

special case the exponential distribution, to data based on at least two threshold levels in order to observe whether the results are sensitive to the threshold level.

[20] This number has been found by substituting the .02821 return value, $k =$ .04181 and $\sigma =$ .01907 into formula (3.5.1) and solve with respect to the unknown cumulative probability, G.

Table 8

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution to the Value Weighted Market index Based on Samples of Positive Extremes for the 1971-1980 Period.*							
Threshold u	No. of exceedances	$\hat{\lambda}(u)$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.9)$	$\hat{x}(.99)$	$\hat{x}(.999)$
.00887 [-1.0]	315	31.5	.41810 (.05175)	.01907 (.00190)	.02821 (.02789) (.02847)	.03898 (.03868) (.03927)	.04309 (.04285) (.04333)
.01309 [-1.5]	151	15.1	.60114 (.10314)	.02705 (.00401)	.03373 (.03316) (.03429)	.04217 (.04165) (.04275)	.04429 (.04401) (.04458)
.01697 [-1.96]	69	6.9	.87940 (.21158)	.03918 (.00838)	.38680 (.03774) (.03962)	.04378 (.04328) (.04429)	.04446 (.04408) (.04484)

* Throughout the appendix, $\hat{\lambda}(u)$ will denote the annual empirical crossing rates over (below) the prespecified positive (negative) threshold level u; $\hat{k}$ and $\hat{\sigma}$ will denote the estimates of the k and σ parameters and x(i) will denote the ith percentile of the fitted extremes. Standard errors for parameter estimates and confidence intervals for quantile values are given in parentheses.

Table 9

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Positive Extremes of the Value Weighted Market index for the 1981-1990 Period.							
Threshold u	No. of exceedances	$\hat{\lambda}(u)$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.9)$	$\hat{x}(.99)$	$\hat{x}(.999)$
.00887 [-1.0]	341	34.1	.16530 (.03223)	.01656 (.00214)	.03172 (.03094) (.03249)	.05339 (.05226) (.05452)	.06821 (.06694) (.06947)
.01309 [-1.5]	163	16.3	.22642 (.08211)	.02244 (.00546)	.04026 (.03780) (.04273)	.06417 (.06093) (.06741)	.07836 (.07507) (.08166)
.01697 [-1.96]	93	9.3	.27441 (.15718)	.02725 (.01020)	.46530 (.04072) (.05233)	.07126 (.06111) (.08144)	.08441 (.07762) (.09112)

In addition, we have computed annual crossing rates over positive extreme return levels using formula (3.3.3). These results are summarized in tables 10 and 11. The results indicate that the stock volatility (frequency with which high return levels were crossed) of positive return extremes was higher in the 80's compared to that of the 70's. These results are, again, independent of the threshold level. As far as the significance of these results is concerned, the crossing rate of threshold level .00887 and return level .18 found in the 80's is higher than its counterpart found in the 70's by about 3.6%.

Table 10

Estimated Crossing Rates Over Various Positive Return Levels for Three Different Threshold Levels of the Value Weighted Market Index for the 1971-1980 Period.					
Threshold	Return Levels				
u	.018	.022	.03	.04	.05
.00887	18.46	13.68	7.11	2.029	.12
.01309	12.46	10.31	6.89	3.31	.86
.01697	6.71	5.96	.4.65	3.01	.48

Table 11

Estimated Crossing Rates Over Various Positive Return Levels for Three Different Threshold Levels of the Value Weighted Market Index for the 1971-1980 Period					
Threshold	Return Levels				
u	.018	.022	.03	.04	.05
.00887	19.12	14.27	8.14	3.59	1.39
.01309	13.02	10.59	7.13	4.02	2.082
.01697	8.95	7.6	5.57	3.55	2.13

The same methodology has been used in order to analyze the volatility of negative extremes. Tables 12 and 13, show the estimated parameters, with their standard errors, plus point estimates and condidences for the .1, .01, and .001 quantiles. The results show that the location parameter, k, did not satisfy the regularity for maximum likelihood estimation in three out of six cases, while in the remaining three the σ parameter was statistically insignificant. These results indicate that the data may be assumed to come from an exponential distribution.

Table 12

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Market Index in the 1971-1980 Period.							
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$	$\hat{x}(.001)$
-.00800 [-1.0]	361	36.1	.51708 (.05342)	.018269 (.00168)	-.02459 (-.02476) (-.02442)	-.03207 (-.03239) (-.03194)	-.03434 (-.03443) (-.03425)
-.01221 [-1.5]	164	16.1	.75839 (.12575)	.02660 (.00408)	-.02896 (-.02930) (-.02863)	-.02869 (-.03419) (-.03385)	-.03490 (-.03501) (-.03493)
-.01609 [-1.96]	80	8.0	1.03079 (.24059)	.03604 (.00914)	-.03169 (-.03195) (-.03144)	-.03466 (-.03474) (-.03458)	-.03493 (-.03508) (-.03479)

Table 13

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Market Index for the 1981-1990 Period.							
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$	$\hat{x}(.001)$
-.00800 [-1.0]	331	33.1	.02948 (02016)	.01479 (.00167)	-.03293 (-.03734) (-.02852)	-.06370 (-.07208) (-.05532)	-.09245 (-.10439) (-.08051)
-.01221 [-1.5]	154	15.4	.04590 (.04611)	.02068 (.00398)	-.04519 (-.05788) (-.03250)	-.08585 (-.09800) (-.09800)	-.12243 (-.15491) (-.08995)
-.01609 [-1.96]	71	7.1	.06136 (.10303)	.02813 (.00866)	-.06042 (-.10163) (-.01891)	-.11288 (-.18753) (-.03823)	-.15843 (-.25935) (-.05751)

The exponential distribution has been fitted to these samples and the estimated parameters, with their standard errors, plus point estimates and confidence intervals for the .1, .01, and .001 quantile values have been summarized in tables 14 and 15. The σ parameter was statistically significant in five fits. The results show that the extreme quantile values are statistically lower in the 80's compared to those of the 70's. All the results are independent of threshold level. In reference to the significance of the upward trend in the magnitude of the extreme stock market drops; we have concluded that the 10th percentile of the extreme stock market drops of level .008 or (.8%) is 8.57% lower in the 80's; based on this result it seems justifiable to say that the most extreme stock market drops were about 8.57% of higher magnitude in the 80's than those in the 70's (-.03054). In addition, we may say that the 10th percentile value in the 70's corresponds to the 12th percentile value of its counterpart in the 80's. Again, based on these results it might seem justifiable to say that the magnitude of the most severe stock market drops was statistically, but not economically, higher in the 80's.

Table 14

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Exponential Distribution Based on Negative Extremes of the Value Weighted Market Index for the 1971-1980 Period

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$	$\hat{x}(.001)$
-.00800 [-1.0]	361	36.1	.01326 (.00187)	-.03054 (-.03098) (-.03009)	-.06107 (-.06197) (-.06018	-.09161 (-.09295) (-.09027)
-.01221 [-1.5]	154	16.1	.00503 (.00198)	-.03986 (-.04163) (-.03808)	-.07971 (-.08326) (-.07617)	-.11957 (-.12489) (-.11425)
-.01609 [-1.96]	80	8.0	.02072 (.01065)	-.04772 (-.05224) (-.04319)	-.09543 (-.11085) (-.08002)	-.14315 (-.16857) (-.11772)

Table 15

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Exponential Distribution Based on Negative Extremes of the Value Weighted Market Index for the 1981-1990 Period.						
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{\sigma}$	x̂(.1)	x̂(.01)	x̂(.001)
-.00800 [-1.0]	331	33.1	.01439 (.00094)	-.03316 (-.03339) (-.03292)	-.06631 (-.06678) (-.06584)	-.09947 (-.10018) (-.09877)
-.01221 [-1.5]	154	15.4	.01984 (.00198)	-.04570 (-.04642) (-.04498)	-.09140 (-.09285) (-.08996)	-.13710 (-.13927) (-.13493)
-.01609 [-1.96]	71	7.1	.02662 (.00390)	-.06130 (-.06037) (-.06024)	-.12260 (-.12047) (-.12047	-.18390 (-.18710) (-.18071)

The crossing rates below very low negative return rates are summarized in tables 16 and 17. The results show that the very extreme negative returns (the ones below -.04) were crossed at a higher rate in the 80's in comparison to their counterparts of the 70's, while all other extreme negative returns (the ones higher than -.04) were more volatile in the 70's in comparison to their counterparts of the 80's. In reference to the significance of these results, we have concluded that there is a significant upward trend in the volatility of the extreme stock market drops of size lower than -4% and evidence of a downward trend in the volatility of the extreme stock market drops of size lower than -4%.

Table 16

Estimated Crossing Rates Over Various Negative Return Levels for Three Different Threshold Levels of the Value Weighted Market Index for the 1970-1980 Period.					
Threshold	Return Levels				
u	-.018	-.022	-.03	-.04	-.05
-.00800	18.96	13.60	5.48	.37	0
-.01221	12.92	10.65	6.45	2.06	0
-.01609	7.57	6.68	4.88	2.61	.26

Table 17

Crossing Rates Below Various Negative Return Levels for Three Different Threshold Levels of the Value Weighted Market Index in the 1980-1990 Period					
Threshold	Return Levels				
u	-.018	-.022	-.03	-.04	-.05
-.00800	16.7	12.27	7.23	3.54	1.71
-.01221	11.61	9.54	6.4	3.84	2.28
-.01609	6.63	5.74	4.29	2.96	2.03

Our results create two puzzles; First, why stock market volatility has been found not to be unusually high in the 80's while extreme value theory has shown statistical evidence of an upward trend in both the magnitude as well as the volatility of the most extreme stock market returns. One possible hypothesis has been proposed to explain this puzzle; the kourtosis in the distribution of returns must have been higher in the 80's in comparison to its counterpart in the 70's. If this hypothesis is true, then monthly volatility (interday volatility) might remain unchanged while the magnitude as well as the volatility of the extreme stock market returns might be rising. Second, what factors might have caused the upward trend in both the magnitude as well as the frequency of the extreme stock market returns in the 80's. Three possible results have been suggested: i) program trading ii) portfolio insurance, and iii) trading in futures and options. These possible causes need to be tested.

Preceeding the October, 1987 stock market crash, many researcers have raised the question whether the start of options trading (April, 1973) has been associated with an upward trend in stock market volatility. We have revisited this concern by analyzing the left tail of the distribution of returns only.

The Generalized Pareto Distribution has been fitted to data based on three different thresholds. The regularity conditions have not been satisfied, for all three fits, in the 1962-1973 period. Consequently, for the 1962-1973 period, the exponential distribution has been used, with estimated parameters and their corresponding errors, plus point estimates and confidence intervals for the .1, .01, and .001 quantile values shown in tables 18 and 19. The σ parameter was statistically insignificant in one fit. The results (exluding the fit where σ was statistically insignificant) show that the extreme quantile values were statistically lower in the period which is associated with the trading of options. As far as the significance of these results is concerned, one might say that for the .008 (or .8%) threshold level the .1 percentile value has been, on the average, about 13.9% higher in the period associated with option trading in comparison to its counterpart prior to the period when options have begun to trade. In addition, one might say that for the .008 threshold level, the -.3205 extreme return (which is the estimated 90th quantile in the period associated with option trading) will correspond to the 7.9 quantile for the period prior to the start of option trading.

Table 18

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Exponential Distribution to Negative Extremes of the Value Weighted Market Index in the July, 1962-Aipril, 1973 Period

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$	$\hat{x}(.001)$
-.008 [-1.0]	224	20.67	.01223 (.0025)	-.02816 (-.02892) (-.02741)	-.05632 (-.05793) (-.05481	-.08449 (-.09372) (-.08222)
-.01221 [-1.5]	91	8.4	.01578 (.00649)	-.03634 (-.03941) (-.03327)	-.07268 (-.07883) (-.06653)	-.10902 (-.11824) (-.09980)
-.01609 [-1.96]	28	2.58	.02048 (.01864)	-.04717 (-.06307) (-.03127)	-.09434 (-.12615) (-.06254)	-.14151 (-.18922) (-.09381)

Table 19

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Market Index for the May, 1973-December,1990 Period.							
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$	$\hat{x}(.001)$
-.00800 [-1.0]	649	36.73	.05316 (01165)	.01478 (.00123)	-.03205 (-.03292) (-.03119)	-.06041 (-.06197) (-.05885)	-.08550 (-.08763) (-.08337)
-.01221 [-1.5]	308	17.43	.07371 (.02015)	.02015 (.00296)	-.04267 (-.04541) (-.03994)	-.07869 (-.08348) (-.07390)	-.10908 (-.11539) (-.10277)
-.01609 [-1.96]	157	8.88	.08903 (.05608)	.02550 (.00584)	-.05310 (-.06122) (-.04498)	-.09636 (-.11022) (-.08249)	-.13160 (-.14943) (-.11376)

Crossing rates below extreme stock market drops have been estimated, with results summarized in tables 20 and 21. The results show a positive association between the start of option trading and the volatility of extreme stock market drops. These results are independent of the threshold level. In reference to the significance of these results, one may conclude that for the -.008 threshold level the crossings below the -.018 return level were, on the average, about 83.5% higher in the period associated with options trading in comparison to the period prior to options trading.

Table 20

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Daily Value Weighted Market Index for the July 1962-May, 1973 Period					
Threshold	Return Levels				
u	-.018	-.022	-.03	-.04	-.05
-.00800	10.05	6.63	2.26	007	0
-.01221	6.44	5.07	2.86	.6	0
-.01609	2.43	2.09	1.48	.68	.0

Table 21

Empirical Crossing Rates Below Various Negative Return Levels for Three Threshold Levels of the Daily Value Weighted Market Index for the July, 1962-December, 1990 Period.					
Threshold	Return Levels				
u	-.018	-.022	-.03	-.04	-.05
-.00800	18.45	13.61	7.79	3.69	1.69
-.01221	13.04	10.46	6.9	4.07	2.31
-.01609	8.33	7.02	5.13	3.37	2.18

The same hypothesis has been tested using monthly returns. The January, 1940 - December 1991 period of the Dow Jones Industrial Average has been divided into two time periods: i) January, 1940 - December, 1973, and ii) May, 1973 - December, 1991. The exponential distribution has been fitted to data based on two different threshold levels. Tables 22 and 23 show the estimated parameters with their standard errors, plus point estimates and confidence intervals for the .1 and .01 quantile values. The sigma parameter has been statistically significant in all fits. The results show again a positive association between the start of options trading and the magnitude of extreme stock market drops. These results are again independent of the threshold level. We conclude that for the -.008 threshold level the .1 percentile of the extreme stock market drops has been lower, on the average, by about 1.8% in the period associated with option trading.

Table 22

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Exponential Distribution to Negative Extremes of the Dow Jones Industrial Average from Jan., 1940 through Aipril, 1973 Period					
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{\sigma}$	x̂(.1)	x̂(.01)
-.01971 [-.5]	76	2.28	.04476 (.00954)	-.10307 (-.10463) (-.10150)	-.20613 (-.20926) (-.20301
-.04371 [-1.0]	30	.9	.06723 (.03464)	-.15481 (-.18336) (-.12626)	-.30963 (-.36673) (-.2525)

Table 23

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Exponential Distribution Based on Negative Extremes Dow Jones Industrial Average from May, 1973 through Jan., 1991 Period.					
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{\sigma}$	x̂(.1)	x̂(.01)
-.01971 [-.5]	51	2.88	.04559 (.01003)	-.10499 (-.11133) (-.09865)	-.20998 (-.22266) (-.19729
-.04371 [-1.0]	18	1.018	.07754 (.0524)	-.17855 (-.23432) (-.12278)	-.35710 (-.46864) (-.24555)

Annual crossing rates below several extreme stock market drops have been estimated, with results summarized in tables 24 and 25. The results, again, show a positive association between options trading and the volatility of extreme stock market drops which is independent of the threshold level. Based on these results, we conclude that for the -.01971 threshold level the crossings below the -.05 return level has been about 30% higher in the period associated with options trading.

In conclusion, based on these results there is evidence that the start of options trading has been associated with increase in both the magnitude as well as the volatility of the extreme stock market returns. The evidence is more obvious for daily returns as opposed to monthly returns. These results seem statistically, but not economically, significant.

Table 24

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Dow Jones Industrial Average from Jan. 1940-May, 1973 Period					
Threshold	Return Levels				
u	-.05	-.06	-.07	-.08	-.09
-.01971	1.3	1.05	.83	.63	.47
-.04371	.85	.77	.70	.62	.55

Table 25

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the the Dow Jones Industrial Average from June 1973-Dec., 1991 Period					
Threshold	Return Levels				
u	-.05	-.06	-.07	-.08	-.09
-.01971	1.69	1.37	1.1	.86	.65
-.04371	.97	.90	.83	.76	.69

Several researchers have tried to examine whether the creation of the Security and Exchange Commission (S.E.C.) has increased public confidence and consequently contributed to reduction in stock volatility. Officer (1973) has shown that this is inconsistent to the fact that postwar volatility levels were very similar to the pre-1929 levels. We examine whether the creation of the S.E.C. has been associated with a statistical decrease in the volatility and the magnitude of the extreme stock market drops.

In order to test the above hypothesis, we have used the monthly returns of the Smith's and Coles and Dow Jones Industrial Average Indices from 1848 through 1991[21]. The time period under study has been seperated into the years: 1848-1928, 1929-1933 and 1934-1991. The Generalized Pareto Distribution has been fitted to this data based on two low threshold levels. Tables 26 through 28 show the estimated parameters, with their corresponding standard errors, plus point estimates and confidence intervals for the .1, and .01 quantile values on these fits.

[21] For the construction of this combined index see section 2.

The estimated parameters are statistically significant in all fits. The results show: i) The magnitudes of the extreme negative returns have been extremely higher in the 1929-1933 time period in comparison to their counterparts of the other two periods. These results are independent of the threshold level. ii) Comparing the pre-1929 and the post-1933 periods, there is statistical evidence that the creation of the S.E.C has been associated with a reduction in the magnitude of the extreme stock market drops. As far as the significance of these results is concerned, one might say that for the -.0197 threshold level, the .1 quantile value was, on the average, 18% higher in the pre-1929 period in comparison to its counterpart of the post-1933 period.

Table 26

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Smith and Cole's and Dow Jones Industrial Average Indices for the July, 1848-December,1928 Period.

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.0197 [-.5]	236 [-.5]	2.96	.35408 (10253)	.09747 (.01831)	-.11110 (-.12863) (-.09357)	-.18346 (-.21078) (-.15614)
-.0437 [-1.0]	110 [-1.0]	1.38	.1861919637 (.04744)	.05933 (.0658)	-.15347 (-.16200) (-.14493)	-.22138 (-.23074) (-.21202)

Table 27

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Smith and Cole's and Dow Jones Industrial Average Indices for the January, 1929-December, 1933 Period.

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.0197 [-.5]	26	5.2	.51486 (.27363)	.15416 (.04581)	-.20792 (-.25085) (-.16500)	-.27146 (-.31248) (-.23045)
-.0437 [-1.0]	20	4	.76855 (.42572)	.21544 (.08262)	-.23256 (-.27190) (-.19322)	-.27218 (-.30169) (-.24267)

Table 28

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution to Negative Extremes of the Dow Jones Industrial Average Indices in the January, 1934-December, 1991 Period						
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.0197 [-.5]	142	2.45	.43842 (.12744)	.06622 (.00987)	-.09600 (-.10105) (-.09094)	-.13098 (-.13621) (-.12575)
-.0437 [-1.0]	56	.96	.99549 (.11008)	.13414 (.01482)	-.12100 (-.12510) (-.11690)	-.13321 (-.13621) (-.12918)

Annual crossing rates, for the same analysis, have been computed, with results summarized in table 29 through 31. The results show: i) The volatility of the extreme stock market drops has been significantly higher in the 1929-1933 period in comparison to its counterpart of the other two periods. These results are independent of the threshold level. ii) The volatility of the pre-1929 period was higher than that of the post-1933 one. Again these results are independent of the threshold level. As far as the significance of these results is concerned, one might say that the volatility of the extreme stock market drops for the -.0197 threshold level and -.05 return level have been higher, on the average, by about 18% in the pre-1929 period in comparison to its counterpart of the post-1933 period.

Table 29

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Dow Jones Industrial Average from July 1848 through December 1928 Period					
Threshold	Return Levels				
u	-.05	-.06	-.07	-.08	-.09
-.01971	1.73	1.43	.1.17	.96	.78
-.04371	1.29	1.16	.	.93	.82

Table 30

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Dow Jones Industrial Average from January 1929 through December 1933 Period					
Threshold	Return Levels				
u	-.05	-.06	-.07	-.08	-.09
-.01971	4.22	3.92	3.63	3.36	3.09
-.04371	3.88	3.70	3.31	3.34	3.16

Table 31

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Dow Jones Industrial Average from January 1934 through December 1991 Period					
Threshold	Return Levels				
u	-.05	-.06	-.07	-.08	-.09
-.01971	1.47	1.21	.97	.76	.59
-.04371	.915	.84	.77	.70	.62

Schwert (1989) has found evidence that stock volatility is higher on average during recessions. He has linked this phenomenon partly to the financial and leverage effects. In the context of our analysis, we ask whether recessions (as opposed to expansions) are correlated with the volatility (frequency) and the magnitude of extreme stock market drops.

For this purpose, the daily returns of the value weighted index for the 1962-1990 period have been employed. The periods of recessions are [22] : July, 1969-May, 1971, June, 1972-December, 1975, and August, 1979-May, 1983, while the periods of expansion are: July, 1962-June, 1969, June, 1971-May, 1971, October, 1975-July, 1979, and June, 1983-December, 1990.

The Generalized Pareto Distribution has been fitted to data, for all the above periods, based on two very low thresholds. Tables 32 through 38 provide the estimated parameters with standard errors, plus point estimates and confidence intervals for the .1, .01, .001 quantile values. The results show that, on the

[22] We have assumed that recessions begin six months in advance to the NBER recessions since the NBER announces that a recession has begun at least six months after it starts.

average, the magnitude of extreme stock market drops have been statistically higher during periods of recessions. The 1983-1990 period, an expansion period, has relatively high magnitude of extreme stock market drops. One interpretation for this is that it might be associated to the start of trade in index options and index futures. Laurence (1990) has found evidence that the start of trade in index options and index futures (1983) has been associated with a statistically but not economically significant increase in stock market volatility. As far as the significance of these results is concerned, one may conclude that, prior to 1983, the magnitude of the negative return extremes of the -.00378 level and .1 quantile value has been, on the average, lower from its preceeding period of expansion by a number that is in the 18-56 percentage range. Therefore, it might seem justifiable to say that the magnitude of the very extreme stock market drops has been , on the average, lower during periods of recession by a number which lies in th 18-56 percentage range.

Table 32

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Index for the July, 1962-June, 1969 Period.

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	301	43	.33294 (.04335)	.00998 (.00094)	-.01606 (-.01629) (-.01582)	-.02351 (-.02377) (-.02326)
-.008 [-1.0]	107	15.29	.62093 (.14431)	.01791 (.00357)	-.02194 (-.02252) (-.02136)	-.02719 (-.02759) (-.02679)

Table 33

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Index for the July, 1969- May, 1971 Period.†						
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	141	73.56	.34562 (.07267)	.01192 (.00172)	-.01893 (-.01959) (-.01827)	-.02747 (-.02819) (-.02675)
-.008 [-1.0]	71	37.04	.54023 (.01713)	.17031 (.00444)	-.02358 (-.02485) (-.02231)	-.03038 (-.03140) (-.02935)

†. This sign denotes a period of expansion.

Table 34

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Index for the June, 1971- May, 1972 Period.						
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	56	56	.80169 (.19097)	.013298 (.00269)	-.01397 (-.01442) (-.01352)	-.01617 (-.01645) (-.01589)
-.008 [-1.0]	23	23	.1.28630 (.04581)	.02119 (.00339)	-.01561 (-.01636) (-.01485)	-.01641 (-.01725) (-.01558)

Table 35

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution Based on Negative Extremes of the Value Weighted Index for the June, 1972- September, 1975 Period.†						
Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	286	85.8	.40222 (.04454)	.01448 (.00129)	-.02175 (-.02198) (-.02151)	-.03036 (-.03059) (-.03013)
-.008 [-1.0]	181	54.3	.55140 (.08642)	.01952 (.00269)	-.02547 (-.02583) (-.02510)	-.03262 (-.03289) (-.03235)

Table 36

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution to Negative Extremes of the Value Weighted Index for the October, 1975- July, 1979 Period

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.01)$	$\hat{x}(.01)$
-.00378 [-.5]	236	61.61	.52825 (.06303)	.01135 (.00116)	-.01512 (-.01528) (-.01497)	-.01961 (-.01973) (-.01948)
-.008 [-1.0]	96	25.04	.88333 (.19598)	.01874 (.00381)	-.01845 (-.01877) (-.01812)	-.02086 (-.02101) (-.02070)

Table 37

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution to Negative Extremes of the Value Weighted Index for the August, 1979- May, 1983 Period.†

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	282	72	.32373 (.01292)	.0431 (.00118)	-.02099 (-.02132) (-.02065)	-.03094 (-.03133) (-.03056)
-.008 [-1.0]	154	39.3	.4955 (.01902)	.08813 (.00280)	-.02614 (-.02667) (-.02561)	-.03450 (-.03494) (-.03406)

Table 38

Parameter and Quantile Estimates for Maximum Likelihood Fits of the Generalized Pareto Distribution to Negative Extremes of the Value Weighted Index for the June, 1983- December, 1990 Period

Threshold u	No. of exceedances	$\hat{\lambda}$	$\hat{k}$	$\hat{\sigma}$	$\hat{x}(.1)$	$\hat{x}(.01)$
-.00378 [-.5]	479	63.87	-.01114 (.01416)	.01004 (.00076)	-.02343 (-.02881) (-.01805)	-.04748 (-.05845) (-.03651)
-.008 [-1.0]	233	31.07	.01458 (.02719)	.02791 (.00204)	-.03451 (-.05081) (-.01821)	-.06788 (-.09966) (-.03610)

Annual crossing rates for the same analysis have been presented in tables 39 through 44. The results indicate that the volatility of the extreme stock market drops has been significantly higher during periods of recession. As far as the significance of these results is concerned, prior to 1983, the volatility of the extreme stock market drops over the -.018 return level for the -.00378 threshold level has been by at least 133% higher, on the average, during periods of recession in comparison to their preceeding periods of expansion.

Table 39

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the July 1962-June, 1969 Period.

Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	6.24	2.59	.08
-.00800	7.70	5.24	1.5

Table 40

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the July, 1969-May, 1971 Period.†

Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	15.08	8.36	1.18
-.00800	19.04	13.4	4.91

Table 41

Estimated Crossing Rates Over Various Negative Return Levels for Three Different Threshold Levels of the Value Weighted Market Index for the June, 1971-May, 1972 Period.

Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	4.93	0	0
-.00800	11.11	5.24	0

Table 42

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the June, 1972-September, 1975 Period.†			
Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	24.6	14.85	3.36
-.00800	29.75	21.08	9.34

Table 43

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the October, 1975-July, 1979 Period.			
Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	7.92	1.74	0
-.00800	12.97	7.39	0

Table 44

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the August, 1979-May, 1983 Period.†			
Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	18.48	10.96	2.65
-.00800	21.38	15.74	7.06

Table 45

Estimated Crossing Rates Over Various Negative Return Levels for Three Threshold Levels of the Value Weighted Market Index for the June, 1983-December, 1990 Period.			
Threshold	Return Levels		
u	-.018	-.022	-.03
-.00378	15.59	10.59	4.87
-.00800	16.07	12.32	7.22

7. Summary and Conclusions:

This thesis proposes: i) a volatility measure and ii) a magnitude measure, for stock market return extremes. Both these measures have been assumed as risk indicators which considerably worry investors, regulators and the public in general. Extreme value theory has been used to estimate these measures through time. The following important findings have been found: (i) The volatility and magnitude of the stock market return extremes has been statistically, but not economically, higher in the 80's in comparison to their counterparts of the 70's. These results are in contrast to what one must have expected based on the study by Schwert (1990a). Schwert has found that stock market volatility has not been unusually high in the 80's. One possible explanation has been proposed to explain this puzzle; the kourtosis in the distribution must have been higher in the 80's in comparison to its counterpart in the 70's. (ii) The volatility and magnitude of extreme stock market drops have been found abnormally high during the Great Depression (1929-1939). (iii) The volatility and magnitude of extreme stock market drops have been found to be positively correlated with i) Recessions, ii) The beginning of options trading and iii) Negatively correlated to the establishment of the Security Exchange Committee. Future work in examining other possible causes in the volatility of stock market return extremes seems promising.

5. References

Bolleslev, T., 1986, Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics, 31, 307-327.

Black, F. and Scholes, M. 1973, The Pricing of Options and Corporate Liabilities, Journal of Political Economy, 81, 637-659.

Black, F., 1976, Studies in Stock Price Volatility Changes, Proceedings of the 1976 Business Meeting of the Business and Econmic Statistic Section, American Statistical Assotiation, 177-181.

Davison, A. C., 1984, Modelling Excesses Over High Threshold, with an Application, Statistical Extremes and Applications, J. Tiago de Oliveira, ed., 461-482. Reidel, Dordrecht.

Davison, A. C. and Smith R. L., 1990, Models of Echeedances over high Thresholds, J. Roy. Statist., Soc. Series B, 52, 393-442.

Engle, R. F., 1982, Autoregressive Conditionally Heteroskedastic with Estimates of the Variance of the U.K. Inflation, Econometrica, 50, 987-1008.

Engle, R. F., Lilien, D. M., and Robins, R. P., (1987), Estimating Time varying risk premia in the term structure, Econometrica, 55, 391-407.

Fama, E. F., 1965, The Behavior of Stock Market Prices, Journal of Business, 38, 34-105.

Fisher, Laurence, 1966, Some New Stock Market Indices, Journal of Business, 29, 191-225.

French, L., 1966, Some New Stock Market Indices, Journal of Business, 29, 191-225.

French, K.R., and Shwert,.G.W., and Stambaugh,R.F., 1987, Expected Stock Returns and Volatilty, 19, 3-29.

Fisher, R. A., and Tippett, L. H. C., 1928, Limiting Forms of the Frequency Distributions of the Largest or Smallest Member of a Sample, Cambridge Philos.

Soc., 24 180-190.

Gallant, A. Ronald, 1981, On the Bias of Flexible Funtional Forms and Essentially Unbiased Forms: The Fourier Flexible Form, Journal of Econometrics, 221-224.

Gnedenko, B. V., 1943, Sur la Distribution Limite du Terms Maximum d'une Serie Aleatoire, Annal. of Math., 44, 423-453.

Gumbel, E.J., 1958, Statistics of Extremes, Columbia University Press.

Higgins, M. L., and Amil K. Bera, 1992, A Class of Nonlinear ARCH models, International Economic Rerview, 33, 137-157.

Hosking, J.R.M., Wallis, J.R. and Wood, E.F. 1985, Estimation of the Generalized Extreme Value Distribution by the Method of Probability-Weighted Moments. Technometrics 27, 251-261.

Hosking, J.R.M, and Wallis J. R., 1987, Parameter and Quantile Estimation for the Generalized Pareto Distribution, Appl. Statistics, 34, 339-349.

Laurence, H., 1989, S&P 500 Cash Stock Price Volatilities, Journal of Finance, XLIV, 1155-1175.

Leadbetter, M. R., Lindgen, G. and Rootzen, H.,1983, Extremes and RelatedProperties of Random Sequences and Series. Springer, New York.

Markowitz, H. M., 1962, Portfolio Selection, Journal of Finance, 7, 77-91.

Merton, R. C., 1980, On Estimating the Expected Return on the Market: An Exploratory investigation, Journal of Financial Economics, 8, 323-361.

Nelson, D. B., 1990, Conditional Heteroskedasticity in Asset Returns: A New Approach, Econometrica, 59, 347-370.

Officer, R. R., 1973, The Variability of the Market Factor of New York Stock Exchange, Journal of Business, 46, 434-453.

Parkinson, M., 1980, The Extreme Value Method for Estimating the Variance of the Rate of Return, Journal of Business, 53, 61-65.

Pagan, A. R., and Ullah, A., 1988, The Econometric Analysis of models with Risk Terms, Journal of Applied Econometrics, 3, 87-105.

Pindyck, R.R., Risk, 1984, Inflation and Stock Market. The American Economic Review, 74, 335-351.

Poterba, M.J.,and Summers Lawrence H., 1986, The Persistence of Volatility and Stock Market Fluctuations, American Economic Review, 76, 1142-1151.

Prescott, P. and Walden, A.T., 1980, Maximum Likelihood Estimation of the Generalized Extreme Value Distribution, Biometrica, 67, 723-724.

Prescott, P. and Walden, A.T., 1983, Maximum Likelihood Estimation of the parameters of the Three-Parameter Extreme Value Distribution from Censored Samples, J. Comput. Simulation, 16, 241-250.

Scholes, M., Williams, J., 1977, Estimating Betas from Non-Synchronous Data, Journal of Financial Economics, 5, 309-327.

Resnick, S., 1987, Extreme Values, Point Processes and Regular Variation, Springer, New York.

Schwert, G.W., 1989, Why Stock Market Volatility Changes over Time, Journal of Finance, 44, 1115-1153.

Schwert, G.W.,1990a, Stock Market Volatility, Financial Analyst Journal, 23, 24-34.

Schwert, G.W.,1990b, Stock Market Volatility and Crash of 1987, Review of Financial Studies, 3, 77-102.

Schwert, G. W., 1990c, Indexes of Stock Prices from 1802 to 1987, Journal of Business, 63, 399-426.

Smith, R.R., 1989, Extreme Value Analysis of Enviromental Time Series: An Application to Trend Detection in Ground - Level Ozone, Statistical Science, 4, 367-393.

Smith, R.R., 1984, Threshold Methods for Sample Extremes, Statistical Extremes and Applications, J. Tiago de Oliveira, ed., 621-638. Reidel, Dordrecht.

Tobin, J., 1958, Liquidity Preference as Behavior Towards Risk, Review of Economic Studies, , 65-66.

Zakoian, J. Michell, 1990, Threshold Heteroskedastic Model, unpublished manuscript, INSEE.