THE EFFECTS OF COVER, COPY, AND COMPARE, PERFORMANCE FEEDBACK AND REWARDS ON THE MATHEMATICAL CALCULATION SKILLS OF STUDENTS IDENTIFIED WITH MATH DIFFICULTY

by

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Abstract

COVER, COPY AND COMPARE AND MATHEMATICAL CALCULATION SKILLS IN STUDENTS WITH MATH DIFFICULTY

By

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This study examined the isolated effects of Cover, Copy and Compare (CCC) and the effects of CCC paired with performance feedback (CCC + PF) and rewards (CCC + RW) on the mathematical calculation skills of first grade students identified with math difficulty. Four research questions were addressed in this study. 1. Does Cover, Copy, and Compare increase first grade students' fluency in addition and subtraction calculation skills? 2. Does Cover, Copy and Compare paired with Performance Feedback have a higher rate of increase of first grade students' fluency in addition and subtraction calculation skills when compared to Cover, Copy, and Compare alone? 3. Does Cover-Copy-Compare paired with a Reward have a higher rate of increase of first grade students' fluency in mathematics calculation skills when compared with Cover, Copy, and Compare in isolation or Cover, Copy, and Compare paired with Performance Feedback? 4. Does Cover, Copy, and Compare increase first grade students' fluency in addition and subtraction skills at a higher rate than a control receiving no intervention?

Eight first-grade students enrolled in General Education in an elementary school in a low-socioeconomic community within a major city in the Eastern United States were

identified with Math Difficulty through a curriculum-based measure (CBM), and were the participants in the study. The students were randomly assigned to one of four treatment conditions: CCC, CCC + PF, CCC + RW and control. An alternating treatment design was used following an assessment of baseline levels that were determined using a CBM probe. The students received the interventions in both addition and subtraction operations over the course of 10 weeks. Rates of digits correct per minute (DCPM) and errors per minute (EPM) were the dependent measures used to indicate gains in calculation skills.

Overall, the results of the study indicated that CCC produced significant decreases in EPM when compared with baseline performance and modest gains in DCPM. Adding PF or RW to CCC did not increase the power of the CCC intervention as hypothesized, although it produced faster response times in some students. The study replicated previous research by demonstrating the CCC is a sound method for improving academic skills.

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CHAPTER I

Introduction

A mathematics disorder is defined as “mathematical ability (as measured by individually administered tests of mathematical calculation or reasoning) that falls substantially below that expected for the individual’s chronological age, measured intelligence, and age appropriate education.” (DSM-IV-TR; American Psychiatric Association, 2000, p. 53). The disorder must significantly interfere with academic achievement, or daily living skills that require mathematical skills (DSM-IV-TR; American Psychiatric Association, 2000, p. 53). During the past two decades, math learning disabilities have received increased attention from educational researchers, evaluators, and teachers. Mathematics learning disability (MLD) is estimated to affect between 5 to 8% of school-age children within the United States (Badian, 1983; Fleischner & Manheimer, 1997; Geary, 2004; NICHD Reading Panel, 2000; Shalev, Auerbach, Manor, & Gross-Tsur, 2000).

The field of mathematics is complex and encompasses many approaches to instruction, making it difficult to identify and study the cognitive phenotypes of children with MLD (Geary, 2004). Mathematics difficulties might stem from or be associated with deficits in some aspects of cognition, some general and some specific to mathematical knowledge, rather than a deficit in instruction (Russell & Ginsberg, 1984). An MLD resulting from a cognitive disability must be distinguished from poor achievement due to inadequate instruction (Geary, Brown, & Samaranayake, 1991). Although difficulties in mathematics are widespread among school-age children, no universally accepted definition of MLD exists and no core deficit has been identified for MLD (Mazzocco &

Myers, 2003). It is possible that MLD subtypes do not even share a common core deficit, because several cognitive functions have been linked to poor achievement, primarily reading-related, memory, visuospatial skills and/or executive skills (Mazzocco & Myers, 2003).

A myth exists in the field of education that holds that there are many more students classified with learning disabilities that affect their ability to learn to read, write, and spell than to do math at a level expected of other students of their age or grade level (Fleischner & Manheimer, 1997). Most students are referred for evaluations to determine whether their difficulties in school are related to their failure to acquire early reading skills or to behavioral issues in class (Fleischner & Manheimer, 1997; Gottlieb & Weinberg, 1999; Soodak & Podell, 1993). There is a considerable amount of evidence that difficulties in mathematics achievement are as prevalent as dyslexia or specific reading disabilities (Kosc, 1974; Norman & Zigmond, 1980).

Until recently, much of the research on students with learning disabilities focused mainly on reading disorders. Research on mathematics learning disabilities is far less extensive than research on reading disabilities (Gersten, Jordan, & Flojo, 2005; Jordan, Levine, & Huttenlocher, 1995; Mazzocco & Myers, 2003). MLD has been studied through intervention studies, however it is first appropriate to discuss assessments for targeted intervention.

Arithmetic is not a single entity. It is made up of components, such as knowledge of mathematical facts, ability to carry out arithmetical procedures, understanding and use of arithmetical principles, such as commutativity and associativity, estimation, mathematical knowledge, and application of arithmetic to the solution of word problems

(Dowker, 2005). Experimental findings conducted with typically developing children (Dowker, 1998; Ginsburg, 1977) have shown that it is possible for individuals to have significant discrepancies between almost any two components of arithmetic (Dowker, 2005). For example, Temple (1991) reported a case study with one child who could carry out arithmetical calculation procedures correctly, but could not remember number facts, and another child who could remember the facts, but could not carry out the procedures (Dowker, 2005). Effective interventions imply some form of either formal or informal assessment to indicate the strengths, weaknesses, and educational needs of a group of children with MLD and a method of evaluating the effectiveness of the interventions to improve performance (Dowker, 2005).

In the late 1970's, Deno and his colleagues at the University of Minnesota Institute developed Curriculum Based Measurement (CBM) for research on learning disabilities (Hosp & Hosp, 2003). CBM is a set of standardized procedures used to assess student performance on long-term goals in reading, spelling, written expression, and math. It is designed to be an objective, ongoing measurement system of student outcomes. CBM helps to facilitate the evaluation of the effectiveness of an individual student's educational program (Deno, 1992; Deno & Epsin, 1991; Thurber, Shinn, & Smolkowski, 2002). Much research (Fuchs & Fuchs, 1986; Fuchs, Fuchs, & Hamlett, 1989; Fuchs, Fuchs, Hamlett, & Stecker, 1990) has shown that measuring the effectiveness of an intervention is accomplished using formative evaluation. CBM is a formative evaluation, which involves the collection of data during instruction as a basis for modifying that instruction (Deno & Epsin, 1991).

Formative evaluation requires frequent and repeated measurement; therefore, the procedures must be technically sound, quick and easy to administer and interpret, and provide useful information about students' performances in basic skill areas (Deno, 1985; Shinn, 1989). Curriculum-based measurement possesses these features. CBM is easy to administer, score, and has good treatment validity, as well as good reliability and validity. CBM assesses student progress toward long-term goals, using frequent monitoring and graphical depiction of students' scores (Stecker, Fuchs, & Fuchs, 2005).

Mathematics Curriculum-Based Measurement (M-CBM) can be used for progress monitoring once students have begun explicit instruction in mathematics to identify at-risk students and monitor achievement interventions (Clarke & Shinn, 2004). M-CBM is conducted by having a student answer computational problems for 2 minutes. The number of correct digits is counted to obtain the digits correct per minute (Hosp & Hosp, 2003). The math probes should be different but equivalent in grade level and should contain at least 25 problems per probe (Fuchs & Fuchs, 1991; Shinn, 1989).

One problem with M-CBM is that, like other achievement tests, M-CBM can be used to detect difficulties in mathematics only when there is sufficient growth in academic skill to enable measurement, and after students have had sufficient instruction in mathematics computation. This means that M-CBM can be used starting in first grade, but prior to this, students will have initial scores of 0 on M-CBM (Clarke & Shinn, 2004).

M-CBM can be used as an outcome measure of an intervention that is used to improve mathematical skills (Deno, 1985; Shinn, 1998, 2002). To study the nature of arithmetical difficulties that children experience and to determine the best interventions to help them, it is important to remember that individuals can show marked differences between almost any two possible components of arithmetic (Dowker, 2005; Geary & Hoard, 2001; Jordan et al., 2003; Russell & Ginsburg, 1984).

Dowker (1998) studied calculation and arithmetical reasoning in 213 children between the ages of 6 and 9. Dowker found that individual differences in arithmetic are relatively different and arithmetic is not unitary. Jordan and Hanich (2000) noted that children with MLD are consistently weaker at retrieving arithmetical facts from memory than at any other aspect of arithmetic. Children with MLD often rely on counting strategies in arithmetic, while their age-mates rely more on fact retrieval (Cumming & Elkins, 1999; Miles, Haslum, & Wheeler, 2001).

Much research in the 1980's on students with learning disabilities shows that teaching mathematics to students with MLD through drill and practice increases retention of math facts (Haring & Eaton, 1978; Pellegrino & Goldman, 1987). Brosvic, Dihoff, Epstein, and Cook (2006) examined the effect of feedback on the acquisition and retention of numerical facts by elementary school students with MLD. The outcome showed that provision of immediate, affirming, and corrective feedback facilitated the acquisition and retention of fact series in the four operations (Brosvic et al., 2006). Research suggests that feedback is most effective when it is provided immediately, and that strategies that use immediate feedback to increase retention of mathematics operations in students with MLD have positive effects (Bangert-Downs, et al., 1991; Brosvic et al., 2006; Codding et al., 2007)

Interventions to improve the mathematical skills of students with MLD are aimed not only at increasing the retention of math facts, but also at increasing fluency. Fluency in computation skills includes being able to automatically compute mathematical facts

(Learner, 2003), and it is obtained when it is faster to solve the problem through recall than to perform the steps needed to solve the problem. In order to increase the speed of performance and increase automaticity of mathematical facts, Delquadri, Greenwood, Stretton, and Hall (1983) suggested that repeated practice is necessary, including increased opportunities to respond (OTR). Successful retention of new information through rehearsal is linked to the number of practice trials (Daly, Hintze, & Hamler, 2000; Delaquadri & Hall, 1984; Logan & Klapp, 1991).

Burns (2005) conducted a study on incremental rehearsal (IR) and the effect of using IR to teach unknown single-digit multiplication facts to children identified as having MLD. IR uses a gradually increasing ratio of known to unknown items that reach, at the final stage, a ratio of 90% known items to 10% unknown items. IR produces many OTRs. Including known items with unknown items allows for a level of challenge so that the learning task will not be too easy or too difficult. Burns taught multiplication facts using IR without showing the answer to the children. The children stated the answer orally and when three errors occurred, the children were placed in the special education resource room activity. The multiplication skills of these students improved following the IR intervention.

In sum, interventions to increase the calculation skills of students with MLD are effective in increasing fluency (Daly, Hintze, & Hamler, 2000; Delaquadri & Hall, 1984; Haring & Eaton, 1978; Logan & Klapp, 1991; Pellegrino & Goldman, 1987). Much research has been conducted using drill and practice methods, and these methods have repeatedly demonstrated increased math skills. Rehearsal strategies show positive results, because research has repeatedly shown that children with MLD have working memory deficits (Jordan & Hanich, 2000; Burns, 2005). Providing immediate feedback when students produce incorrect responses increases retention of mathematics skills (Bangert-Downs, et al., 1991; Brosvic et al., 2006; Codding et al., 2007). Immediate feedback provides students with MLD with more opportunities to respond, which increases the amount of practice, and therefore retention.

It is clear that drill and practice methods effectively increase student performance in mathematics. Drill and practice methods have two potential limitations, however. First, drill and practice methods can be burdensome for teachers to implement, because they require increased levels of individualized attention (Grafman & Cates, 2010). Second, students may not find these methods enjoyable, and student engagement is related to preference for academic tasks (Greenwood, Delquadri, & Hall, 1984; Skinner, 2002). Teachers and clinicians may prefer to use procedures that do not require as much individualized attention, but still consist of increased opportunities for students to respond and to be provided with feedback (Grafman & Cates, 2010).

Self-managed instructional strategies are one set of procedures that may be more enjoyable to students and also present increased opportunities to respond. These procedures focus on antecedent-response-consequence chains (Skinner, 1998) that provide students with the stimulus, opportunities to respond, and to evaluate their response and make corrections as necessary (Grafman & Cates, 2010). Cover, Copy and Compare (CCC) is a self-managed academic intervention that has been demonstrated to be effective for spelling (McAuley & McLaughlin, 1992; Murphy, Hern, Williams, & McLaughlin, 1990; Pratt-Struthers, Bartalamay, Williams, McLaughlin, 1989), geography (Skinner, Belfiore, & Pierce, 1992), science (Smith, Dittmer, & Skinner, 2002), and mathematics (Codding et al., 2007, 2009; Skinner, Shapiro, Turco, Cole, & Brown, 1992).

Skinner and colleagues (1989) studied the effects of a CCC procedure on single digit by single digit multiplication performance of primary and secondary students with behavior disorders. The authors demonstrated how the CCC procedure increased both accuracy and fluency in all participants in the multiple-baseline design.

CCC can be used to improve accuracy and fluency in responding for children with learning problems across a variety of mathematical calculation skills (Skinner et al., 1989).

CCC provides a series of learning trials through five steps: (a) look at the mathematics problem with the answer, (b) cover the mathematics problem with the answer, (c) record the answer, (d) uncover the mathematics problem with the answer, and (e) compare the answer to the recorded answer (Skinner, McLaughlin, & Logan, 1997). Students engage in repeated trials of this procedure, which result in improved accuracy and fluency of mathematical calculation skills. CCC is an academic intervention that has been used across many skills, such as spelling and mathematics, with students identified at-risk for failure and those with an identifiable learning disability.

Purpose of Present Study

The primary purpose of this dissertation was to compare the isolated effects of CCC with the combined effects of CCC with either immediate performance feedback or a reward (reinforcer) on the curriculum based measurement (CBM) of mathematical calculation skills of students at risk for MLD (e.g., combining CCC with either performance feedback or a reward will improve the accuracy and fluency of mathematics skill to a greater degree than CCC alone). Many students with MLD show deficits in working memory and fact retrieval (Geary, 2004; Jordan & Hanich, 2000). Providing immediate feedback, which is affirming and corrective, can facilitate in the acquisition and retention of numerical facts by elementary students with MLD (Bangert-Downs, et al., 1991; Brosvic, Dihoff, Epstein, & Cook, 2006; Codding, 2007). Teaching mathematical facts to students with MLD through drill and practice alone may not improve retention as much as combining drill and practice methods, such as CCC, with affirming or corrective feedback (Codding, 2009; Eckert, Dunn, & Ardoin, 2006; Skinner, 1998).

The first goal of this study was to examine the change in fluency of mathematics computation skills of the child at risk for MLD following either CCC or CCC combined with immediate performance feedback or a reward. Fluency in computation skills includes the automatic computation of mathematical facts (Learner, 2003) and measured high rates of responding. Fluency is obtained when it is faster to solve the problem through recall than to perform the steps needed to solve the problem. An increase in fluency can be obtained through repeated practice and rehearsal of the calculation skill (Daly, Hintze, & Hamler, 2000; Logan & Klapp, 1991).

It was hypothesized that students at risk for MLD would show an increase in fluency of mathematics computation skills over their baseline performances when computing single digit addition and subtraction problems following practice of computations using CCC. A second goal of the study was to determine whether the combination of CCC and immediate performance feedback would result in greater increases in fluency over baseline beyond the increases achieved by CCC alone. It was hypothesized that students at risk for MLD would show greater increases in fluency of digits correct over their baseline performances when performing mathematical computations in single-digit addition and single-digit subtraction when immediate performance feedback was combined with CCC.

A third goal of this study was to determine whether there would be greater increases in fluency of digits correct over their baseline performances for children at risk for MLD when computing single-digit addition and single-digit subtraction problems when CCC was combined with a reward.

CHAPTER II

Literature Review and Rationale

This chapter reviews the natural development of mathematical understandings, mathematical cognition and development, and mathematics as a subject in school. This is followed by the research that examines the diagnostic criteria of Mathematics Learning Disorder, a phenotype of MLD, and the subtypes of MLD. A review of the interventions that have been effective in increasing mathematical computation skills in students with MLD follows. The chapter then discusses Curriculum Based Measurement and how it is used to measure the progress of academic interventions. The remaining portion of the chapter examines self-managed interventions and focuses on Cover, Copy, Compare as an intervention for students with MLD.

Natural Development of Mathematical Understanding

The development of mathematical understanding and competency does not begin when a child enters school, but rather begins in early childhood (Krajewski & Schneider, 2009). Infants and young children begin developing what is called number sense, in which children begin to understand numbers and numerical relationships (Malofeeva, Day, & Saco, 2004). Fennell and Landis (1994) define number sense as awareness and understanding about what numbers are, their relationships, their magnitude, and the relative effect of operating on numbers, including the use of mental mathematics and estimation. Students must therefore understand how numbers relate to one another. Number sense is also the foundation from which all other mathematical concepts and ideas arise, and number sense is a necessary condition for learning formal arithmetic in the early elementary grades (Fennell & Landis, 1994; Gersten & Chard, 1999; Griffen, Case, & Siegler, 1994).

Number sense is facilitated by a child’s environment. The environmental conditions that promote the development of number sense are mediated by the informal teachings of parents, siblings, and other adults in the child’s life (Gersten & Chard, 1999). Children may also enter school knowing that 8 is bigger than 5. Number sense not only leads to an automatic use of math knowledge and information, but it is a vital component of the solution of basic arithmetic computations (Gersten & Chard, 1999; Griffen et al., 1994).

For example, Griffen et al. (1994) found that entering kindergarteners differed in their responses when asked questions such as “which number is bigger, 5 or 4?”, even when student abilities in counting and working simple arithmetic problems in the context of visual materials were controlled for. Children of high socioeconomic status (SES) answered the question correctly 96% of the time, compared with children of low SES who answered the question correctly only 18% of the time (Griffen et al., 1994). SES is typically defined by family income, level of poverty in the child’s neighborhood, and educational attainment by parents (Clements & Samara, 2008).

Besides number sense, children develop verbal subtizing, which is a term used to refer to young children’s ability to identify the number for small sets of objects without counting (Jordan & Levine, 2009; Pressley & McCormick, 1995). Many 2-year-olds can report to their parents when there are two toys, or two cookies. Three-to-4-year-olds can also point out three or four people (Pressley & McCormick, 1995). The identification of small sets of numbers associated with the objects or people that are present is called subtizing, and leads to identification of larger sets of objects when counting skills begin to develop.

Counting is a critical component in learning mathematics and is key in the extension of number understanding beyond small numbers (Ginsburg, 1989; Jordan & Levine, 2009). Children begin to learn the “count words”, such as “two” to label small quantities soon after they learn to talk (Fuson, 1988). Many pre-school aged children learn the whole number counting sequence from 1 to 10, and some even acquire the sequence from 1 to 20 before entering Kindergarten (Pressley & McCormick, 1995). Most children develop knowledge of three important counting principles before they enter Kindergarten (Gellman & Gallistel, 1978), including the principles of one-to-one correspondence (i.e., each item can only be counted once), stable order (i.e., the count words must be used in a consistent order), and cardinality (i.e., the final number in the count indicates how many items are in the set) (Jordan & Levine, 2009; Pressley & McCormick, 1995). Many five-year-olds recognize that if the count of two sets of objects ends in the same number, the two sets of objects have the same number of objects. Understanding of cardinality and equivalence of two sets is a cognitive prerequisite for learning simple addition and subtraction (Pressley & McCormick, 1995).

Children entering Kindergarten and Grade 1 will have many of the fundamental understandings of mathematics just described. Their counting principles will expand during these years from being able to count in one-to-one correspondence to learning to count backwards, to count by twos, and to detail objects sets greater than 10 (Jordan & Levine, 2009). Kindergarten and Grade-1 students will begin to learn addition and two types of addition problems. Joining problems involves putting together two quantities, for example, “James had 5 marbles. Jessie gave him 6 more marbles. How many marbles did James and Jessie have altogether?” (Pressley & McCormick, 1995). Part-part-whole problems require analyzing an existing quantity and then breaking it down into components. For example, “There are 5 boys and 7 girls on the soccer team. How many children are on the team altogether?” (Pressley & McCormick, 1995). Children will attempt to solve problems using several different strategies. Some use fingers to count, while others may use concrete objects to solve the problems. A child who already has knowledge of the number fact, such as 5 + 6 = 11, based on prior experience will use his knowledge of the basic fact to solve the problem and won’t need to rely on finger-counting strategies (Carpenter & Moser, 1982; Pressley & McCormick, 1995).

Just as children will apply their knowledge of numbers to learn to solve addition problems in the early grades, they will also learn to solve subtraction problems. Just as addition problems require several strategies, solution of subtraction problems require the application of different types of strategies. One such type of strategy to solve subtraction problems is separation (i.e., one set of objects is separated from another). For example, the problem might read, “Tom has 13 marbles. He gave 5 marbles to his sister Connie. How many marbles does Tom have left?” (Pressley & McCormick, 1995). Other types of subtraction problems are join/missing addend problems. These problems are structured by requiring the child to figure out the missing addend. For example, “Tom had 5 marbles. His sister Connie has 13 marbles. How many more marbles does Connie have than Tom?” (Pressley & McCormick, 1995). A third type of subtraction problem will involve a combine problem, which requires a specification of a total, with one of the numbers that contributes to the total given and the other number missing. For example a combine problem would be, “There are 13 marbles in the bag. Five of them are red, and the rest are blue. How many blue marbles are in the bag?” (Pressley & McCormick, 1995).

All of the problems discussed in the previous two paragraphs are word problems. The National Council of Teachers of Mathematics recommends that word problems be given to young children in the early elementary grades (NCTM, 2006). Children often use fingers as concrete representational aids to solve word problems. As children use strategies to work addition and subtraction problems, they develop associations between the problem statements and the answers they generate, such as 2 + 5 = 7 is strengthened when a child solves 2 + 5 using fingers for counting (Pressley & McCormick, 1995; Siegler & Shrager, 1984). Unfortunately, children do not always produce the correct answer to problems, and as a result, associations with incorrect answers are formed. For example, the association between 2 + 5 and 6 is strengthened when a child erroneously produces 6 as an answer to the problem 2 + 5. It is important that the most frequent association is also the correct response. The correct response is strengthened externally by the classroom teacher who provides students with, and shows complete number facts to students during lessons. When the associative strength between the problem and the correct answer is stronger than any other number-problem association, the student elicits the correct answer and no longer needs concrete aids or counting to solve problems. The student can “look it up” in her long-term memory (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981).

In sum, students develop mathematical skills from an early age, even before they attend school. Number sense and subtizing are the beginnings of understanding of numbers and their relationship to concrete objects and sets. These skills are developed in infancy through interactions with parents, siblings and other adults in a child’s life. When students come to school, formal instruction in mathematics focuses on further developing these skills and applying the basic understanding of numbers to concrete operations, such as addition and subtraction in the early grades. Once these operations are learned, basic number facts become automated associations and students no longer need strategies, such as finger counting, to perform arithmetic tasks. Rather, they can retrieve these facts from their long-term memory and focus on learning higher-level math skills as they progress through elementary school.

Mathematical Cognition

Mathematics is a highly interconnected and cumulative subject area and numbers pervade all areas of mathematics (NCTM, 2006). It is important to understand mathematical cognition and the natural development of mathematical thinking in order to understand a learning disorder of mathematics. Schoenfeld (1985, 1992) provides a model of mathematical thinking and knowledge that separates mathematics into four components of information processing: (1) knowledge base, (2) problem-solving strategies, (3) monitoring and control, and (4) beliefs and affects. A fifth component is instructional practices that foster effective mathematical cognition (Pressley & McCormick, 1995).

Mathematical knowledge includes both declarative knowledge, which is factual knowledge, and procedural knowledge, or knowing how to perform a task (Kolers, 1975; Scheffler, 1965). A student who has little difficulty with mathematics will be able to articulate his or her declarative knowledge of mathematical concepts and procedural knowledge when solving problems (Pressley & McCormick, 1995). For example, a student who is asked to solve a geometric problem, will be able to state that the length of a circle is called a diameter, and also be able to explain how to calculate this length when given other properties and information about the circle and its size. This student has both the understanding of the facts of a circle, as well as of the procedures he or she must carry out to calculate these facts.

Schoenfeld (1992) also recognized that problem solving depends on the short-term capacity of memory. Long-term memory of mathematics is well organized within the brain and is not as limited in capacity as short-term memory. Good problem solvers are therefore able to hold more information within their short-term memory and their long-term stores of mathematical knowledge are better organized (Cooney & Swanson, 1990; Pressley & McCormick, 1995; Schoenfeld, 1992). When working memory capacity is required, it becomes more difficult for learners to solve problems. Those problems requiring computations that have not become automatized are more demanding than those that require only calculations that are automatized (Fayl, Abdi, & Gombert, 1987; Pressley & McCormick, 1995). Gagne (1983) stated that if computational skills were mastered to an automatic level, students would be able to direct energy and attention to the complex aspects of math problem solving. When students can recall math facts rapidly as they work on a word problem, their short-term memory demands are reduced. That is, having to reflect on the answer to 3 + 2 = 5 consumes short-term capacity (Pressley & McCormick, 1995).

Zentall (1990) assessed the retrieval time and accuracy of math computations and word problems by students with learning disabilities, attention-deficit-disorders, and typically-developing seventh and eighth graders. Zentall used a measure of basic skill and retrieval time, to discriminate between children with LD, ADD, and typical children to assess the relation between computational retrieval time and problem solving performance. Zentall (1990) observed that only speed of math fact retrieval predicted performance on word problems, which was consistent with the hypothesis that when short-term memory is strained, word problem solving performance declines.

While problem solving in mathematics, a student must also possess a method to monitor his cognitions and determine which strategy to choose when attempting to solve a problem (Pressley & McCormick, 1995). Monitoring is an aspect of metacognition that contributes to students becoming self-regulated learners (Ramdass & Zimmerman, 2008; Schoenfeld, 1979). Strategy training must occur if students are to be aware of what strategies work best for which problems. Students will not be able to apply the strategies they are taught if they do not see their value (Ramdass & Zimmerman, 2008). Monitoring is a large component of mathematical cognition and good mathematics instruction, because teachers assist students' choice of strategies to solve problems and when to implement them. Students then learn to ask questions and monitor their own performance during problem solving. (Schoenfeld, 1992).

Students' construct beliefs about mathematics, which are often based on the experiences they have had during mathematics instruction, are the last part of mathematical cognition according to Schoenfeld (1992). Common beliefs, such as "there is only one way to solve any mathematics problem correctly", which is usually the rule that the teacher has demonstrated to the class, often discourage reflective problem solving (Pressley & McCormick, 1995). Students beliefs about their own abilities to solve mathematics problems, will in turn affect their self-efficacy belief (i.e., their belief that they are capable of doing well on an academic task (Bandura, 1977a, 1986; Schunk, 1990, 1991; Zimmerman, 1989a, 1989b, 1990a, 1990b). Unrealistically low self-efficacy beliefs can become responsible for student avoidance of challenging academic courses, such as mathematics (Ramdass & Zimmerman, 2008).

In summary, mathematical cognition is composed of four parts of cognitive processing. Students must first gain a knowledge base in mathematics, which begins in early infancy; is fostered by parents, siblings, and teachers; and grows as students attend and progress throughout elementary school. They will gain strategies for mathematics problem solving through instruction in mathematics, and it is important that teachers and instructors emphasize to students how to choose strategies to solve particular problems. Once students have learned the strategies, they must monitor their strategy use and progress in order to facilitate questions about problems and learn how to monitor their own progress. Students' beliefs about their abilities will affect not only their performance, but also whether or not they attempt difficult academic tasks, such as mathematics instruction. Those students who become frustrated with continued attempts and repeated failures, may be lacking in skills and possibly have a Mathematics Learning Disorder.

Mathematics as a School Subject

Mathematics is the study of measurement, properties, and relationships of quantities and sets, using numbers and symbols (American Heritage Dictionary, 2010). As a school subject, these terms are a part of standards and a mathematics curriculum that becomes increasingly more complex at each successive grade level. The Common Core

State Standards for both Mathematics and English Language Arts were developed by a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The standards were developed in collaboration with teachers, school administrators, and experts within education who provided constructive criticism to develop the standards. Forty out of the 50 states in the United States of America have adopted these standards as a guideline for implementing mathematics instruction in grades K-12 (NGA Center, 2010).

Most students enter elementary school in Kindergarten, with some mathematical knowledge. In early childhood settings, such as pre-school and Kindergarten, mathematics should concentrate on (a) number, which includes whole number, operations, and relations and (b) geometry, spatial relations and measurement, with more mathematics time devoted to number than to other topics (NGA Center, 2010). Students learn to represent and compare whole numbers initially with sets of objects, and to describe shapes and space (NGA Center, 2010). Students in Kindergarten use written numbers including numerals, to represent quantities and to solve quantitative problems such as counting objects within a set, counting out a given number of objects, comparing sets or numerals, and modeling simple joining and separating situations with sets of objects. This leads to eventually recognizing and solving simple addition and subtraction equations, such as 5 + 2 = 7 and 7 – 2 = 5 (NGA Center, 2010). Students in Kindergarten also learn cardinality, counting order, and comparisons of numbers and quantities, such as less than and greater than. Geometry in Kindergarten focuses on recognizing shapes and creating and composing shapes. Students begin to work with numbers 11-19 to gain foundations for place value (NGA Center, 2010).

In Grade 1, students build on their knowledge bases of mathematics and focus on four critical areas: (a) developing understanding of addition, subtraction, and strategies for addition and subtraction with numbers 11-20; (b) developing understanding of whole number relationships and place value, including grouping in tens and ones; (c) developing understanding of linear measurements and measuring lengths in units; and (d) reasoning about attributes of and composing and decomposing of geometric shapes (NGA Center, 2010). Students in first grade begin adding and subtracting whole numbers based on their knowledge of numbers. They make connections between counting and addition and subtraction, such as adding two is the same as counting by two's, or two numbers ahead of where they started. First grade students also begin to understand the concepts of place value by10. In the area of measurement, first grade students begin to learn to tell and write time and represent and interpret data on pictorial graphs (NGA Center, 2010). When students have difficulty attaining these goals in the early childhood grades of Kindergarten and first grade, it becomes difficult to succeed when the demands of the mathematics curriculum becomes more challenging.

In second grade, instructional time is focused on four critical areas: (a) extending understanding of base-ten notation; (b) building fluency with addition and subtraction; (c) using standard units of measure; and (d) describing and analyzing shapes (NGA Center, 2010). Students in the second grade begin to expand their understanding of counting and learn to count by fives, tens, and hundreds. They begin to compare these units and understand the relationships between these units by comparing them. Students in second grade are taught to use a ruler and understand standard units of measure, such as centimeter and inch, and begin to understand concepts pertaining to shapes by analyzing the sides and angles of shapes.

In third grade mathematics, students' instructional time is spent on four critical areas. First, students work on developing understanding of multiplication and division strategies for multiplication and division within 100. Students work on finding either an unknown factor in multiplication or an unknown dividend in division. They also work on learning and understanding the relationship between division and multiplication and how the two operations are related. Second, students in third grade learn about fractions and develop an understanding of fractions, with a focus on unit fractions (fractions with numerator 1). Fraction models are used to represent how fractions are a part of a whole unit. They learn to compare fractions, such as ½ is greater than 1/3. A third part of third grade math is developing an understanding of the structure of rectangular arrays and area. Students break down rectangular areas into smaller units, and link these units to multiplication. They begin to understand the concepts of two-dimensional area. The last objective of third grade math is describing and analyzing two-dimensional shapes. Students learn to classify shapes by their sides and angles and link these classifications with their definitions of shapes (NGA Center, 2010).

In the fourth grade, math is composed of a focus on developing fluency in multi-digit multiplication and division. Place value to 1,000,000 is studied and students' understanding of place value is applied to multiplication and division. Students also develop an understanding of fraction equivalence, that two different fractions can be equal (e.g., 2/4 and ½), and develop methods for recognizing equal fractions. They learn to multiply fractions by a whole number and continue to build knowledge about how fractions are decomposed into unit fractions or composed into whole numbers. Another focus of math in the fourth grade is for students to understand two-dimensional shapes and their properties (NGA Center, 2010).

Fifth grade is usually the last grade in elementary school. In fifth grade math, students are expected to focus on developing fluency with addition and subtraction of fractions, as well as developing an understanding of multiplication with fractions and of division of fractions in some cases. Division skills are also expanded to include two-digit divisors. Students learn to integrate decimals into the place value system to develop an understanding of operations with the hundredths place value (e.g., 0.01). Students in the fifth grade also begin to develop an understanding of volume of shapes (NGA Center, 2010).

Elementary mathematics skills begin with early understandings of numbers, such as number sense and counting. With each successive grade level, students build on the skills they have obtained in math. Addition and subtraction skills begin in the early childhood grades of Kindergarten and first grade with single digits, and expand as the students progress in each successive grade level to two-digit computations, and multi-digit computations. Students then learn the prerequisite skills for multiplication and division and expand these operations to multi-digits as well. Geometry is a part of math skills, beginning with recognizing shapes in the early grades, and progressing to understanding angles, area, and the volume of three-dimensional shapes. Mastering basic math skills is important for achievement in each successive grade in elementary school, and for further study in mathematics and everyday life. Those students who struggle with math skills may benefit from extra academic instruction in math, or may be diagnosed with a Mathematics Learning Disorder (MLD) if their performance is substantially lower than expected (Geary, Hamson, & Hoard, 2000).

Diagnosing MLD

According to the DSM-IV-TR, the diagnostic criteria for 315.5 Mathematics Disorder involves mathematical ability, as measured by individually administered standardized tests, which is substantially below that expected given the person's chronological age, measured intelligence, and age appropriate education (DSM-IV-TR, 2000). The disorder must significantly interfere with academic achievement or activities of daily living that involve mathematical ability. If there is a sensory deficit present, the mathematical difficulties are in excess of those usually associated with it (DSM-IV-TR, 2000). This definition of Mathematics Disorder (MD) is based on a discrepancy between ability and achievement.

Using the DSM-IV-TR definition of MD calls for diagnostic use of standardized achievement tests in combination with measures of intelligence. (Geary, 2004). A score lower than the 20^{th} or 25^{th} percentile on a mathematics achievement test, combined with a low average or higher IQ score has been the typical criteria for diagnosing MLD (Geary, Hamson, & Hoard, 2000). However, standardized achievement tests include questions that cover a variety of mathematical topics and tasks, and children with MLD often have more specific deficits in some of these areas. They may perform in the average range in other areas examined. The overall score of a child on a standardized achievement test who in fact does have MLD is obtained from the average of the scores across different mathematical items, and the total score may inflate the competency of this child in some areas while underestimating her competency in others (Geary, 2004).

In school settings, School Psychologists diagnose MLDs and eligibility for special education services according to the regulations of the Individuals with Disabilities Education Act (IDEA), which was most recently reauthorized in 2004 with final regulations published in 2006. Children who present with difficulties in mathematics, according to IDEA, fall under the category of Specific Learning Disability. Under IDEA, the term ‘specific learning disability’ means

(1) A disorder in one or more of the basic psychological processes involved in understanding or using language, spoken or written, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell or do mathematical calculation.

(2) Includes such conditions as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.

(3) Does not include a learning problem that is primarily the result of visual, hearing, or motor disabilities, of mental retardation, emotional disturbance, or of environmental, cultural or economic disadvantage (IDEA, 20 U.S.C. § 1401 [30]).

Before the reauthorization of IDEA in 2004, a severe discrepancy between intellectual ability and academic achievement was required in order for a student to be identified with a Specific Learning Disability. As part of the current IDEA, school districts are not required to take into account a severe discrepancy between intelligence and achievement when determining whether a student presents with a Specific Learning

Disability (SLD) and if he qualifies for special education services. Key additions to the procedures for identification of specific learning disabilities are those that do not require states to use a severe discrepancy between intellectual ability and achievement 34 CFR 300.309, permit the use of a process based on the child's response to scientific 34 CFR 300.8(c)(10), research-based intervention, and permit the use of other alternative research-based procedures for determining whether a child has a SLD 34 CFR 300.8(c)(10). In updating IDEA, Congress indicated a desire to see school districts begin to use procedures to identify SLD that are more relevant to instruction (Cortiella, 2006). IDEA also includes an additional provision stating that schools can use the student's response to research-based intervention within the evaluation process.

Because measures that are designed to diagnose MLD specifically are not available, research-based interventions are invaluable in diagnosing MLD. Children with MLD can present as treatment resistant to intervention and this characteristic can be used as a diagnostic criterion for MLD. An example of such a process is Responsiveness to Intervention, also called Response to Intervention or RTI.

RTI integrates assessment and intervention to identify students who are at-risk for poor learning outcomes, monitors student progress and provides evidence-based interventions. The intensity and nature of the interventions are adjusted depending on student responsiveness and can help to identify students with SLD and other disorders within the school environment (Artiles & Kozleski, 2010) as well as determine the need for special education services (Cortiella, 2006).

Despite the differences in defining and identifying MLD, the consensus in the research is that students at-risk for MLD should be identified at an early age and interventions should be consistent with classroom instruction and provide consistent monitoring of student progress.

Phenotype of MLD

The complexity of the field of mathematics makes it difficult to define a cognitive phenotype of MLD. Determination of this phenotype is complicated further by the difficulty of distinguishing low achievement due to poor instruction from an actual learning disability (Geary, Brown, & Samaranayake, 1991). The approach that instruction takes, whether it emphasizes conceptual understanding or procedures, also affects whether an observed deficit would be considered a learning disability (Geary, 2004).

Many children with MLD have difficulties with basic arithmetic fact retrieval from their long-term memory, and these types of difficulties are pervasive in children with MLD even when they are provided with intensive instruction on basic facts (Geary, 2004; Howell, Sidorenko, & Jurica, 1987). Several procedural deficits and characteristics are present when a child has MLD as well as retrieval deficits. Children with MLD tend to use immature strategies, such as finger counting when engaging in problem solving. Some studies have focused on the competency of children with MLD children when solving multi-step arithmetic problems.

Russell and Ginsburg (1984) studied fourth-grade children with MLD with normal intelligence and fourth-grade children with matched intelligence with no difficulty with mathematics. The children with MLD were found to have “insightful” solutions, which can help to shortcut problem solving and can solve simple forms of word problems, but they have difficulty with complex word problems (Russell & Ginsburg, 2000). Russell and Ginsburg also found that children with MLD have a generally adequate informal knowledge of mathematics. The study intended to assess knowledge of number facts, which are crucial for calculation and instruction (Russell and Ginsberg, 1984) and are a "rote" aspect of mathematical knowledge. It is assumed that number facts would therefore be relatively simple for a child with MLD to learn. The results of the study showed that a "rote" task of addition facts was the most difficult for the children with MLD (Russell & Ginsburg, 1984).

Children with MLD have difficulty retrieving basic arithmetic facts from their long-term memory more than they do performing any other aspect of arithmetic (Jordan & Hanich, 2000; Jordan Hanich, & Kaplan, 2003; Jordan & Montani, 1997), which will in turn affect their performance on subsequent higher-level tasks, as most of instruction time is spent on basic fact retrieval. The specific relationship between working memory and difficulties in carrying out and completing arithmetic tasks is not yet fully understood. The deficit involves the representation of number words and the support of these representations when performing procedures, such as counting (Geary, 2004; Siegler & Shrager, 1984). For example, children state "/eyt/" or "eight" when asked to solve 5 + 3 (Geary, 2004). Once the representation is formed in long-term memory, it supports the use of memory-based problem-solving processes (Geary, 2004).

Children with MLD use finger counting as a strategy for solving arithmetic problems so that the representation is their fingers and the demands on the working memory are reduced (Geary, 1990; Geary, Bow-Thomas, Liu, & Siegler, 1996). Children with MLD are also known to undercount or over count during problem-solving, which is related to the deficits in working memory that are part of the diagnosis of MLD (Geary 1990; Hanich, et al., 2001). Children with MLD easily lose track of how many fingers they have counted, and how many remain to be counted (Geary, 2004), and rely much more on counting strategies in arithmetic at ages when their age mates are relying more on fact retrieval (Cummings & Elkins, 1999; Miles, Haslum, & Wheeler, 2001; Russell & Ginsburg, 1984, Yeo, 2003). Fact retrieval occurs when children pull the representation of the answer to a problem from their long-term memory (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981). For example, when given the problem 2 + 2 = 4, the student is able to answer automatically, rather than counting on her fingers, or requiring other types of strategies to produce the answer.

The prevalence of MLD was studied through experimental measures administered to more than 300 children from well-defined populations, that is all fourth graders in an urban school district in the United States (Badian & Ghublikian, 1983). The experimental measures focused mainly on number and arithmetic competencies and the deficits that are associated with dyscalculia (Geary & Hoard, 2002; Geary, 2004). Performance that differed significantly from age-related norms and was similar to the performance associated with dyscalculia has been used as an indication of MLD and suggests that 5% to 8% of school-age children exhibit MLD (Auerbach, Manor, & Gross-Tsur, 2000; Badian, 1983; Geary, 2004; Shalev, Fleischner & Manheimer, 1997).

Subtypes of MLD

According to Geary (2004) MLD may be divided into three subtypes, which are specifically procedural, semantic memory, and visuospatial. Procedural deficits are present when children with MLD engage in problem solving, especially when solving multi-step arithmetic problems. Russell and Ginsberg (1984) found that fourth-grade children with MLD committed more errors than their IQ-matched typically achieving peers when solving problems such as 45 x 12 or 126 + 537. The errors were those of misalignment of numbers when writing down partial answers or errors while carrying or borrowing from one column to the next (Russell & Ginsburg, 1984). Procedural deficits also involved the use of developmentally immature strategies when solving problems, such as finger counting, which is used by younger, typically achieving students (Geary, 2004). The procedural errors of children with MLD while solving complex arithmetic problems, as described by Russell and Ginsburg (1984), may result from difficulties in monitoring and coordinating the sequence of steps in solving the problem, and in turn suggest a pre-frontal and executive dysfunction (Gross-Tsur, Manor, & Shalev, 1996; Russell & Ginsburg, 1984; Welsh & Pennington, 1988). Deficits in the central executive area of the brain and associated areas of the prefrontal cortex that support inhibitory mechanisms result in retrieval errors, because children with MLD cannot inhibit irrelevant associations from entering their working memory (Bull, Johnston, & Roy, 1999; Welsh & Pennington, 1988). Being unable to block out irrelevant information and associations in turn makes mathematical problem solving less efficient for children with MLD.

The semantic memory subtype of MLD consists of difficulties in retrieval of basic facts, such as answers to mathematical problems. Children with this subtype of MLD display a high error rate for mathematical facts (Geary, 2004). The deficit seems to involve the information representation in the language system, which is the semantic system that represents the articulation of number words and supports procedural abilities, such as counting (Geary, 2004). Wilson and Swanson (2001) examined whether mathematics disabilities are due to domain-general or domain-specific working memory deficits. They examined the relationship between verbal and visual-spatial working memory and mathematical computation skill in both children and adults with and without disabilities in mathematics. Results showed that verbal and visual-spatial working memory composites predicted mathematics performance, and these results held when reading ability and gender were accounted for in the analysis. Working memory resources play a crucial role in mathematics performance (Geary, 2004; Geary, Hamson, & Hoard, 2000; Mazzocco & Hanich, 2010; Wilson & Swanson, 2001)

Some studies of children with MLD have identified another form of retrieval deficit. The retrieval of mathematical facts may become disrupted due to the hindrance of retrieval by unrelated associations. Geary, Hamson, and Hoard (2000) conducted a study in which children were given arithmetic tasks and asked not to use counting strategies (e.g., using their fingers) to solve simple arithmetic problems. Children with MLD and RD committed more retrieval errors than their typically achieving peers, with the most common error being the next number in the sequence of one of the addends in the problem. For example, if the problem given was 6 + 2, the most common responses were 7 and 3, the numbers that follow 6 and 2, respectively in counting sequence (Geary et al., 2000). This result demonstrates a basic fact retrieval error, because the student did not know the fact automatically, and produced an incorrect response when she was told not to use counting strategies to compute the result.

The visuospatial deficit subtype has not been studied extensively. Visuospatial systems are involved in areas of mathematics, such as geometry and solving complex word problems (Geary, 1996). Hanich, Jordan, Kaplan, and Dick (2001) found that children with MLD differed from their peers on an estimation task, specifically approximate arithmetic. On this task, the students were required to estimate results, such as 9 + 8 = 20 or 30, and the children must form a “mental number line” to estimate the answer (Dehaene &Cohen, 1991). This type of approximation is a visuospatial ability, independent of semantic or language representation (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). When simple arithmetic is involved, many children with the procedural or memory forms of MLD do not differ from other children in basic visuospatial abilities (Geary et al., 2000).

In all three subtypes, the developmental delays and deficits are related to a combination of disrupted functions in the central executive area of the brain, attentional control, and poor ability to inhibit irrelevant associations, and difficulties with information representation in the language system (Geary, 2004; Geary, 1993; Mazzocco, 2001; Shalev, et al., 1998). Children with MLD who display the semantic memory subtype present mainly with deficits of fact retrieval. The procedural subtype of MLD results in deficits when carrying out procedures to solve mathematical problems, as well as continued immature strategy use. Delays in the visuospatial system can also result in MLD, but the mechanisms of this pathway are not fully understood. In sum, the three subtypes of MLD result in deficits in both carrying out mathematical problems and storing mathematical facts in long-term memory to retrieve them when needed.

Educational Features of Students with MLD

Given the myth in the field of education that many more students have learning disabilities that affect their ability to learn to read, spell, and write, rather than to do math, most students are referred for evaluation to determine if their difficulties in school are related to delays in early reading skills (Fleischner & Manheimer, 1997; Gottlieb &

Weinberg, 1999; Soodak & Podell, 1993). A number of studies have indicated that poor achievement in arithmetic may be largely attributable to the same language-based deficits that underlie poor reading achievement (Share, Moffitt, & Silva, 1988). Children with poor arithmetic achievement also tend to be poor readers (Satz, Taylor, Friel, & Fletcher, 1978). Common verbal factors may underlie both arithmetic and difficulties experienced by children with both poor arithmetic and poor reading achievement (Geary, 1993; Geary & Hoard, 2001; Share et al., 1988). Geary (1993) posited that there is a relationship between deficits in processing words and accessing arithmetic facts from long-term memory. Children who have arithmetic difficulties often have difficulties in storing arithmetic facts, or in accessing them in their long-term memory (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997). Arithmetic fact retrieval is based on counting, which involves number words and the use of phonological skills. When counting, number words are activated and the association in long-term memory between the problem and answer could be represented in part by the same phonetic and semantic systems that support word recognition (Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Geary & Hoard, 2001). There is also a subgroup of children who have specific difficulties in mathematics with normal levels of reading. This differentiates children with MLD and those with MLD with co-morbid reading disorder (Rourke, 1993; Rourke & Conway, 1997; Rourke & Finlayson, 1978)

Rourke and Finlayson (1978) studied children with learning disabilities with above average reading ability, but below-average mathematics achievement. These children had different strengths and weaknesses compared with children with low achievement in both reading and mathematics. The group with specific mathematics difficulties showed poor non-verbal skills relative to their verbal skills. The group with both mathematics and reading difficulties displayed a crossover effect, in which their performance on non-verbal tasks was superior to their performance on verbal tasks (Rourke & Finlayson, 1978). According to Rourke (1993) and Rourke and Conway (1997) the presence or absence of reading disabilities (RD) with MLD is associated with dissimilar patterns of neuropsychological and arithmetical performances. This claim further suggests different hemispheric dysfunctions (Rouselle & Noel, 2007). Rourke (1993) and Rourke and Conway (1997) found that children with MLD/RD mainly exhibit verbal deficits, which are presumably linked to left hemispheric dysfunctions, while children with MLD only manifested problems in mainly non-verbal tasks, which is linked to right hemispheric dysfunctions.

A study conducted by Share et al. (1988) sought to find evidence of the crossover effect that was observed by Rourke and his colleagues (1978) using longitudinal data from a large sample that was homogenous in age. Achievement tests were administered at age 11 to divide the group into arithmetic-and-reading disabled, specific arithmetic disabled, and non-disabled children who were used as a control group. The study attempted to identify the factors that are associated with arithmetic-and-reading disability and specific math disability. Data were taken from several measures of general speech, language, intelligence, and motor development at ages 9, 11, and 13. Neuropsychological measures were administered that required subjects to use language, and nonverbal measures were administered as well.

Share et al. (1988) concluded that nonverbal deficits may have a causal role in specific arithmetic disability in boys. Gender results were analyzed separately because it has been shown that combining boys and girls may cause gender-related differences or distort within-gender results (Share, et al., 1988). Boys with specific arithmetic disabilities showed a reverse pattern of strengths and weaknesses compared to boys with arithmetic-and-reading disabilities. The group with specific arithmetic disabilities also displayed significantly lower non-verbal skills than boys of the control group (Share et al., 1988). Girls with arithmetic-and-reading disabilities showed a similar pattern to the boys with arithmetic-and-reading disabilities. The crossover effect was absent for girls. The girls with specific arithmetic disability did not differ from the control group. The data from this study supported Share et al.'s (1988) hypothesis that nonverbal deficits have a causal role in MLD in boys; however, other factors may have played a role in the specific arithmetic disabilities in girls. The study also concluded that deficits in the language area alone are not sufficient to explain arithmetic difficulties in children with reading deficits. Other factors, such as motivation for mathematics learning, may play a role in the performance of girls (Share et al., 1988).

Research on children with MLD suggests that deficiencies are prevalent in two areas of mathematical thinking: retrieval of number facts and the ability to solve story problems (Russell & Ginsburg, 1984). Work on the acquisition of number-fact skills in first and second grade students has shown that although children with MLD use the same type of calculation strategies (counting with fingers and verbal counting) as children without disabilities, they make more retrieval and counting errors and use less mature strategies for longer periods of time (Jordan & Hanich, 2000). The children with MLD show deficits in using fact-retrieval as a strategy and these deficits tends to persist throughout elementary school (Jordan & Hanich, 2000).

Geary, Bow-Thomas, and Yao (1992) contrasted counting knowledge of children with MLD and co-morbid reading disorder (RD) with their typically achieving peers. Typical development of children's understanding of counting emerges from several principles, cited by Gelman and Gallistel (1978). These principles are: one-to-one correspondence (one word tag is assigned to each counted object), stable order (the order of the word tags must be invariant across counted sets), cardinality (the value of the final word tag represents the quantity or items in the counted sets), abstraction (objects of any kind can be collected together and counted), and order irrelevance (items within a given set can be tagged in any sequence).

Geary et al. (1992) conducted a study that focused on Gelman and Gallistel's (1978) principles. In the study, the children watched a puppet count a set of objects. The puppet sometimes counted correctly, and sometimes violated one of Gelman and Gallistel's (1978) counting principles. The children had to determine whether the puppet's counting was "OK" or "not OK and wrong." Results showed that children with MLD/RD identified correct counts, violations of the counting principles, and understood that counting from right-to-left was equivalent to standard counting from left-to-right. Many of the children with MLD/RD did not understand the order irrelevance principle and believed that adjacency was necessary for correct counting. Children with MLD/RD identified counts as correct when the first object was counted twice, suggesting that MLD/RD children had difficulties using their working memory system to hold information while simultaneously monitoring the act of counting.

Geary et al. (1992) found that children with MLD/RD and those with MLD only differ from children with RD only and from typical children when given adjacency trials in the first and second grade, and on double-counting trials (double counting of the first item) in the first grade. Children with MLD/RD and MLD only did not understand all the counting principles, and difficulties with working memory impeded their ability to perform as well as typically developing or children with RD only on counting tasks.

In addition to deficits in counting tasks, and basic addition and subtraction fluency, children with MLD may also fail to develop number sense. Number sense refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to make comparisons (Gersten & Chard, 1999). Number sense can be difficult to define, but easy to recognize. Case (1998) stated that children with a good number sense can invent their own procedures for conducting numerical operations, can represent the same number in multiple ways, and can recognize number patterns. They can also talk in a way about the general properties of a numerical problem without actually completing the calculations.

Number sense is acquired through interactions with parents and siblings before children even enter kindergarten (Gersten & Chard, 1999). Number sense leads to an automatic use of math information and is also a main component in the ability to solve basic arithmetic computations. Gersten and Chard (1999) believe that building number sense in students with learning disabilities would enable them to have a better awareness of numbers and their relationships. Because the concept of number sense is likely related to semantic representations of information, number sense would therefore be a deficit found in students with both MLD and MLD/RD (Geary, et al., 2000; Gersten & Chard, 1999; Jordan & Montani, 1997).

Studies have suggested that arithmetic and reading may depend on similar cognitive predictors (Geary, 1993; Jordan, Kaplan, & Hanich, 2002). Hecht, Torgesen, Wagner, and Rashotte (2001) found that the same phonological processing abilities that are considered to influence growth in reading skills contribute to the development of general computation skills. Children with both MLD and RD have been found to have greater deficits on non-verbal tasks than verbal tasks (Rourke & Finlayson, 1978). Children with combined reading and arithmetic disabilities not only have more generalized verbal and non-verbal problems, but also are most impaired when compared to groups with reading-only or arithmetic-only disabilities (Badian, 1999; Shafrir & Siegel, 1994).

Children with MLD only present with greater non-verbal deficits than children with combined MLD/RD. Students with MLD present with working memory deficits in storing or accessing arithmetic facts (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997), which affect mathematical performance. Students with MLD display deficits in understanding basic counting principles that can last throughout elementary school. Students with MLD are also thought to be slower in computing basic computations than their typically achieving peers, and therefore interventions with MLD may focus on increasing computation skills and fluency.

Academic Assessment of MLD and Curriculum Based Measures

Error analysis is used to determine whether errors in a student's computation are based on a systematic misunderstanding of processes or procedures (Ashlock, 1990). An error analysis can be conducted on one or a few examples, but one can only conclude that there are systematic misunderstandings if there are a sufficient number of examples

(Fleischner & Manheimer, 1997). Clinical interviews are procedures used by evaluators and teachers to have students talk through the processes they used in computing correct and incorrect answers (Fleischner & Manheimer, 1997). Both procedures require a substantial number of examples of student performance and time to analyze the root of the incorrect computational patterns. The educator must have sufficient knowledge in mathematics or incorrect analysis may occur (Fleischner & Manheimer, 1997).

A method of assessing student growth and performance in basic skill areas, such as reading, math, and spelling that does not involve error analysis is Curriculum Based Measurement (CBM). CBM was developed at the University of Minnesota Institute. During the 1980s, interest developed in alternative approaches to assess the outcomes of schooling. Approaches that were related directly to a school's curriculum and teacher's daily instruction were the most widely popular (Deno, 1992). CBM is a set of standardized procedures used to assess the level and trend of academic performance and growth in basic skill areas of reading, mathematics, written expression, and spelling (Deno, 1985; Shinn, 2002, 1989). These procedures are often used to screen for academic problems within student groups, identify instructional placements, evaluate student response to instruction, and make other educationally relevant decisions (Christ, Scullin, Tolbize, & Jiban, 2008; Fuchs, Deno, & Mirkin, 1984). The purpose of CBM is to enable teachers to improve student performance and to allow teachers to create a database of each student's performance so that the effectiveness of each student's individual education plan can be assessed (Deno, 1985, 1992, 2003; Stecker, Fuchs, & Fuchs, 2005). Because there are so many tests available to teachers to assess students, one might wonder why CBM was developed.

Several reasons support the development of CBM. Instruction in basic skills lacks clarity of focus, and both teachers and students are uncertain of what are the key indicators of growth (Deno, 1985, 1992; Stecker et al., 2005). Most standardized tests measure student outcomes on different subskills, rather than examine if the student is fluent in basic skills. CBM was created to measure the fluency of the skill. When asking teachers to provide the individual data on key indicators of student growth, there may be differences in what teachers consider a key indicator. CBM allows for consistency in the "vital signs" necessary to track student's growth (Deno, 1992) and whether the most recent instructional program was effective in bringing about the student growth (Stecker et al., 2005). Current achievement tests are designed to compare a student's performance to their relative standing within the population of students of the same age. The most reliable score that is obtained from a standardized achievement test is a percentile rank, which is useful when a student's achievement status within his or her school or the general population is of interest. When information about the student's growth in fluency and proficiency is needed, it cannot be obtained using a standardized achievement test. Standardized achievement tests also rely on face validity (i.e., whether the test item looks like it measures what it is supposed to measure). Although a test may have face validity, it may not measure the construct that it is supposed to measure, and therefore standardized achievement tests are not applicable to measuring fluency and individual growth within the classroom (Deno, 1992).

CBM assesses student progress toward long-term goals, such as general outcomes rather than consecutive objectives (Stecker et al., 2005). When using CBM, alternate forms of short tests are developed that examine performance toward the long-term goal and not just the content or skills that the student is currently learning within his/her classroom. To determine progress in a skill area, repeated measurements on equivalent forms of the same task across periods of time are administered (Deno, 1992). The change in student performance is measured by the rate of change exhibited in repeated measures of performance on a task of the same difficulty (Deno, 1992).

CBM has been referred to as a general outcome indicator, because it is flexible with respect to the curriculum the evaluator uses to create the CBM measure. The setting where it is conducted or the materials used do not constrict CBM. CBM can be used to measure the effectiveness of changing a student's program from one setting to the next or using a different set of materials in the same setting (Deno, 1992, 2003). Another important feature of CBM is that frequent monitoring of students progress is displayed on a time-series, equal-interval graph (Stecker et al., 2005). The data collected in CBM shows how a student performs over time, because the content and level of difficulty of the assessment tasks remain the same. CBM can be used to predict whether students are on track to meet their long-term goals by analyzing the data graphically.

The first experimental study that detected significant achievement effects in reading in which teachers monitored students' progress using CBM was conducted by Fuchs, Deno, and Mirkin (1984). Fuchs et al. (1984) conducted a large-scale investigation with 39 special education teachers in New York City who either used CBM to monitor special education student progress and performance in oral reading, followed by adjusted instruction, or did not use CBM. Special educators were assigned randomly to either (a) a progress monitoring condition, where teachers measured oral-reading fluency twice weekly, compared the slope of progress for every 7 to 10 data points against a goal line, and made instructional modifications; or (b) a conventional special education evaluation condition where the teachers used their typical procedures for measuring student progress, such as teacher-made periodic tests, informal observations, and workbook samples. The teachers who did not implement CBM created goals for the students' Individualized Education Plans (IEP) and monitored the progress as they wished. The teachers in the CBM condition wrote goals and objectives as a part of the special education students' Individualized Education Plan, and developed CBM systems to match the goals.

Fuchs et al. (1984) found significant effects on student reading when CBM and an ongoing measurement and observation system was implemented. The teachers in the CBM group also improved in their structure, while the contrast group teachers' structure decreased. Contrast teachers were also less specific in describing students' performance levels and maintenance of goals. CBM allowed the teachers to assess and re-evaluate goals more frequently and may have signaled more accurate assessment of the students' reading levels (Fuchs et al., 1984).

CBM has some limitations, despite its usefulness in guiding instructional practice and interventions. CBM requires a great deal of time on the part of the teacher to develop CBM tests, administer and score them, and to graph and use the data (Stecker et al., 2005) Wesson, King, and Deno (1984) examined whether teachers had heard of direct and frequent measurement, what percentage of teachers use frequent measurement, what factors inhibited teachers use of frequent measurement techniques, and for those teachers that did use direct and frequent measurement, what percentage of time did they allocate to measurement. The results of the survey of 136 special education teachers from all regions of the United States indicated two primary factors that inhibited the use of direct and frequent measurement. Many teachers did not know how to implement this type of measurement, even if they had heard of direct and frequent measurement, they had not been trained in its use (Wesson et al., 1984). The largest group of respondents who chose not to measure a student's behavior using direct measurement, believed that direct and frequent monitoring was too time consuming. However, teachers who did use the assessment procedures were inaccurate in their estimates of the time that was involved (Wesson et al., 1984). Teachers need training and experience in CBM procedures in order for them to understand the benefit of the techniques and accurately assess the amount of time it will require (Wesson et al., 1984).

CBM requires frequent data collection and careful monitoring of student progress. Teachers' simple collection of CBM data may not be powerful enough to affect student achievement (Stecker et al., 2005). Teachers must use the data to evaluate their instructional effectiveness and make program modifications accordingly. Raising goals when teachers underestimate student performance also appears to affect student growth (Stecker et al., 2005). Teachers may underestimate actual student progress and fail to raise goals spontaneously when students present with faster rates of learning than expected (Fuchs, Fuchs, & Hamlett, 1989). Without teachers' modification of goals during CBM procedures, student achievement will not reach its potential rate. Specific training in CBM and how to assess student performance spontaneously is necessary in order for CBM to be effective.

CBM is central to the RTI framework introduced in the reauthorization of IDEA (Shinn & Bamonto, 1998) to aid in evaluation of students based on instruction, rather than use the discrepancy model to identify academic disorders. Educators, teachers, and psychologists can use CBM to monitor student progress and make educational decisions about individuals (Shinn & Bamonto, 1998). CBM uses standardized procedures to make formative evaluations of students' progress and is a brief and efficient progress-monitoring tool.

In summary, CBM is an effective way to measure student progress during instruction and progress towards a long-term goal, rather than a specific objective. CBM uses frequent monitoring, which allows for instruction to be tailored to the students' progress, if they are not meeting the long-term goal. CBM is a formative evaluation, which measures progress as instruction progresses, rather than a summative assessment, which measures skills after instruction is completed. CBM is a useful tool since it can be used in many different subject areas to measure student progress if teachers and educators receive proper training.

CBM Application to Mathematics

When CBM is utilized to assess student progress in mathematics, it can be used to make educational decisions, (Allinder, 1996; Stecker & Fuchs, 2000), modify instruction for class groups (Fuchs, Fuchs, Hamlett, & Philips, 1994), modify instruction for students with special needs (Fuchs, Fuchs, & Hamlett, 1989, Fuchs, et al., 1994), and identify strengths and weaknesses (Fuchs et al., 1989; Fuchs, Fuchs, Hamlett, & Stecker, 1990). When Mathematics CBM (M-CBM) is used to assess computational fluency performance, it is quantified as the number of digits written correctly per a unit of time (Christ & Johnson-Gros, 2005). Digits correct are defined as any digit written by the student that is in the correct place value within the solution of a mathematics problem.

Whenever the final answer is correct, the response is scored with full credit for all correct digits (Christ & Johnson-Gros, 2005).

Hintze, Christ, and Keller (2002) examined the psychometric properties of single skill (e.g., 1x1 digit addition) and multiple-skill (e.g., 1x1 digit addition and 1x1 digit subtraction measured together) computational assessments and found that one 2-minute single-skill administration of M-CBM procedures was sufficient to interpret performance when assessing a single-skill. M-CBM is conducted by having a student answer computational problems for 2 minutes (Hintze et al., 2002; Hosp & Hosp, 2003). The teacher or examiner will then count the number of digits correct completed.

Math is generally considered more skill-specific than reading. The content for M-CBM tests is derived by determining the grade-level skills that are important within the student's curriculum (Stecker et al., 2005). M-CBM assessments consist of different but equivalent math probes (sheets) that include at least 25 problems per probe (Fuchs & Fuchs, 1991). The math problems represent the skills in the curriculum that the students are expected to master by the end of the school year. The problems on each CBM probe are different from each other, because they contain different numerals, but are the same type of problem and consist of the same number of problems, which makes them equivalent and representative of the curriculum. The problems in a CBM probe are generated randomly and are assigned in random order on each page of the probe.

CBM math probes are administered to students with specific directions. Students are told that the digit will be scored as correct as long as it is the correct numeral in the right place (Stecker et al., 2005). The total number of digits correct is a more sensitive index of change in student performance, and is typically used to plot student progress in

CBM. Assessments can be developed using other grade-level skills, such as items pertaining to money, measurements, and geometry for higher-level students (Stecker et al., 2005).

Christ and Vining (2006) proposed that M-CBM procedures could be devised as either a subskill mastery measure (SMM) or a general outcomes measure (GOM). SMMs are used to assess a narrow range of skills that usually develop within a small time period and within a single instructional unit. SMMs assess a single skill, such as single-digit-by-single-digit addition. GOMs are used to assess a broad range of skills that develop over the course of many instructional units that may be taught over an academic year. GOMs assess multiple skills, such as those acquired in a particular grade.

In sum, CBM can be applied to mathematics by assessing a specific skill. The students' progress in reaching a specific level of fluency and accuracy is measured by the number of digits correct within the time allotted for each math probe. M-CBM can be used to assess a specific math skill, or the broad range of math skills that students are expected to develop over an academic year.

Reliability and Validity of M-CBM

CBM is often used as a tool to guide educational decisions. M-CBM is useful in planning and adapting instruction to the needs of students, and although M-CBM has some limitations, it is relevant and useful especially when applied as a subskill mastery measure (SMM) (Christ et al., 2008). SMMs are used to assess the level or trend of achievement for a narrow range of skills (Fuchs & Deno, 1991).

At the elementary level, most studies using M-CBM rely on curriculum sampling measures or a mix of curriculum sampling and robust indicators (Foegen, Jiban, & Deno, 2007). When studies include curriculum-sampling measures, the most common types of progress measuring were Monitoring Basic Skills Progress measures (MBSP; Fuchs, Hamlett, &Fuchs, 1998, 1999) or a sampling of computation skills within a grade span (Christ et al., 2005). To the extent that a curriculum constitutes the construct of interest, M-CBM that is representative of the curriculum sample exhibits content validity. Construct validity is established by the magnitude of evidence for criterion and content validity (Christ et al., 2008). M-CBM assesses performance within a narrow range of mathematical skills, whereas most published achievement tests, and curriculum materials include a wider range of skills. Criterion-related validity is higher among assessments that target computation exclusively and the low construct representation of M-CBM relative to overall mathematics performance should be considered (Christ et al., 2008).

Thurber, Shinn, and Smolkowski (2002) used confirmatory factor analysis procedures to determine what constructs M-CBM actually measures in the context of a range of other mathematics measures. In other validity studies of M-CBM, the emphasis has been on concurrent validity. The relation of M-CBM with norm referenced math tests provides modest support for validity, with few reported correlations exceeding .60 and a median correlation of .43 with the Problem-Solving subtest and .54 with the Math Operations subtest of the Metropolitan Achievement Tests (Marston, 1989). Thurber et al. (2002) sought to examine the relation of M-CBM to the constructs of general mathematics achievement, computation, and application from a theoretical perspective. The potential influence of reading in mathematics assessment was also studied. Evidence of high alternate form reliability for M-CBM was observed with a median correlation of .91 among the three forms used (Thurber et al., 2002). Lower than expected interscorer agreement coefficients of .83 were also observed. Christ et al. (2008) reviewed seven studies of M-CBM and found that interscorer reliability coefficients ranged from .60 to 1.00, with median values from .83 to 1.00 (Evans-Hampton, Skinner, Hennington, Sims, McDaniel, 2002; Fuchs, Fuchs, & Hamlett, 1989; Hintze, Christ, & Keller, 2002; Phillips, Hamlett, Fuchs, & Fuchs, 1993; Stecker & Fuchs, 2000; Thurber et al., 2002). These interrater coefficients are generally within the accepted range, although lower than the researchers' expectations.

M-CBM was found to correlate highly with other measures of basic facts computation (median r = .82) and more modestly with commercial measures of math computation (median r = .61). Performance on M-CBM was also less related to tests conceptualized as measuring math applications (median r = .42). This study indicated that there is a need for additional research to improve the technical properties of M-CBM, because the interscorer agreement was too low, and M-CBM was the only measure used that sampled specific skills at a specific grade-level. All other mathematics measures used in the study sampled a range of across-grade types of problems (Thurber, et al., 2002).

Reliability refers to the consistency of measurements. Fuchs et al. (1989) examined the internal reliability of CBM-M used in a study to examine the effects of alternative goal structures within CBM. The study sought to examine whether more dynamic goals implemented with CBM would produce great content mastery. The internal consistency reliability found within this study was .93 (interscorer agreement) (Fuchs et al., 1989; Philips et al., 1993). For grade level 2, alternate-form reliability was found to be .86, and intercorrelations among the Stanford Tests and correlations with the

Otis-Lennon School Ability Tests were between .62 and .78 (Fuchs et al., 1989; Thurber et al., 2002).

Effective Interventions to Increase Mathematical Computation Skills in Students with MLD

Much research in the 1980's on teaching mathematics to students with MLD concentrated on increasing automaticity in the areas of basic arithmetic facts and operations. Tournaki (2003) studied the differential effects of teaching addition through strategy instruction versus drill and practice to students with and without learning disabilities. Forty-two second grade general education students and 42 students classified with MLD according to the New York State criteria, were taught basic, one-digit addition facts through a direct instruction strategy, drill and practice, or control. All students with MLD were receiving math instruction in their self-contained classroom. Students who were included in the study were those who had an accuracy score between 50% and 70% on the pretest to ensure that their performance was low enough to allow for room for improvement, but high enough to ensure that they had some understanding of the concept of addition.

Instruction through drill and practice entailed the students being given a lesson and asked to work as fast as they could. Two forms of the lesson were alternated until the student reached criterion on one of them, or until the 15 minutes expired. Feedback was not given while the students were working on the lesson. After the lesson was complete, the errors were marked and the students were asked to recompute the problems they missed. Answers were provided if an error occurred at this point.

In the strategy instruction group, each session began with demonstration of the strategy and then students were asked to compute each item of a lesson and to work as fast as they could. When a student made an error, the lesson was stopped and the strategy was reviewed. Tournaki (2003) found that the strategy instruction method was more effective than drill and practice for improving addition skills in students with MLD. The students with MLD improved in both conditions from pretest to posttest. The students in the strategy instruction condition performed better when given a transfer task.

Cognitive research on learning indicates that feedback is important because it helps the student benefit from practice (Bangert-Drowns et al., 1991; Butler & Winne, 1995; Kluger & deNisi, 1996). Thorndike’s (1913) studies of feedback indicate that learners can make better use of feedback when they engage in practice with specific tasks (Roschelle et al., 2010). For feedback to be most effective in improving learning, the feedback must provide cues to the learner on what they need to know based on what they know now (Hattie & Timperley, 2007). When the learner must input an overt response, such as a correct answer, when given multiple opportunities to respond, strong effects of feedback are demonstrated (Clariana & Lee, 2001).

Brosvic, Dihoff, Epstein, and Cook (2006) examined the effect of feedback on the acquisition and retention of numerical facts by elementary students with MLD. The students included in this study showed high rates of inaccurate responding and needed high rates of verbal prompting to maintain responding in their learning environments. These learning difficulties suggested that the Immediate Feedback Assessment Technique (IF AT) that provides corrective information to increase the acquisition of mathematics facts and uses rote memorization as a strategy once concepts and the function of concepts are learned will assist learners in rote memorization of basic math facts (Brosvic et al., 2006).

Three studies were conducted by Brosvic et al. (2006) to examine how different methods of delivering feedback affect the acquisition of fact series by elementary school students classified with MLD. In the first study, 40 third-grade students classified with MLD and 40 typically achieving third-grade students were provided with immediate feedback following instruction in each of the four operations: addition, subtraction, division, and multiplication. If a student gave a correct response, the student was given feedback indicating it was correct. For some students the feedback was provided with a Scantron form, where a star was drawn underneath a correct response, and no star was drawn when an incorrect response was given. In the educator feedback condition, feedback was recorded on a Scantron form and the educator provided verbal feedback. An incorrect response was followed by two additional opportunities to provide a correct response before the examiner provided the correct response.

In the second study, participants with MLD were the same as those described in the first study. Performance on the arithmetic operation was evaluated during five baseline sessions and participants received additional instruction on the concept and function of the arithmetic operation that was being taught. One half of the participants who received feedback in the first study continued to receive feedback in the same format. The remaining half of the students were given immediate feedback provided by an educator or the IF AT that provides individualized performance feedback coupled with the opportunity to answer until correct using simple paper-and-pencil media (Dihoff, Brosvic, & Epstein, 2003). During a second intervention period, those receiving the delayed feedback or Scantron forms were given immediate feedback with the other half receiving feedback from the IF AT. Five maintenance sessions were then conducted.

The results of study 1 and study 2 did not differ as a function of the source of the feedback provided, educator or IF AT. Delayed feedback was no more effective than control procedures. The third study was conducted to compare the effectiveness of auditory feedback provided by an educator and the visual feedback provided by an IF AT form. The Write-Say method was used to combine the visual and auditory methods. In this method, the student is provided feedback from the educator when the student makes a correct response. When the student makes an incorrect response, the participant is told that the response is incorrect, asked to review the problem, and continue to respond until correct. When the correct answer is obtained, the student writes it five times on a tablet and repeats it orally five times. The participants in this study included students enrolled in third grade classes who were classified with MLD.

The consistent outcome of studies 1, 2, and 3 was that providing immediate affirming and corrective feedback facilitated the acquisition and retention of fact series in the four operations (Brosvic et al., 2006). Research suggests that feedback is most effective when it is provided immediately and this proposes that strategies using immediate feedback to increase retention of mathematics operations in students with MLD will have positive effects.

Retention of math facts must also result in fluency. Fluency in computation skills includes being able to compute mathematical facts automatically (Learner, 2003), and it is obtained when it is faster to solve problems through recall than to perform the steps needed to solve the problem. For example, when given the problem 8 + 5, recalling the answer, 13, is faster than counting on one's fingers to produce the same result, especially when given a sheet of similar computations to complete in a specified time. In order to increase the speed of performance and to increase automaticity, Greenwood, Delquadri, and Hall (1984) suggested that repeated practice is necessary, including increased opportunities to respond (OTR). The amount of new information that is retained through rehearsal is linked to the number of trials during practice (Daly, Hintze, & Hamler, 2000; Logan & Klapp, 1991). Burns' (2005) study on incremental rehearsal (IR) and the effect of using IR to teach unknown single-digit multiplication facts to children with MLD addressed the use of OTR to increase fluency. IR uses a gradually increasing ratio of known to unknown items that reaches 90% known items to 10% unknown items at the final stage. IR produces many OTRs and includes known items within the unknown items to allow for a level of challenge so that the learning task will not be too easy, or too difficult.

The study was a multiple baseline design with three subjects in the third grade that received mathematics instruction for one hour each day in a special education resource room. Two of the students received an additional hour of special education due to deficits in reading and one student received the additional hour for instruction for written expression. The remaining instructional time was spent in a general education classroom. Fluent computation is an important goal for mathematics according to the National Council of Teacher of Mathematics (NCTM) Standards for Mathematics and children with MLD often display difficulties with math fluency (Miller & Mercer, 1997, NCTM, 2006). The students' progress was monitored with weekly curriculum based measurement (CBM; Deno, 1985) fluency probes. Participants in the study were taught unknown single-digit multiplication facts in a one-to-one format.

Single digit multiplication facts were identified as known or unknown by presenting each of the 100 single digit facts to the child, one at a time, and identifying if the student was able to give a correct answer within 2 seconds. Unknown facts were those to which a student gave an incorrect answer, no answer, or the correct answer after 2 seconds, and were confirmed by presenting the new fact to the student again and asking for the correct answer. The facts were taught through IR. The fact was shown without including the answer and the child was asked to restate the fact orally and provide the correct answer. The unknown fact was rehearsed within a series of presentations of known and unknown facts. When three errors occurred, instruction stopped and the child was placed in the special education resource activity. IR was an effective intervention for increasing the fluency of single-digit mathematics facts among third grade children in this study. IR is considered an intensive intervention, since it is provided in a one-to-one format, making it difficult to implement with a large number of students.

Flores (2009) studied the use of the concrete-representational abstract (CRA) instructional sequence on the computation performance of students with MLD and those identified at-risk for failure in mathematics. The instructional sequence of CRA involves three phases. The first phase is the concrete phase in which manipulatives are used to demonstrate the meaning of a particular concept and the teacher models their use, followed by guided independent practice. During the second phase, pictures are used to represent numbers and students learn a mnemonic strategy to help with the computation process. In the abstract phase, students continue to memorize and learn the operation or procedures automatically (Flores, 2009). The results of Flores' study indicated that students made academic gains, and there was an immediate change in student performance between baseline and treatment conditions for all students. Butler, Miller, Crehan, Babbitt, and Pierce (2003) compared two instructional methods for teaching equivalent fractions, concepts, and procedures: CRA instructional sequence and representational-abstract sequence, which omitted the use of manipulatives. Both types of instruction increased student performance, and the students in the CRA group displayed higher mean performances on posttest achievement measures.

Burns (2005), Flores (2009), and Babbitt and Pierces (2003) all used instructional methods to increase fluency of mathematics skills in students identified with low achievement in mathematics. The instructional methods in all the studies focused on determining the effects of fluency on mathematical computation skills and were administered in an individual format. Burns (2005) increased the OTR and demonstrated that unknown facts could become known facts with enough practice. Burns' study emphasized the need for increased practice to obtain fluency. Flores (2009) illustrated how teacher involvement followed by guided independent practice can improve fluency.

Instructional studies to increase fluency in computation skills require a great deal of individualized instruction. Students can be given instruction from the teacher, followed by guided practice in some interventions, to help students become more autonomous. Many of the interventions described thus far are cumbersome and require individual teacher attention. Immediate corrective feedback used as a part of the interventions was found to be effective in increasing fluency as well. Since many instructional strategies are time-consuming for teachers to implement, finding a strategy that incorporates student awareness of her own performance and student involvement is important. The next section describes such a strategy.

Self-managed strategies. Several different types of instruction to improve mathematics skills for students with MLD have been described thus far. Mathematics is a large part of the school curriculum, and students who cannot add and subtract are likely to encounter difficulty in school and life, as mastery of these skills is needed in many situations (Miller & Heward, 1992). To instruct students in mathematics skills and prevent skill-deficits from persisting, the instructional strategies described, such as peer tutoring (Philips, Hamlett, Fuchs, & Fuchs, 2002), drill and practice methods (Tournaki, 2003), and feedback (Brosvic, Dihoff, Epstein, & Cook, 2006), have been studied and shown to be effective.

Instructional procedures that increase student engagement and responding are likely to increase learning (Berliner, 1984; Greenwood, Delquadri, & Hall, 1984). The strategies described thus far do increase student engagement; however, simple engagement is insufficient for efficient learning to occur, because inaccurate responding may occur (Grafman & Cates, 2009; Skinner, 1998). Consequences that are contingent on responding are also important (Belfiore, Skinner, & Ferkis, 1995), and therefore, many of the procedures that are used to increase the academic performance of students focus on the completion of antecedent-response-consequence (ARC) chains (Skinner, 1998). The procedures provide students with an academic stimulus, such as a mathematics problem; wait for a response, which is either written or verbal; and then provide a consequence, most often in the form of feedback that is contingent on the response produced (Grafman & Cates, 2009).

Self-managed strategies work in the same manner as other ARC procedures; however, they focus on providing students with the instructions and knowledge on how to provide self-administered ARCs. That is, the student learns through instructions how to provide himself with an academic stimulus, opportunities to respond, opportunities to evaluate his response, and opportunities to make corrections as necessary (Grafman & Cates, 2009). A self-managed strategy involves teaching a student to self-evaluate her responses, while performing an academic task.

Self-evaluation is a key regulatory process that involves setting and using standards to judge the quality of one's performance, and to be effective, evaluations of one's performance must be reasonably accurate (Zimmerman, 1998). Schunk (1996) found that when students self-evaluate their capabilities or progress in learning a particular task, they develop a higher level of competence in that skill area, and their self-efficacy beliefs are strengthened. Self-efficacy refers to the belief in one's capability to organize and perform a set of activities necessary to complete a task at a specific level of competency (Bandura, 1986, 1997). Bandura (1997) also hypothesized that self-efficacy beliefs increase one's motivation and eventually one's success on challenging tasks. Self-managed strategies increase student's motivation by actively involving the student in evaluating his or her performance. If the student becomes more motivated to engage in the task, the opportunity to respond will also increase. Greenwood, Delquadri, and Hall (1984) suggested that the increased OTRs would result in increased practice, and therefore an increase in performance. Self-managed strategies provide active engagement in the task (e.g., Mathematics calculation), which will lead to increases in fluency and automaticity of math skills.

Cover-Copy-Compare. Cover-Copy-Compare (CCC) is a self-managed academic intervention that can be used to improve accuracy, fluency, and maintenance across students, academic skill domains, and settings (Skinner, McLaughlin, & Logan, 1997). In the simplest application of CCC, students are required to look at an academic stimulus (e.g., math problem), cover the stimulus, respond by copying the stimulus, and evaluate the responses by comparing it to the original stimulus (Skinner et al., 1997). If the student uncovers the stimulus and determines that his last response was accurate, the student will move on to the next item and repeat the CCC procedure. If the student determines that he or she provided an incorrect response, the student will perform an error correction procedure before moving on to the next item (Skinner et al., 1997).

CCC has been shown to be effective for increasing performance across curricula, setting, and subjects (Skinner et al., 1997). Struthers, Bartalamay, Bell, and McLaughlin (1994) examined CCC paired with public posting, a form of performance feedback, on students' spelling performance. Students received a star on a classroom poster for every day they spelled 80% of their words correctly. CCC increased spelling scores for 7 of 8 students, with an average of 81% correct, and the addition of public posting, led to an average of 94% correct.

CCC is useful for students with low rates of accurate responding, but not for students with higher skill fluency, even inadequate skill fluency (Codding, Shyiko, Russo, Birch, Fanning, & Jaspen, 2007). Once students' performance comes under the control of a stimulus, natural reinforcement for continued practice may decrease (Skinner et al., 1997). CCC in isolation may improve children's mathematics fluency and promote maintenance of skills. CCC is usually conducted with students who have low rates of accurate responding when performing mathematical calculations, such as 2 correct digits per minute (Codding et al., 2007).

Codding, Chan-Ianetta, Palmer, and Lukito (2009) studied the effects of CCC alone and combined with two forms of goal setting to a control condition on the mathematics fluency of 173 third-grade students. Two goal-setting strategies were used: one in which students set goals for the number of problems correct (GSC) and another in which goals were framed as attempting to reduce the number of incorrect problems (GSE). The calculation skills targeted were: double-digit addition, double-digit addition with regrouping, single-digit subtraction (minuends to 10), single-digit subtraction (minuends 11 to 18), and 2 x 1 digit subtraction with regrouping. The skills were selected according to teacher requests and NCTM standards (Codding et al., 2009; NCTM, 2006). The computation fluency intervention was administered class wide to a third-grade class. Codding et al. (2009) found that those students in the CCC + GSC condition had significantly faster progress and higher final scores than those in the control and CCC + GSE groups. Significant differences were also found between students who received CCC + GSC and the students who received CCC alone. Skinner et al. (1997) postulated that CCC does improve students' computation performance, but that additional reinforcement may be needed to enhance the effects of CCC on fluency. Focusing the students on goal-setting to improve their rate of correct answers may have been more motivating than focusing on reducing error-rate in the Codding et al. (2009) study.

The findings of Codding et al. (2009) indicate that providing explicit fluency aims and feedback that targets errors may be counterproductive. Providing performance feedback on the rate of responding and the accuracy of the response may not only be necessary (Codding, et al., 2007), but may improve the rate of reaching fluency (Bangert-Drowns et al., 1991; Codding et al., 2007). After students reach 100% accuracy on mathematics facts, natural reinforcement for continued practice of the skill may diminish (Codding et al., 2007).

Performance feedback provided in interventions is a procedure that provides information to students regarding their performance on the academic tasks and may serve as motivation for children to exceed their previous performance (Shapiro, 2004). Skinner et al. (1993) examined CCC with concealed responding on three students' completion of division facts. Performance feedback and goal setting were used for one student who was unable to master a set of division problems within the nine sessions of CCC. When performance feedback and goal setting were added, the student was able to reach mastery levels for problems correct per minute on three different problem lists after 12 and 13 sessions. This is evidence that CCC and performance feedback are two interventions that can be combined to increase fluency.

Summary and Rationale for Research

Mathematics achievement is a key educational concern in the United States (Jordan, Glutting, & Ramineni, 2010). Foundations for mathematics learning are established before children even begin school, and there is a good reason to believe that mathematics interventions should be aimed at the early childhood grades, such as Kindergarten and first grade, before children fall seriously behind in school (Gersten, Jordan & Flojo, 2005; Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, & Ramineni, 2007). Number sense is one of the earliest forms of mathematical learning, which begins to develop in infancy (Dehaene, 1997; Dowker, 2005; Malofeeva, Day, & Saco, 2004).

Number sense refers to a precise representation of small numbers and leads to secondary symbolic (numeral) and verbal (number words) representations of quantities and numbers (Feigneson, Dehaene, & Spelke, 2004).

Once number sense is developed, children begin learning to map number words onto small sets and recognize quantities (Gellman & Gallistel, 1978; Pressley & McCormick, 1995). During pre-school and Kindergarten, most children learn to count in one-to-one correspondence and learn the rules of cardinality, in which the last number when counting a set of objects indicates the number of items in that set (Gelman & Gallistel, 1978; Jordan & Levine, 2009; Pressley & McCormick, 1995). These early counting principles allow children to learn to enumerate any object or entity, in either a right to left direction, or left to right (Jordan et al., 2010). Learning difficulties in mathematics have been traced to weaknesses in intermediate number competencies related to counting, number comparisons, and set transformations (Geary, 1990; Mazzocco & Thompson, 2005).

Facility with addition and subtraction combinations is a hallmark of elementary school math (Jordan, Hanich, & Uberti, 2003). Deficient math fact mastery is a feature of math difficulties and disabilities (Gersten et al., 2005). Children use a variety of strategies to solve addition and subtraction problems, such as finger counting, guessing, or approximating answers based on their knowledge of numbers (Jordan, Kaplan, Ramineni, & Locuniak, 2008). As students progress through elementary school, basic mathematical facts, such as the answers to single-digit addition and single-digit subtraction problems become automatized. That is, students can find the answers in their long-term memory representations (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981). Students who do not develop basic fact retrieval skills, and begin to show difficulties with mathematical computations may have a math learning disability (MLD).

A review of the research reveals that many students with MLD have a deficit in working memory that affects their performance on basic math computations. Specifically, children with MLD have difficulties retrieving basic arithmetic facts from their long-term memory, which will in turn affect their performance on subsequent higher-level tasks (Geary, 2004). Students with MLD also need repeated practice and multiple trials to increase their opportunities to respond, which leads to increased fluency and accuracy (Daly, Hintze, & Hamler, 2000; Logan & Klapp, 1991). Strategy instruction has been found to increase mathematics competencies in students with MLD, but little research has focused specifically on computation skills.

The studies that have focused on improving mathematical computation skills used strategies such as drill and practice, performance feedback, and rehearsal strategies. These types of interventions are time-consuming and cumbersome for teachers to implement. Self-managed strategies help students to learn how to improve their skills and monitor their own performance. Combining CBM with a self-managed strategy allows teachers to track students' progress throughout the intervention, and adjust the interventions as necessary. CCC is a self-managed strategy that can be used across several academic skills to improve accuracy, fluency, and maintenance across students, and settings (Skinner et al., 1997).

CCC provides an opportunity for increased responding and increases in accuracy and fluency. CCC can be used to improve mathematics skills in both the early and late elementary grades. CCC is used to target specific skills within the curriculum and can be applied to mathematical computations. If computation skills are not mastered in the early elementary grades, children will continue to fall further behind in math (Gersten et al., 2005; Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, & Ramineni, 2007). CCC can also be paired with other interventions, such as performance feedback to increase fluency (Skinner, 1993; Codding et al., 2007; Codding et al., 2009) and maintain skills over time. Providing performance feedback can help students to self-monitor their work, a key cognitive process of becoming a better learner (Ramdass & Zimmerman, 2008). Using CCC to improve mathematical skills of students has been shown to be effective with higher-grade students (Codding et al., 2007; Codding et al., 2009). This study is aimed at first grade students to improve their levels of fluency in mathematics computation skills in order to prevent failure in mathematics in higher-level grades, as the skills expected become more challenging.

Educational Implications

Mathematical computation fluency is an early childhood mathematics skill necessary for successive math performance in the higher grades. The literature that addresses improving mathematical calculation skills has focused on time-consuming drill and practice methods to improve skill levels. The results of this study will expand the research literature and extend our knowledge of how increasing opportunities to respond can increase math skill level and fluency in the early childhood grades. In addition to the investigation in changes in fluency and skill level, the current study may shed light on how school psychologists can assist teachers in planning effective interventions for students at-risk for mathematics failure and those students struggling to grasp basic math readiness and calculation skills.

Research Questions

The present study intended to answer the following questions:

Question 1. Does Cover-Copy-Compare increase first grade students' fluency in addition and subtraction calculation skills?

Question 2. Does Cover-Copy-Compare paired with Performance Feedback have a higher rate of increase of first grade students' fluency over their baseline performance in addition and subtraction calculation skills when compared to Cover-Copy-Compare in isolation?

Question 3. Does Cover-Copy-Compare paired with a Reward have a higher rate of increase of first grade students' fluency over their baseline performance in mathematics calculation skills when compared with Cover-Copy-Compare used alone or Cover-Copy-Compare paired with Performance Feedback?

Question 4. Does Cover-Copy-Compare increase first grade students' fluency over their baseline performance in addition and subtraction skills at a higher rate than a control condition where students receive no intervention?

Hypotheses

The researcher designed this dissertation to determine if Cover, Copy, and Compare (CCC), a self-managed academic intervention for increasing skills across several academic domains, would improve the mathematical computation skills over baseline performance in single-digit addition and subtraction for students identified with Math Difficulty in the first grade. Second, the researcher wanted to determine whether pairing CCC with performance feedback (PF) would increase the number of digits correct per minute (DCPM) that students are able to compute, and decrease the number of errors per minute (EPM) compared to baseline performances. Third, the researcher sought to know whether CCC paired with a reward (RW) would have a greater effect than CCC administered alone or CCC paired with PF on the number of DCPM the student can produce compared to baseline performances. Digits correct per minute (DCPM), errors per minute (EPM), and curriculum based measurement (CBM) were used to assess mathematical computation skills performance. Based on the literature review, this dissertation investigated the following research hypotheses:

HO1: Participants identified as at risk for MLD will display an increase in DCPM and decrease in EPM compared with their baseline performances when the intervention CCC is implemented.

HO2: Participants in the CCC + PF condition will demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions relative to their own baseline performances and to the performance of participants who received CCC in isolation.

HO3: Participants in the CCC + RW condition will demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions when relative to their own baseline performances and to the performances of participants who received CCC in isolation.

HO4: Participants in the CCC in isolation intervention condition, the CCC + PF intervention condition, and the CCC + RW intervention condition will all demonstrate significant increases in DCPM and significant decreases in EPM when compared with their own baseline performance and the baseline performance of the participants in the control condition.

CHAPTER III

Method

This chapter provides a description of the participants, recruitment procedures, setting, materials/measures, interventions, and procedures of the current study.

Participants

Eight first grade students identified with difficulty in mathematics (i.e., at risk for MD) were the participants in this study. The researcher selected this grade level because previous research in mathematics using CCC as an intervention has focused on higher-grade level students (Codding et al., 2009; Codding et al., 2007; Grafman & Cates, 2010), and the researcher wanted to determine the usefulness of CCC with younger students. Furthermore, it is important for children in early childhood grades to master basic calculation skills before difficulties in math arise in successive grade levels (Gersten et al., 2005; Jordan et al., 2010; Jordan, Kaplan, & Ramineni, 2007).

A single-subject design was used in this study. A single-subject experimental design is a type of experimental study conducted with a limited number of participants (Rassafiani & Sahaf, 2010). A key feature of a single-subject design is the experimental manipulation of a treatment. The experimenter seeks replication of the treatment's impact on a dependent variable. Performance is then compared between or within subjects across design phases (Hitchcock & Natasi, 2010). A single-case design was employed in this study because previous research using CCC as an intervention (Codding, 2009; Codding et al., 2007; Grafman & Cates, 2010) used older students (grades 3 and higher). It was hypothesized that students as young as 6 years of age would need more support in learning and self-managing this type of academic intervention. CCC has not been applied frequently to younger students in the area of mathematics in the literature; so a single-case design allowed for individualized attention during the intervention sessions.

In single-subject research it is usual to include between 3 and 8 participants in a single study (Horner et al., 2005; Rassafiani & Sahaf, 2010). Forty-three students were administered a pre-test, and 30 students qualified for the study. The researcher selected the participants based on the following criteria: (a) student performed below 70% on a teacher-administered pre-test (defined as at-risk for failure in this subject area); (b) student was not receiving special education services, was not an ELL student (English Language Learner), and was enrolled in a general education program; (c) the general education class in which the student was enrolled follows the New York City Department of Education mathematics curriculums, “Everyday Math” or “Envisions Math”. Eight participants were used in this study to ensure more than one student per treatment condition should a participant have withdrawn from the study.

The participants began instruction in single-digit addition and single-digit subtraction as part of their math curriculum in September. The study began at the end of October after almost two months of instruction in the subject area. During the course of the study, instruction in single-digit addition and subtraction continued in the participants’ classrooms.

Operational definition of at risk. This study’s participants were considered at-risk because they scored 70% or lower on a CBM in math. Their general education classroom teachers also defined the students as having difficulty with basic mathematical calculation skills in addition and subtraction, which prevented them from succeeding in higher-level math skills. These students were not mandated for special education services. The New York City Department of Education requires teachers to collect student data regarding academic achievement. These data were not provided to the principal investigator nor used as part of this study.

Recruitment Procedures

Two principals of schools near the researcher's workplace were contacted via a recruitment letter (See Appendix B), which described the procedures, time commitment, students required, and consent and assent procedures. One principal declined to allow the researcher to conduct her study in his school. The other principal from an elementary (Grades Pre-K-5) school located in a low-socioeconomic community within a major city in the Eastern United States consented to allow the researcher to conduct the study within his school. The principal consented for the research to occur during an after-school Y program to avoid students missing academic instruction time. Recruitment fliers were placed in the mailboxes of the first grade General Education teachers within the school (See Appendix D). The fliers contained a description of the subjects who should be recruited and gave an operational definition of at-risk. The recruitment fliers stated that if the teacher consented to participate, she should contact the principal investigator.

Two first grade teachers contacted the principal investigator and agreed to participate in the study. The flier explained the procedures of a teacher-administered pretest. Once a teacher consented to participate in the study, she assisted in administering a pretest that consisted of 36 math problems of addition and subtraction of single-digit numbers on several sheets. The students were timed and asked to complete as many problems as possible, and were given two minutes to do so. The pre-test was administered as part of a differentiated math lesson. Forty-three students, from two different first grade classes took the pre-test. The principal investigator scored the pre-tests, and students who scored below the criterion level of 70% correct or less of the attempted problems on the pre-test measure were recruited for the study by consent forms sent home to their parents via the students' teachers (see Appendix E). Thirty students scored below the criterion level and were recruited to participate. The first eight students to return the signed consent form were asked for their assent to participate in the study.

The parents were given a self-addressed stamped envelope in which to return the consent form to the researcher. Once parental consent was obtained, student assent was also obtained from each student individually with the student assent form (see Appendix F). The principal investigator obtained student assent at the beginning of the first intervention session. The student met with the principal investigator individually in the school's guidance office and the assent form was read to the participant. The participant printed his or her name after the script on the assent form was read to them and they willingly agreed to participate. A copy of the signed consent form was provided to the parents for their records.

School and Participant Demographic Information and Descriptions

The eight participants selected for the study attended a Public Elementary School in a large city in the Eastern United States, which services grades Pre-K through grade five. The total school register was 563 students, with a gender ratio of 52.40% female (295 students) to 47.60% male (268 students). The ethnic breakdown of the student population was 34.81% Hispanic (196 students), 0.53% Indian or Alaskan Native (3 students), 7.64% Asian (43 students), 45.83% Black (258 students), 11.01% White (62 students) and 0.18% Multi-Racial (1 Student). Twenty-Five, or 4.44% of the total student population were classified as English Language Learners (ELL). Out of the 563 students, 80.99% (456 students) are instructed within the General Education environment, 9.59% (54 students) are instructed in the least restrictive environment within Special Education and 9.41% of the students (53 students) are instructed within the most restrictive environment within Special Education.

The eight participating students ranged in age from 6 years to 7 years old. Five of the students in the study were female, and three students were male. Five of the students selected for the study were black, two students were Hispanic, and one student was Asian.

Student 1. Student 1 was a 6-year-old Asian female who turned 7 during the course of this study. Student 1 presented with a high level of motivation and excitement to participate within the study. She appeared to enjoy the one-to-one attention she received from the principal investigator. Student 1 received CCC and CCC + PF. During the CCC + PF, she showed positive affect when receiving gold stars placed next to correct responses. Student 1 did not display any behavioral concerns during the study. She did not receive any academic support services and was enrolled within a General Education first grade class.

Student 2. Student 2 was a 7-year-old Black male. Student 2 did not receive any academic intervention services and was enrolled in a General Education first grade class. He was slow-to-warm-up to the principal investigator and after he did establish rapport with the investigator, his affect would change frequently from session to session, and within an intervention session. Student 2 appeared happy before the intervention sessions and while walking to the guidance room. When presented with the addition condition, Student 2 attempted and completed all the math probes and packets requested of him. During the CCC + PF condition with addition, receiving the feedback of gold stars placed next to his correct responses did not appear to motivate him, nor did it upset him. During CCC + PF with subtraction, Student 2's affect became distraught and upset and he shut down. After one or two pages with incorrect responses, he refused to complete the remainder of the CCC + PF subtraction packet. This behavior persisted for two intervention sessions. Student 2 was asked if he wanted to continue to participate in the study and he stated, "Yes." He became upset at the subsequent intervention session and this date was skipped. The next session, Student 2 resumed his participation and his affect was more positive. Student 2's affect was directly related to performing incorrect responses and having them highlighted during the performance feedback subtraction condition.

Student 3. Student 3 was a 7-year-old female youngster, enrolled in a General Education first grade class. Student 3 established rapport with the investigator very quickly and was extremely talkative during the intervention sessions. She appeared to engage in attention-seeking behaviors and thrived on receiving one-to-one attention. Her motivation level towards the tasks presented to her in the study was high. She attempted and completed all probes and packets asked of her, although at times she appeared to rush through to complete her work as quickly as possible. Student 3 received CCC and CCC + RW. Adding a reward did not appear to increase her motivation level for correct responses, because she was already motivated to complete the tasks. Student 3 became excited and happy when she produced a correct response and disappointed when she had an incorrect one. The process of self-checking her work appeared motivating in itself to her. She also appeared to enjoy receiving a reward, however.

Student 4. Student 4 was a 7-year-old Hispanic male youngster, enrolled in a first grade General Education class. He established rapport easily with the examiner and enjoyed the one-to-one attention he received during the intervention sessions. He often wanted to engage in conversation with the investigator. Student 4 presented with a moderate motivation level. He attempted and completed all the probes and packets that were part of the study, but he worked at a slow pace. Student 4 received the treatments CCC and CCC + RW. He enjoyed receiving a reward, but it did not appear to increase his motivation, or his pace. He appeared happy when he completed problems correctly, but was not overly dismayed by incorrect responses. Student 4 had several absences during the course of the study, more so than any other participant. He was absent for two weeks due to a stomach illness. The intervention sessions resumed when he returned to the after-school program.

Student 5. Student 5 was a Black 7-year-old male youngster, enrolled in a General Education first grade classroom. Student 5 presented with a happy affect and established rapport easily with the investigator. He was quiet and did not engage in spontaneous conversation and was remained quiet even when conversation was initiated with him. He smiled frequently. Student 5 attempted and completed all the packets and probes that were presented to him as part of the study. Student 5 received CCC and his motivation level remained constant throughout all of the intervention sessions. No behavioral issues were apparent during the study.

Student 6. Student 6 was a 6-year-old Black female youngster, who turned 7 during the course of this study. Student 6 enjoyed her participation within the study and was motivated to attend all intervention sessions. She attempted and completed all the packets and probes that were asked of her. Student 6 showed a moderate motivation level and worked at a moderate pace. She appeared happy when she had a correct response, and showed some disappointment when her responses were incorrect. No significant behavioral difficulties were apparent. Student 6 received CCC and her affect and motivation appeared constant throughout the study.

Student 7. Student 7 was a 7-year-old Hispanic youngster, enrolled in a General Education first grade class. Student 7 presented as highly distractible during the study. She was in the control condition, and therefore only worked with the investigator for a limited number of sessions. When she saw the investigator pick up other participants, she would run over and ask when she would be taken out again to participate in the study. Student 7 struggled throughout the study with the problems presented to her. Due to behavioral difficulties, as well as academic delays she was being considered for a referral by her classroom teacher for an evaluation to receive special education services.

Student 8. Student 8 was a 7 year old, Black female youngster enrolled in a General Education first grade class. Student 8 presented with high levels of motivation and appeared eager to please. She enjoyed the one-to-one attention she received from the principal investigator. Student 8 had a calm demeanor and smiled frequently. She attempted all the probes presented to her and worked carefully. Student 8 was in the control condition. She asked the investigator in between sessions when she would be working with the investigator again, indicating that she enjoyed her participation in the study.

Intervention Setting

Each intervention session was conducted in the psychologist's office in the guidance suite of the public school that was selected for the study. The sessions occurred during the after-school program when the students were not engaged in academic instruction time. The principal investigator conducted the sessions individually, and each intervention session lasted approximately 15 minutes.

Materials and Measures

In order to assess students' mathematics skills, CBA in Mathematics (CBA-Math) packets were created from http://interventioncentral.org. Two single skills probes were created: one for addition skills (i.e., 1 x 1 digit without regrouping) and one for subtraction skills (i.e., 1 x 1 digit subtraction). Each packet worksheet (probe) contained 36 problems. The performance on these probes was used as a baseline measure. The CBA-Math was administered as the baseline before the intervention sessions began. For each baseline measurement a new probe was generated.

Curriculum-Based Measurement (CBM) probes. Changes in skill level were measured using Curriculum Based Measurement (CBM). CBM was conducted in math by having each student answer computational problems for 2 minutes. The examiner counts the number of digits correct (DC) (Hosp & Hosp, 2003). The problems contained in the CBM were grade equivalent and contained 36 problems within each probe (The CBM probes are different, but equivalent, math sheets that cover math skills taught during that school year) (Hosp & Hosp, 2003).

Independent variables. The intervention conditions, CCC (Cover, Copy, Compare), CCC + PF (Performance Feedback), CCC + RW (Reward), or control, were the independent variables in this study.

Dependent measures. The number of digits computed correctly per minute (DCPM) and the number of errors per minute (EPM) served as the dependent measures for each session. The total number of correct computations during the 15-minute session was also a dependent variable in this study. The responses scored as correct were (a) individual digits (even if the number was reversed or rotated) and (b) digits written below the line. Digits that are incorrect are scored as incorrect. The amount of time it took the student to complete the probe within each intervention condition was recorded. The student may have completed the probe in less than the 15 minutes allotted for each session. The total time to complete the probe was divided by the total number of correct digits or errors during the session to determine the DCPM and EPM.

Experimental Design

A single-subject, reversal design (ABAB) was employed. This design involved a baseline assessment (A), followed by a period of intervention (B), ending with another baseline (A). The second intervention phase (B) followed, and another baseline (A) completed the intervention sequence within each operation. This design aims to build a causal relationship, and its ABAB sequence plays a role in forming a conclusion on the effectiveness of the sequence (Rassafiani & Sahaf, 2010).

Variability, or stability, of data refers to fluctuations in the student's performance. Baseline data should be stable, representing the natural occurrence of behavior, and should not vary more than 50% from the mean of the baseline in order for the treatment phase to be initiated (Barlow & Hersen, 1984). For the current study, data were collected during the baseline phase until stability was reached or at least five data points were collected (Alberto & Troutman, 2003), after which the intervention phase was initiated.

Procedures

The students' performances on the CBA-Math served as a pretest measure, providing a baseline level of their skills in mathematical computations. The students were given two minutes for each CBA or CBM probe administered during the baseline phases, both during the original baseline phase, and during the subsequent baseline phases throughout the study. The CCC probes were administered during the intervention sessions and the participants were given 15 minutes to complete each CCC, CCC +RW, or CCC + PF probe.

The students met individually with the experimenter, two times a week for 15-minute sessions for 10 weeks. The intervention procedure was explained to the students during the first session, and each session thereafter if necessary. At the start of the session, the examiner provided the following instruction to the students: "Today we are working on a way to improve your math skills. I am going to show you a way that you can help yourself to improve in math. This is called Cover, Copy, and Compare. I would like you to practice addition by following these five steps." The procedures consisted of the following five steps:

1. Look at the problem with the answer.
2. Cover the problem and the solution with an index card.
3. Write the answer on the right side of the page.
4. Uncover the problem and solution.

5. Evaluate the response.

Each CCC probe contained 36 addition problems (1 + 1 digit without regrouping) or 36 subtraction problems (1 – 1 digit without regrouping). When accuracy in addition or subtraction was achieved for each student, the second intervention condition was implemented. Accuracy is defined as 30 correct digits per 15-minute intervention session. Meta-analytic research has found strong effects from studies using an accuracy criterion of 70%-85% known items for mathematics (Burns, 2004; Burns, VanDerHeyden, & Jiban, 2006). Obtaining 30 correct digits within a 15-minute intervention session is equivalent to an accuracy rate of 83%. Using this criterion to describe accuracy indicates that the student “knows” 83% of the digits they are presented with.

Fluency is defined as the number of digits correct per minute. Fluency is accuracy plus speed (Burns et al., 2006). Fluency is used to determine the optimal challenge in mathematics instruction and level of instruction, because fluency incorporates time, rather than just the number of correct problems (Burns et al., 2006; Deno & Mirkin, 1977) Fluent computation is also a goal for mathematics instructions (National Council of Teachers of Mathematics, 2000). Improving students’ fluency shows that students have mastered known items and have also improved their rate of responding.

The CCC probes that were generated were of alternating forms, with similar problems presented in a random order. The students were presented with these equivalent probes until accuracy, as defined above, was achieved.

Each participant in the experimental condition (Students 1 – 6) received either CCC and CCC, or CCC and either CCC + PF or CCC + RW. Each participant received the treatment interventions for both mathematical operations. The interventions were counterbalanced as follows. The treatment conditions (CCC; CCC and CCC + PF; CCC and CCC + RW) were written on separate slips of paper and the examiner drew the slips to determine which condition each student would be randomly assigned to. Following this, the words, “addition” and “subtraction” were written on slips of paper and placed inside a hat. The examiner drew the slips to determine the mathematical operation that each student would be assigned to initially. The intervention procedure was then repeated with each student with the alternate mathematical operation.

Each participant received an initial and a second intervention. The initial intervention was done using either addition or subtraction problems. The second intervention was conducted with the mathematical operation that the student did not receive initially. In other words, if a student did addition problems during the initial intervention procedure, he or she did subtraction problems during the second intervention procedure.

The initial intervention procedure for student 1 and student 2 was: (a) Baseline, (b) CCC, (c) Baseline, (d) CCC + PF. The initial intervention procedure for student 3 and student 4 was: (a) Baseline, (b) CCC + RW, (c) Baseline, (d) CCC. The initial intervention procedure for student 5 and student 6 was: (a) Baseline, (b) CCC, (c) Baseline, (d) CCC. The initial intervention procedure for student 7 and student 8 was: (a) Baseline, (b) No Intervention, (c) Baseline, (d) No Intervention. Table 1 gives the experimental design. The order of the interventions was reversed for each student for the second intervention sequence. For example, Table 1 shows that the intervention sequence for student 1 for subtraction probes was Baseline, CCC, Baseline, CCC + PF.

When student 1 received addition probes during the second intervention, the intervention sequence was Baseline, CCC + PF, Baseline, CCC.

Table 1

Intervention Sequence for Each Participant

Participants	Baseline	Treatment	Baseline	Treatment
1 & 2	Addition	CCC	Baseline	CCC + PF
3 & 4	Addition	CCC + RW	Baseline	CCC
5 & 6	Subtraction	CCC	Baseline	CCC
7 & 8	Subtraction	No Txt	Baseline	No Txt
1 & 2	Subtraction	CCC + PF	Baseline	CCC
3 & 4	Subtraction	CCC	Baseline	CCC + RW
5 & 6	Addition	CCC	Baseline	CCC
7 & 8	Addition	No Txt	Baseline	No Txt

Note. The mathematical operations listed above were those that were implemented first for each subject. The operation was determined by random drawing. The treatment sequence for the second mathematical operation was the reverse of the sequence used for the first mathematical operation.

The following presents a description of each of the treatments.

Cover, Copy, and Compare (CCC). This intervention consisted of the administration of the CCC probes following an explanation of the five-step CCC procedure to the student (see above). The student then completed the CCC probe worksheet and the principal investigator recorded the number of digits correct per minute.

Cover, Copy, and Compare and Performance Feedback (CCC + PF). This intervention consisted of the administration of the CCC probes following an explanation of the five-step CCC procedures to the student. The student completed each CCC probe and the principal investigator provided performance feedback after completion of the probe. The feedback consisted of gold stars that the investigator placed as visual feedback next to each problem that the student completed correctly, while the student self-evaluated his or her responses.

Cover, Copy, and Compare and Reward (CCC + RW). This intervention consisted of the administration of the CCC probes following an explanation of the five-step CCC procedures to the student. After the student completed each CCC probe, the principal investigator gave a reward to the student.

The reward was age appropriate, such as a sticker, pencil, or eraser, which each student chose before the intervention sessions began. What serves as a reinforcer may be different for each student (Alberto & Troutman, 2003). The principal investigator conducted multiple stimuli presentations, in which several age appropriate items were presented to the two students in this treatment condition. The students were asked to choose the item they would like to receive (Windsor, Piche, & Locke, 1994). Once a student selected an item, it was removed and the selection process continued until all of the items were selected. This allowed the researcher to determine which reward would be most effective for each student.

Control Condition. In this condition there was no intervention. The students in this condition were assessed with the same baseline procedure at the beginning of the study until a stable baseline was obtained. The second baseline was administered to the students when the students in the treatment conditions reached the second baseline point.

Mastering of all items. When a student mastered all items on the CCC probe, or on the CBM measurement, the EPM was scored as 0 and the DCPM was scored as 36 to indicate that the student had completed all problems within the probe correctly. The student was considered to have reached fluency. The student then moved on to the treatment sequence for the second mathematical operation condition.

Data Analysis

The investigator presented the results of this study graphically, and analyzed the data visually. Researchers using a single-case design often use visual analysis to determine whether their data suggest a relation between the independent variable and the outcome variable, and the strength of that relation (Hersen & Barlow, 1976; Kazdin, 1982; Kennedy, 2005; Kratochwill, 1978; White & Haring, 1980). The number of DCPM or EPM for each student participant for each intervention session was recorded. The average gains in DCPM for each intervention condition was computed for each student. The difference between the DCPM for each student in each phase of the intervention and in both operations was compared with his or her baseline performance to determine the average gain in DCPM. The student's decrease in EPM was also computed in the same way.

The change from the baseline level during each intervention session was displayed on a graph and the trend of performance, variability, level of performance, immediacy of the effect, and consistency of data patterns across similar phases was analyzed from these graphs.

CHAPTER IV

Results

This chapter describes the results obtained from the study. Descriptive statistics, trend lines, and comparisons between DCPM and EPM within each condition were used to answer the research questions and test the hypotheses being posed by this study. Visual analysis of the graphs created was used in the analysis of data.

This chapter begins with the study hypotheses, followed by the results of testing of these hypotheses. The dependent variables in this study were digits correct per minute (DCPM) and errors per minute (EPM) for each condition (Cover, Copy, and Compare (CCC); Cover, Copy, Compare, and Performance Feedback (CCC+PF); Cover, Copy, Compare, and Rewards (CCC+RW); and Control). Trend lines were then created to display the differences in DCPM and EPM for each subject in comparison to his or her baseline measurements. The chapter ends with the explanation of the results.

Hypothesis Testing

Four hypotheses were tested in this study. Hypothesis 1 stated that participants identified with math difficulty would display an increase their DCPM and decrease EPM, compared to their own baseline performances when the intervention CCC is implemented. Hypothesis 2 stated that during the CCC + PF condition participants would demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions when compared with their own baseline performances and the performance of participants during the isolated CCC intervention condition. Hypothesis 3 stated during the CCC + RW condition participants would demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions when compared with their own baseline performances and the performance of participants during the isolated CCC intervention condition. Hypothesis 4 stated that participants in the CCC intervention condition, the CCC + PF intervention condition, and the CCC + RW condition would all demonstrate significant increases in DCPM when compared with their own baseline performance and the baseline performance of the participants in the control condition.

Results of Testing the Hypotheses

Figures 1-12 present graphic data for each participant's performance. Figures 13-16 present control participants' data. Readers can refer to the graphs during the following presentation of the results. Readers are reminded that the baseline and the data points following the intervention phases are CBM results.

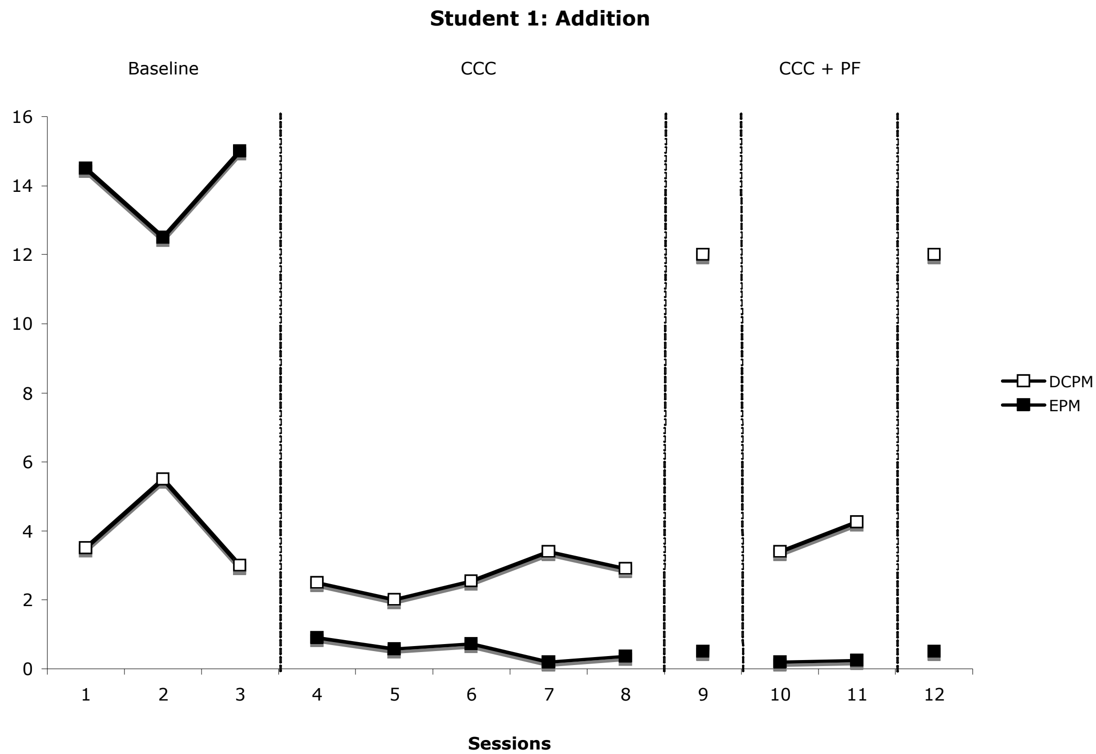

Figure 1

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 1 in addition, across baseline and intervention phases.

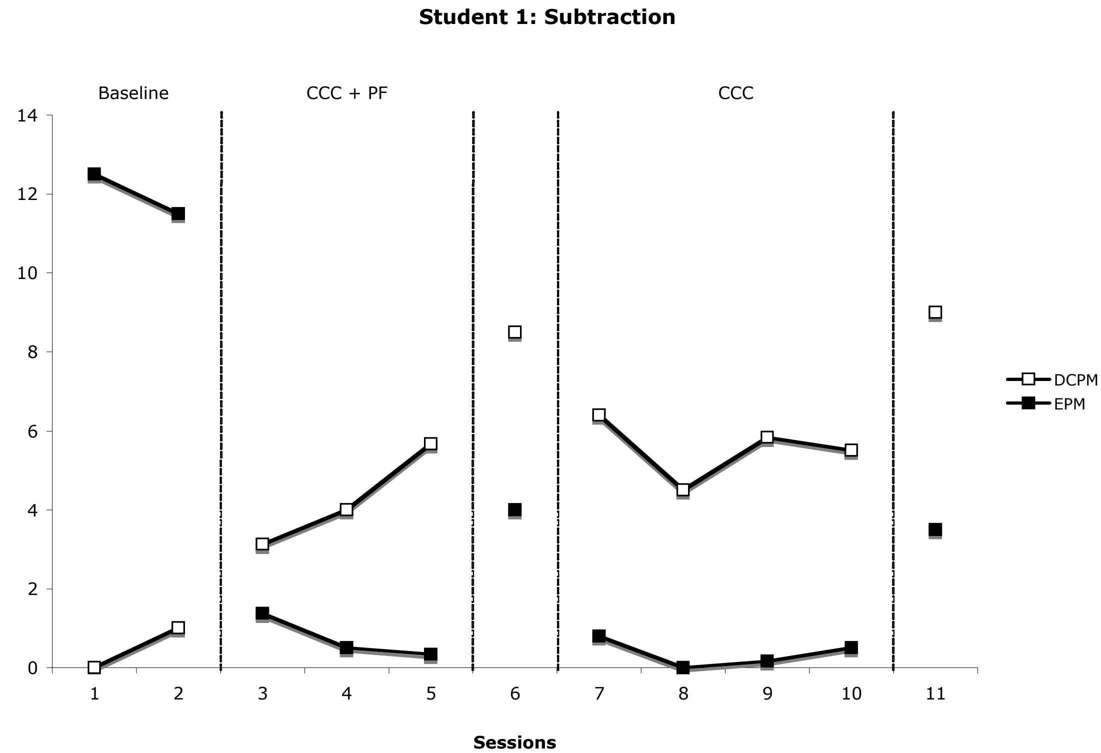

Figure 2

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 1 in subtraction, across baseline and intervention phases.

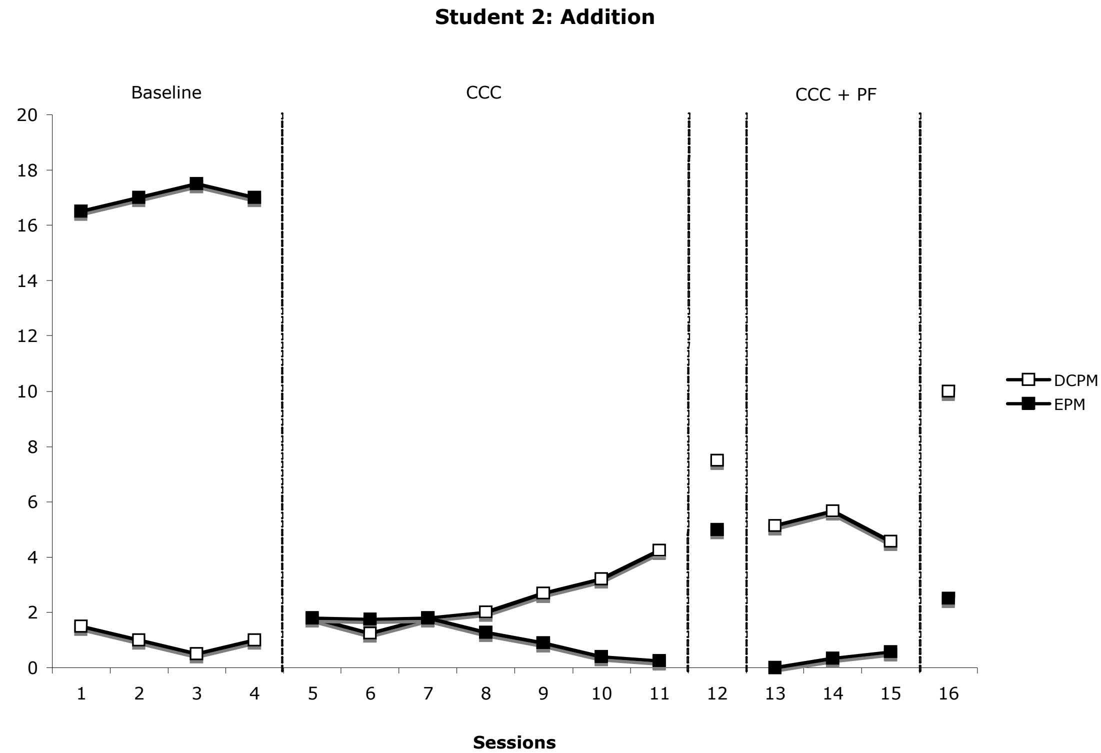

Figure 3

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 2 in addition, across baseline and intervention phases.

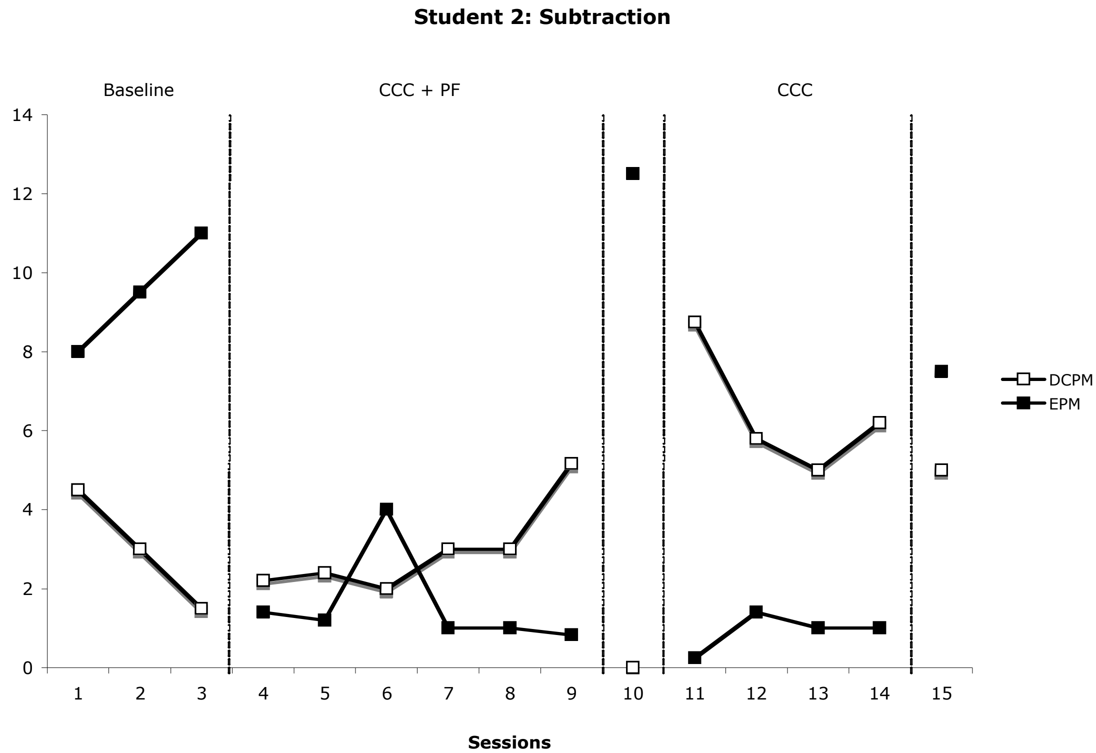

Figure 4

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 2 in subtraction, across baseline and intervention phases.

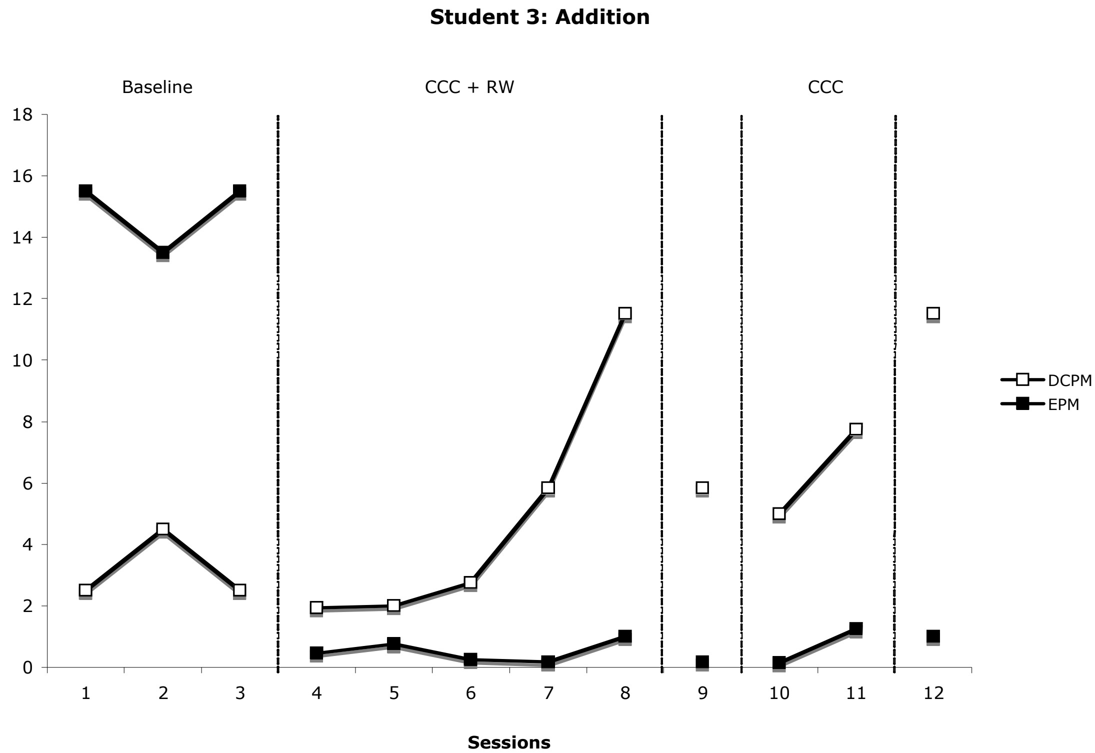

Figure 5

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 3 in addition, across baseline and intervention phases.

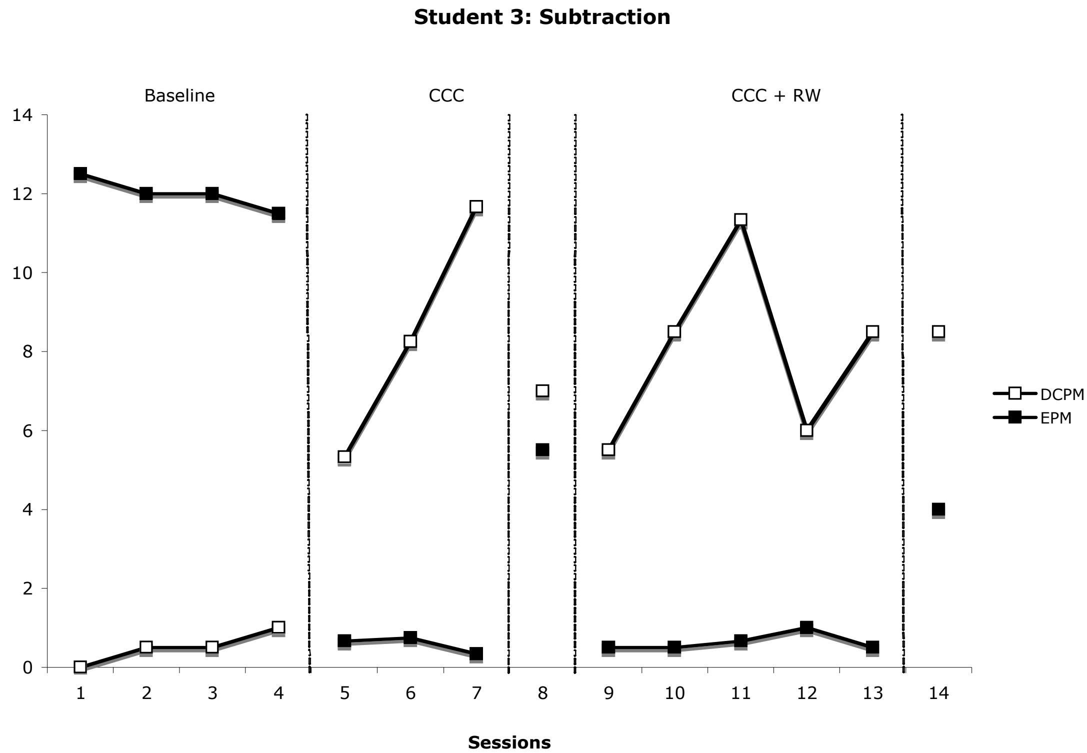

Figure 6

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 3 in subtraction, across baseline and intervention phases.

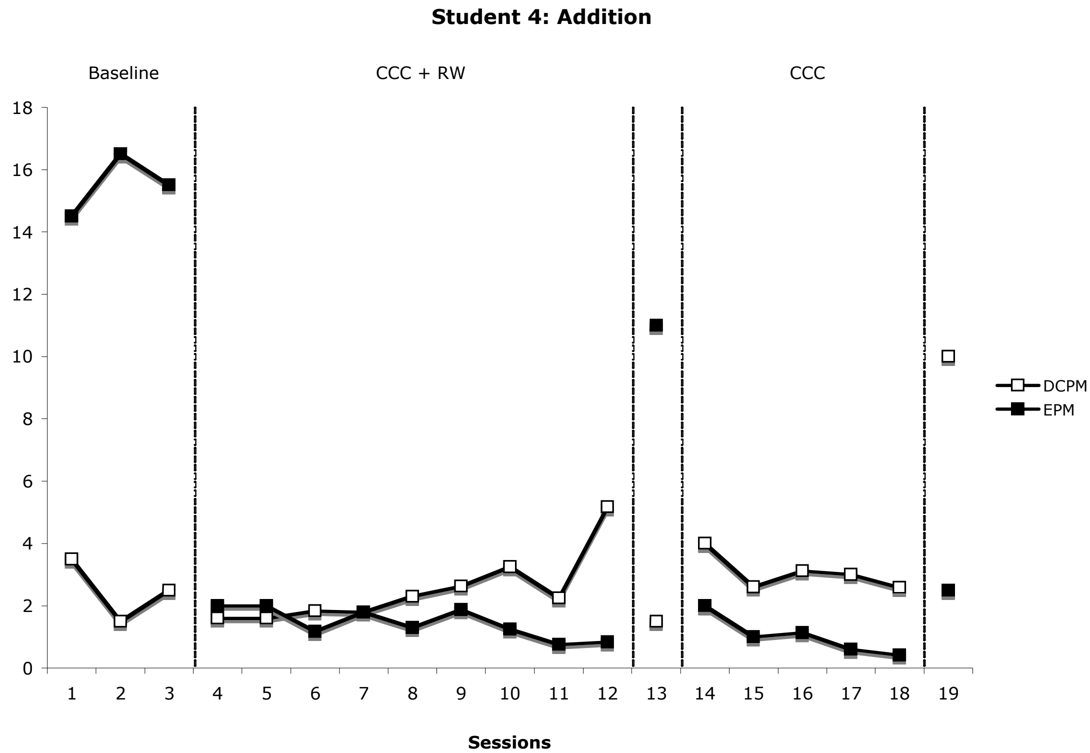

Figure 7

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 4 in addition, across baseline and intervention phases.

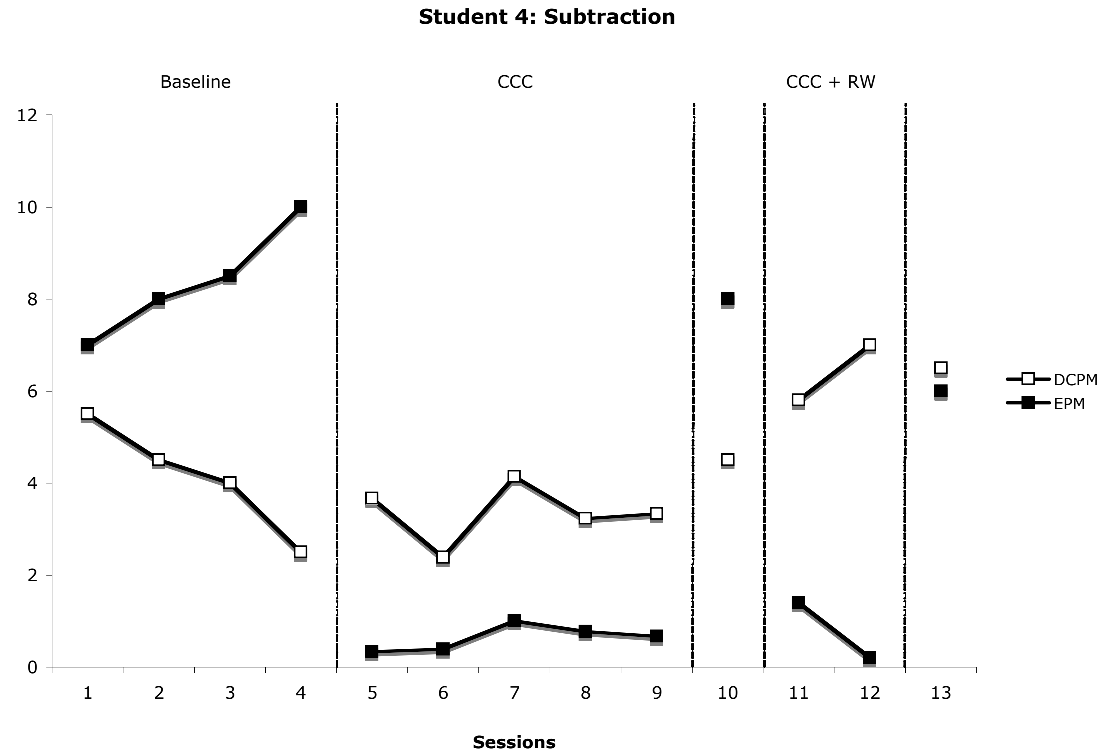

Figure 8

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 4 in subtraction, across baseline and intervention phases.

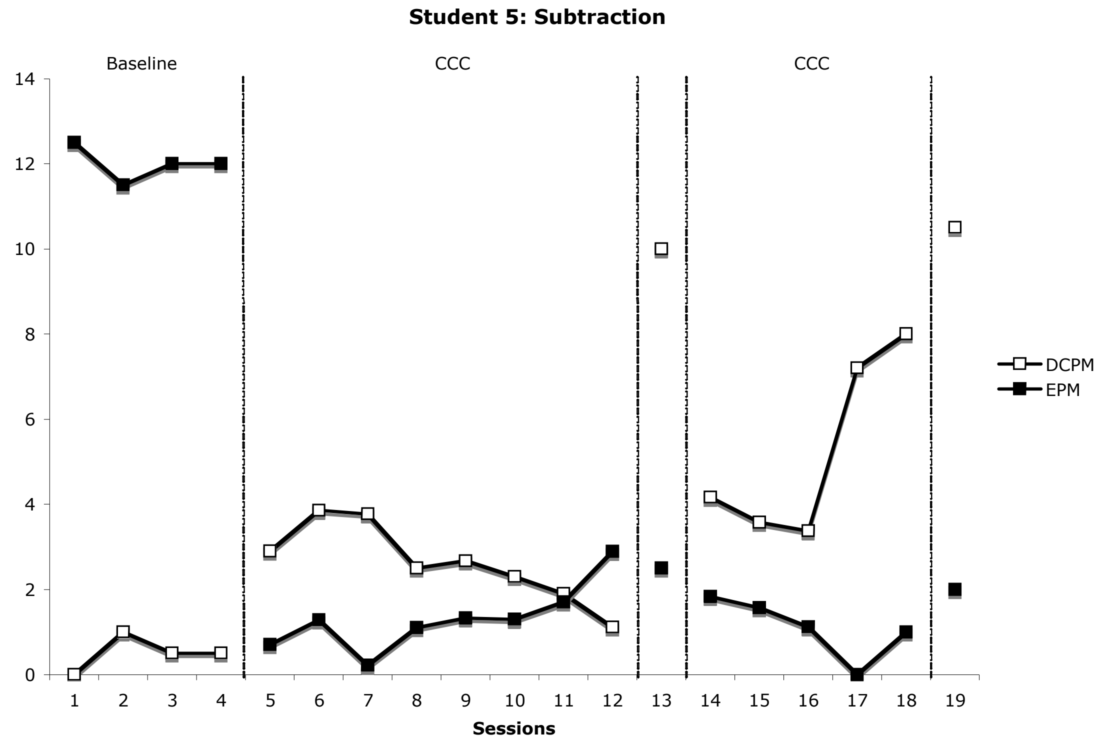

Figure 9

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 5 in subtraction, across baseline and intervention phases.

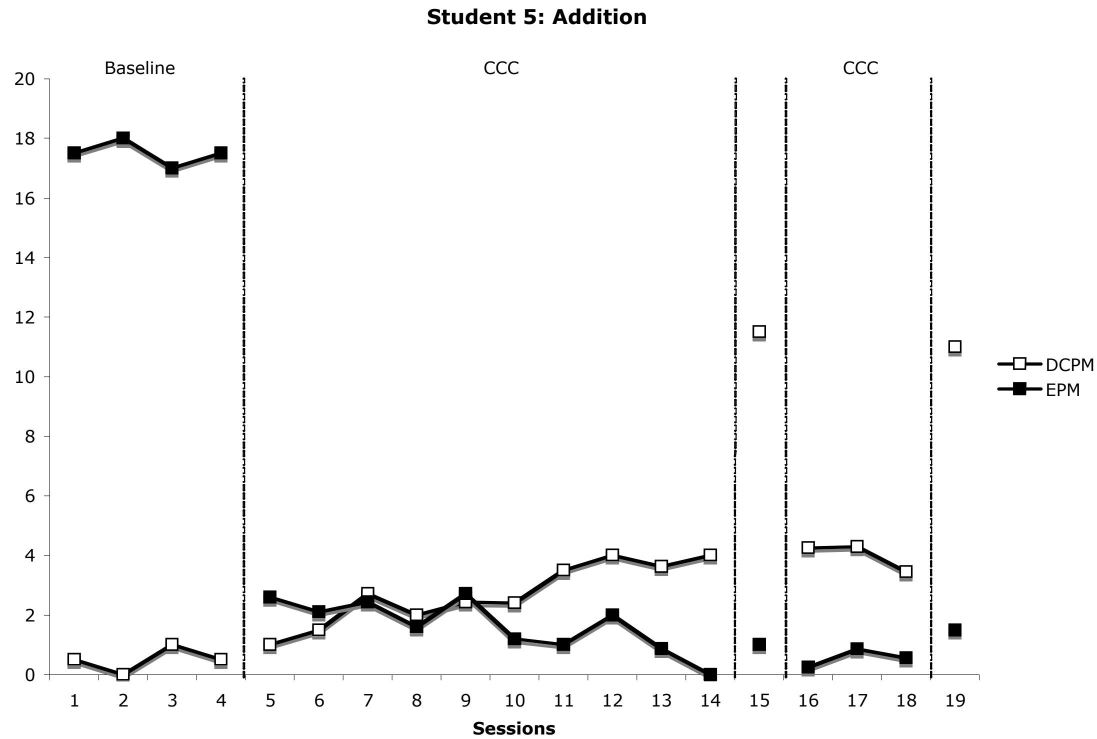

Figure 10

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 5 in subtraction, across baseline and intervention phases.

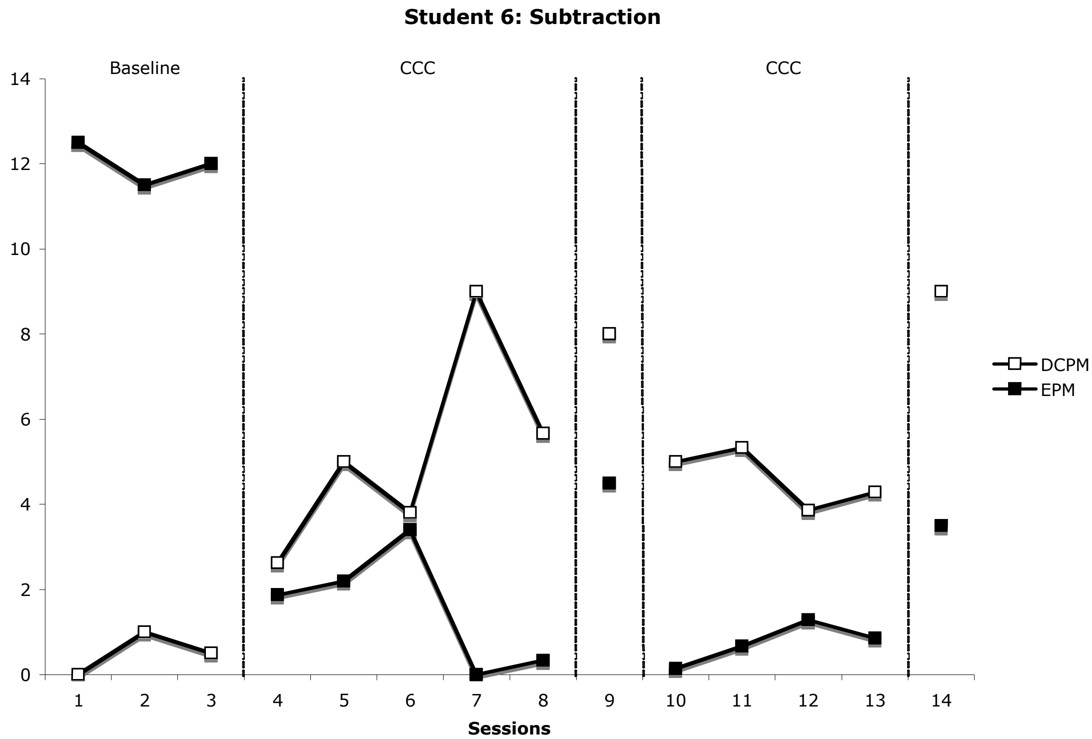

Figure 11

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 6 in subtraction, across baseline and intervention phases.

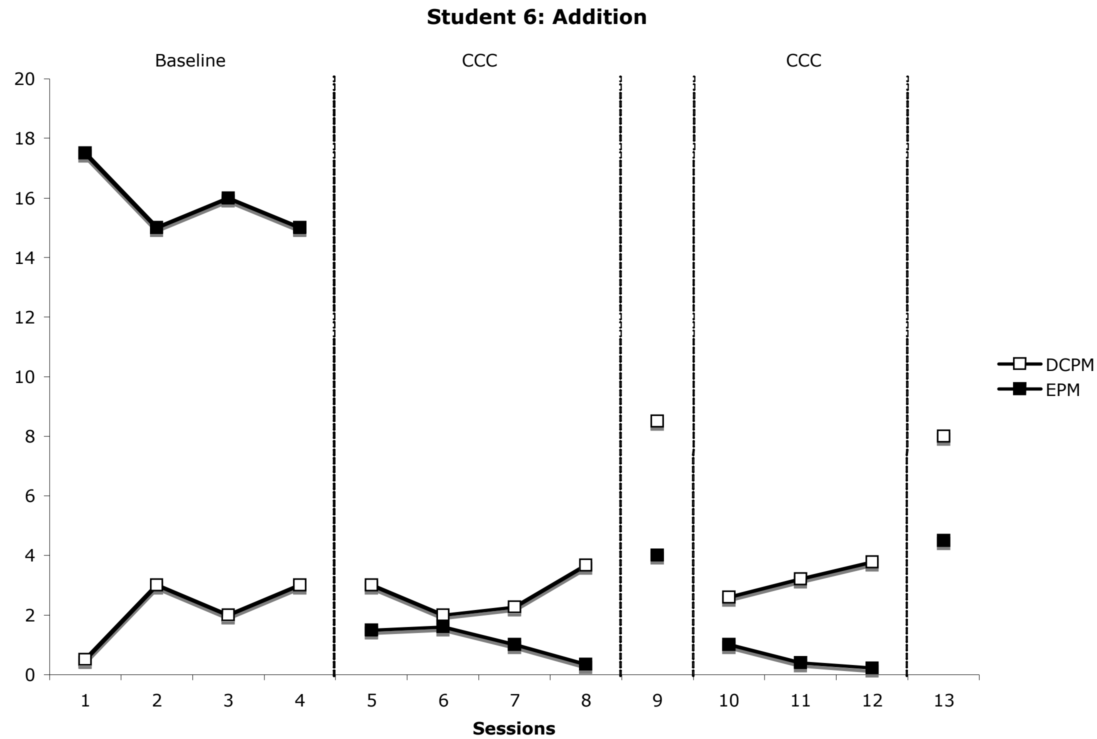

Figure 12

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 6 in addition, across baseline and intervention phases.

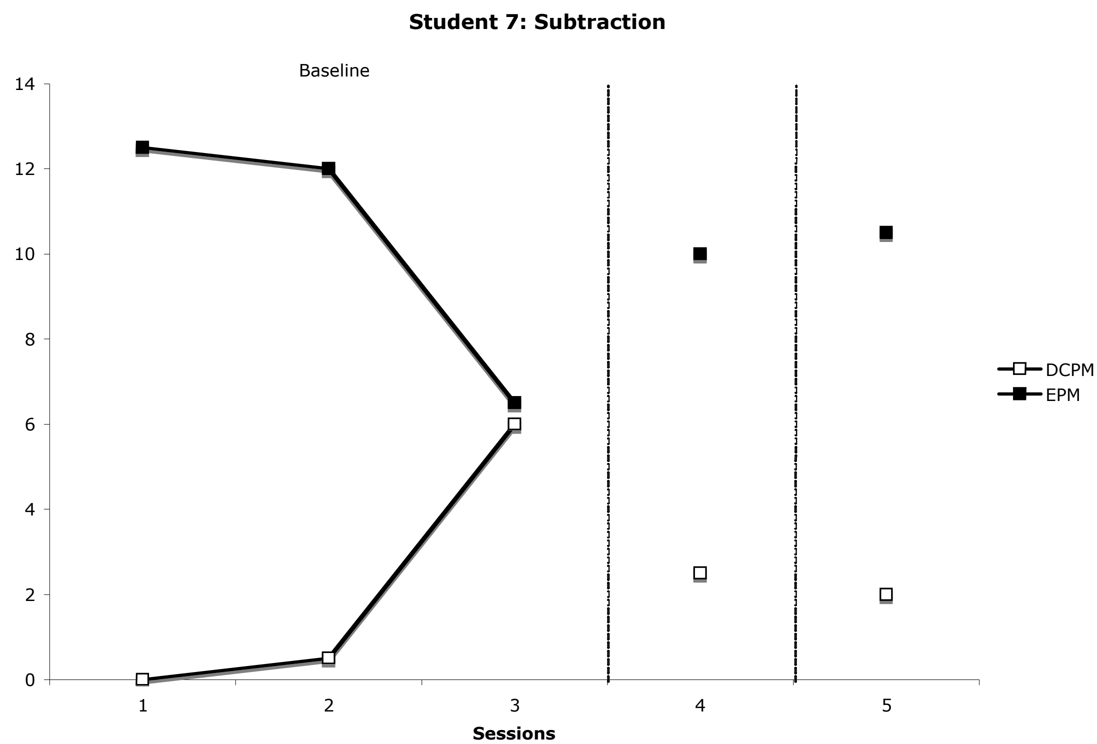

Figure 13

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 7 in subtraction, across baseline phases.

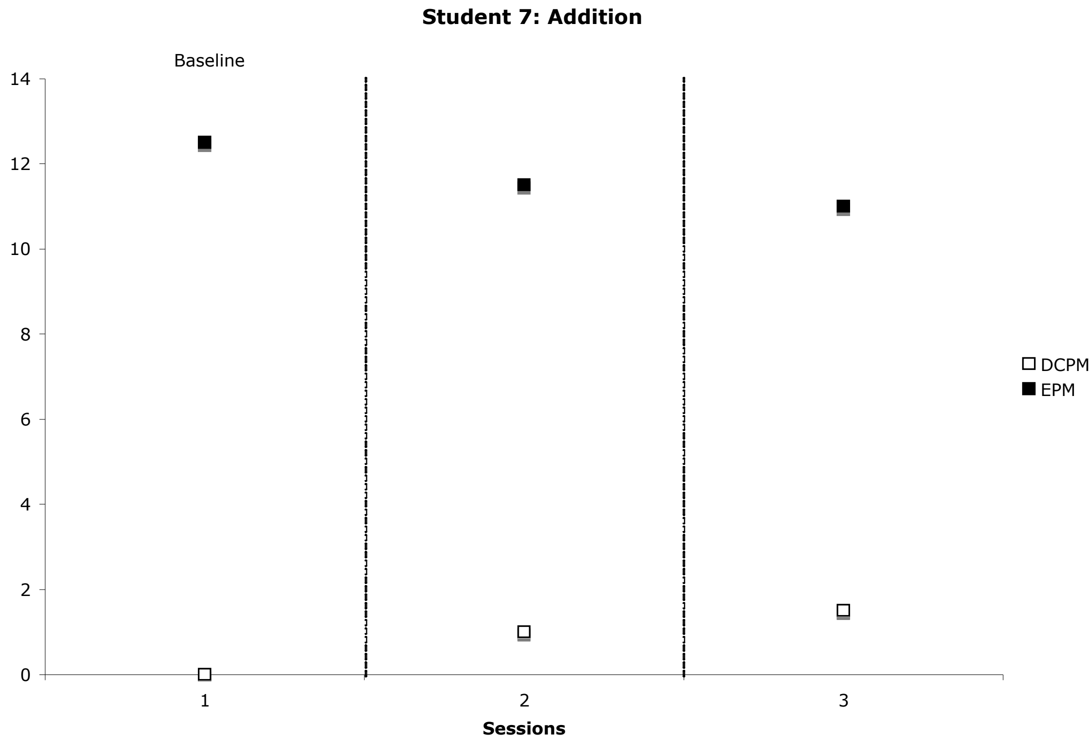

Figure 14

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 7 in addition, across baseline phases.

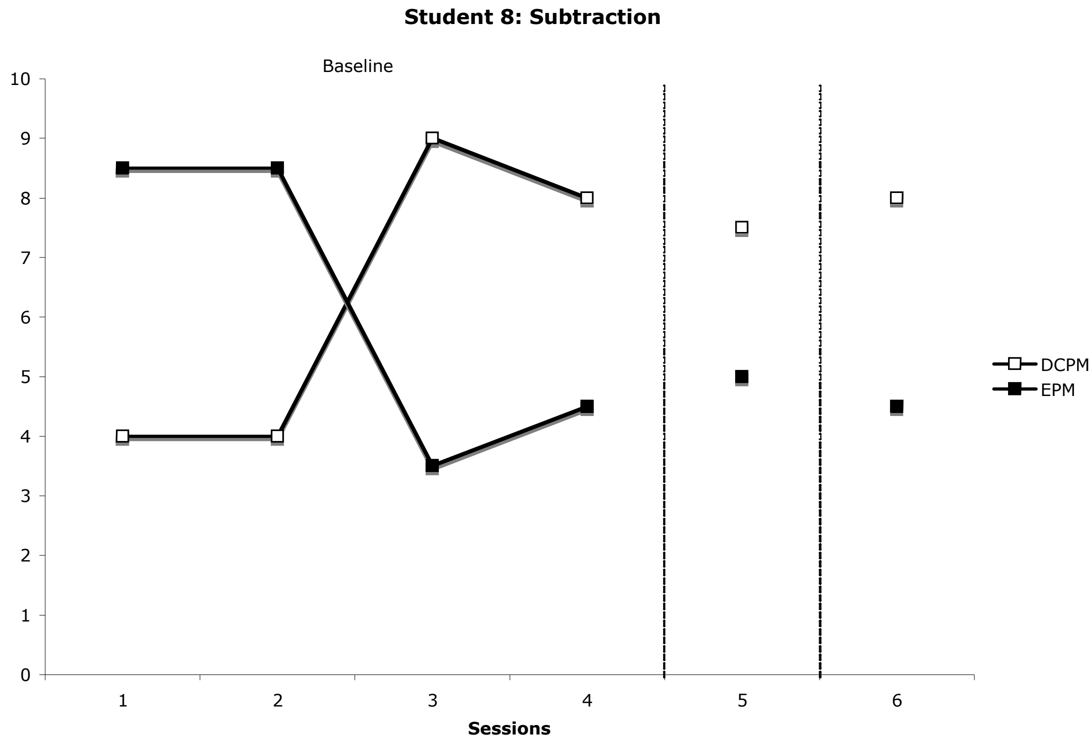

Figure 15

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 8 subtraction, across baseline phases.

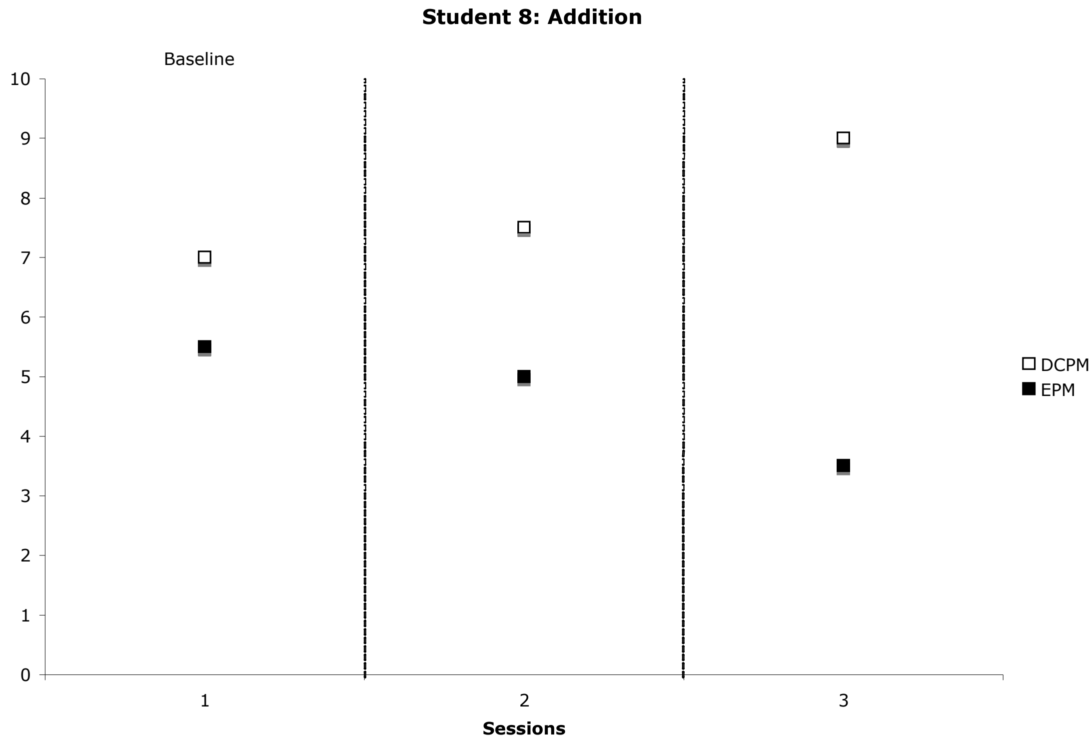

Figure 16

Digits Correct Per Minute (DCPM) and Errors Per Minute (EPM) for Student 8 in addition, across baseline phases.

Hypothesis 1. Hypothesis 1 stated that participants who are at risk for MLD would increase their DCPM and decrease their EPM relative to their own baseline during the CCC intervention phase. This hypothesis was tested by examining the performances of students who received the CCC intervention directly after baseline assessment. Baseline performances are represented in the first column on the left of the figures listed in this paragraph. CCC intervention performances are presented in the second column from the left in the figures. The students who received the CCC intervention right after baseline are Student 1 for addition (Figure 1), Student 2 for addition (Figure 3), Student 3 for subtraction (Figure 6), Student 4 for subtraction (Figure 8), Student 5 for subtraction and addition (Figures 9 and 10), and Student 6 for subtraction and addition (Figures 11 and 12).

Each of these students demonstrated dramatic decreases in EPM from their baseline performances during CCC. Specifically, Student 1 had an average of 14.0 addition errors during baseline, but averaged only 0.55 addition errors during CCC (Figure 1). Student 2's addition errors declined to 1.17 during the CCC intervention from his baseline average of 17 (Figure 3). Students 3 and 4 (Figures 6 and 8) demonstrated comparable decreases in subtraction errors from baseline during CCC implementation, producing 11 and 7.8 fewer errors from baseline averages of 12 and 8.4 respectively. Figures 9 through 12 show that Students 5 and 6 also substantially decreased the number of both subtraction and addition errors from their baseline performances during CCC intervention. Their performances during CCC showed between 16 and 10 fewer errors during CCC than these students made during baseline. Taken together these data provide strong support for the effectiveness of CCC in decreasing student EPM for both addition and subtraction problems.

The same graphs show that CCC was not as effective in improving students' DCPM as it was in reducing their EPM. In fact, Subject 1's addition DCPM (Figure 1) declined from 4 during baseline to 2.7 during CCC. Subject 4 (Figure 8) averaged about 4 subtraction DCPM during baseline and 3.35 DCPM during CCC. In addition, after initially improving, Subject 5's (Figure 9) CCC subtraction DCPM declined to near baseline levels of 0.5-1 during CCC, and Subject 6 (Figure 12) showed no improvement over baseline in addition DCPM during CCC. For just over half the CCC interventions, however, participants demonstrated modest gains over baseline performances during

CCC with both addition and subtraction problems (Figures 2, 3, 6, 10, and 11). These gains tended to trend upward from as low as 0 DCPM (Figure 10, Subject 5, subtraction) during baseline to as high as 11.5 DCPM (Figure 6, Subject 3, subtraction). However, all participants averaged less than 10 DCPM during CCC.

In summary, all the students who received CCC immediately following baseline displayed a significant decrease in EPM over their own baseline levels during the CCC intervention phase, which supports Hypothesis 1. Students 2 and 3 displayed results that fully support Hypothesis 1 by showing an increase in DCPM and a decrease in EPM within both the addition and subtraction conditions during the CCC intervention phases. Hypothesis 1 was partially supported by the data for Students 1, 4, 5, and 6, because these students did not display increases in DCPM in both operation conditions (addition and subtraction) during CCC. The average gains over baseline levels in DCPM per student were very similar and there was little difference between individual students (see CCC conditions and baselines in Figures 1-12).

Hypothesis 2. Hypothesis 2 stated that participants in the CCC + PF condition would demonstrate significant increases in their DCPM and significant decreases in their EPM when their performances were compared with their own baseline performances and the performances of participants in the CCC in isolation intervention condition.

This hypothesis was tested by comparing the performances of students who received CCC+PF directly after baseline, with the performances of students who received CCC directly after baseline. Students 1 and 2 received CCC+PF for subtraction (Figures 2 and 4 respectively), Student 3, 4, 5, and 6 received CCC for subtraction (Figures 6, 8, 9, and 11 respectively). Student 5 and 6 received CCC for addition (Figures 10 and 12).

Results of Hypothesis 1 with these students found large decreases in both addition and subtraction EPM during CCC compared to baseline. These decreases represented a floor effect. Figures 2 and 4 demonstrate a similar drop in subtraction EPM over baseline performances for the CCC+PF intervention. Thus, the CCC+ PF intervention did not lead to greater decreases in EPM than did the CCC alone condition.

Results of Hypothesis 1 indicated that the CCC intervention lead to modest increases in addition and subtraction DCPM over baseline in about half the cases. Figures 2 and 4 show similar modest gains in DCPM over baseline for the CCC+PF condition. Thus, the CCC+PF intervention did not lead to greater gains in DCPM than did the CCC alone condition. The results do not support Hypothesis 2.

Hypothesis 3. Hypothesis 3 stated that participants in the CCC + RW condition would demonstrate significant increases in DCPM and significant decreases in EPM over baseline levels in both the addition and subtraction conditions when compared with gains over baseline levels of participants in the CCC in isolation intervention condition. Students 3 (Figure 5) and 4 (Figure 7) received the CCC + RW intervention condition for addition. Student 3, 4, 5, and 6 received CCC for subtraction (Figures 6, 8, 9, and 11 respectively). Student 5 and 6 received CCC for addition (Figures 10 and 12). Results of Hypothesis 1 for these students found large decreases in both addition and subtraction EPM during CCC compared to baseline. These decreases represented a floor effect. Figures 5 and 7 demonstrate a similar drop in subtraction EPM over baseline performances for students in the CCC+RW intervention. Thus, the CCC+RW intervention did not lead to greater decreases in EPM than did the CCC alone condition.

Results of Hypothesis 1 indicated that the CCC intervention lead to modest increases in addition and subtraction DCPM over baseline in about half the cases. Figures 5 and 7 demonstrate similar gains in DCPM over baseline. Thus, the CCC+RW intervention did not lead to greater gains in DCPM than did the CCC alone condition. The results do not support Hypothesis 3.

Hypothesis 4. Hypothesis 4 stated that participants in the CCC intervention condition, CCC + PF intervention condition, and CCC + RW intervention condition would all demonstrate significant increases in their DCPM and significant decreases in EPM over their baseline levels in both the addition and subtraction conditions and when compared with the baseline performance of the participants in the control condition. Results of the first three hypotheses taken together indicated that the CCC alone, CCC+ PF, and CCC+ RW were similarly effective in decreasing students' EPM and increasing their DCPM.

Students 7 and 8 were in the control condition and did not receive the CCC intervention, nor did they receive the CCC + PF or CCC + RW intervention conditions. These students were administered a baseline assessment of both addition and subtraction over several data points to assess their DCPM and EPM in both operation conditions. Students 7 and 8 were also administered a baseline CBM measure following each intervention condition, in order to make comparisons of gains in DCPM and decreases in EPM between the students and intervention conditions.

Within both the addition operation condition and subtraction operation conditions, Student 7 did not show differences between her initial baseline performance and the baseline administered following each intervention condition. Student 8 did not show a difference between her initial baseline performance and the baseline administered following each intervention condition within both addition and subtraction operations.

Overall, results of testing the hypotheses revealed that the intervention phases (CCC, CCC + PF, and CCC + RW) increased the DCPM and decreased the EPM for the students who received the intervention phases in comparison to the students in the control conditions. Thus, Hypothesis 4 was supported.

Return to baseline and the second intervention. I intended to examine the relative effectiveness of interventions by instituting a second baseline followed by a second intervention and a third baseline. The third, fourth, and fifth panels of Figures 1-12 present the data for these conditions. Readers will notice that the second and third baselines were only 1 session in length.

First, I examined students' baselines relative to their original baseline at the far left of each figure. Interestingly, the DCPM and EPM second and third baseline performances of four of the six participants (Subjects 1, 3, 5, and 6) did not return to their original baseline levels (see Figures 1, 2, 5, 6, 9, 10, 11, and 12). Instead, their second baselines showed DCPM and EPM that were more similar to those they displayed during the first intervention condition. This suggests that participants learned from their first intervention to decrease their EPM and increase their DCPM, and they continued this performance even when the intervention conditions were withdrawn. Subjects 2 and 4 sometimes continued their intervention performances through the second and third baselines and sometimes did not (see Figures 3, 4, 7, and 8). Thus, in general for the majority of participants, the first intervention initiated a pattern of responding that carried through the second baseline, the second intervention, and the third baseline.

Student Accuracy. In this study, accuracy was defined as 30 digits correct within the 15-minute intervention period. Although the all the students who received an intervention phase (CCC, CCC + PF, CCC + RW) made only modest gains in their DCPM, the students displayed accuracy during the intervention phases. All the students displayed a decrease in the number of sessions it took them to reach accuracy during the second intervention phase within each operation condition. For example, Student 1 (see Figure 1) reached accuracy during the fourth session of CCC in addition (Figure 1, panel 2), and displayed accuracy in the first session of CCC + PF within addition. She maintained performing at accuracy levels during the CCC + PF intervention phase within addition (Figure 1, panel 4). Performing at accuracy levels appeared to transfer to the subsequent intervention phase within each operation condition.

Students 1, 2, 3, 4, and 6 all displayed accuracy within both the addition and subtraction operations, and both intervention phases. Once accuracy was reached, they maintained performance at this level during the remaining intervention sessions within each operation condition (Figures 1-8, 11, 12).

Student 5 displayed difficulty maintaining performance at accuracy levels within the subtraction condition. He reached accuracy during the third session of CCC within subtraction, and then regressed to below accuracy levels for the remainder of the first CCC intervention phase within subtraction (Figure 9, panel 2). Student 5 did regain accuracy during the second CCC intervention phase within subtraction during the fourth intervention session. Student 5 displayed accuracy within addition during the first CCC intervention phase after 10 sessions and maintained this performance during the second intervention phase (Figure 10), as did the other students in the study.

Supplemental Results

Students 1-4, who received the CCC + PF or CCC + RW intervention conditions, displayed a decreasing trend in the amount of time spent on each intervention session during the intervention phases, as can be seen in Figures 17-20. The students were given up to 15 minutes per intervention session. The greatest amount of time taken to complete an intervention worksheet probe during any session by any of the participants was 14 minutes (Student 1, during CCC + PF, addition condition). The least amount of time taken to complete an intervention worksheet probe by any of the participants who received an intervention was 3 minutes (Student 3 during CCC, subtraction condition). This observed decreasing trend in time spent on problems during each intervention session shows that as students learned the directions of the task and understood the intervention, they were able to complete the worksheet probe at a faster pace. A practice effect was observed for the CCC + PF and CCC + RW intervention conditions.

Student 5 (Figure 21) who received the CCC in isolation intervention did not demonstrate this decreasing trend, but rather showed a stable amount of time spent on the intervention sessions. Student 6 (Figure 22), who received the CCC in isolation condition, displayed a lower amount of time spent on intervention sessions within the subtraction operation condition, than within the addition operation condition, which was administered second. Student 6 did not display a decreasing trend in time spent on each intervention condition within each operation or when the two operation conditions were compared.

These results indicate that, although CCC+PF and CCC+RW do not provide different DCPM and EPM than CCC, they are associated with faster performances for some students.

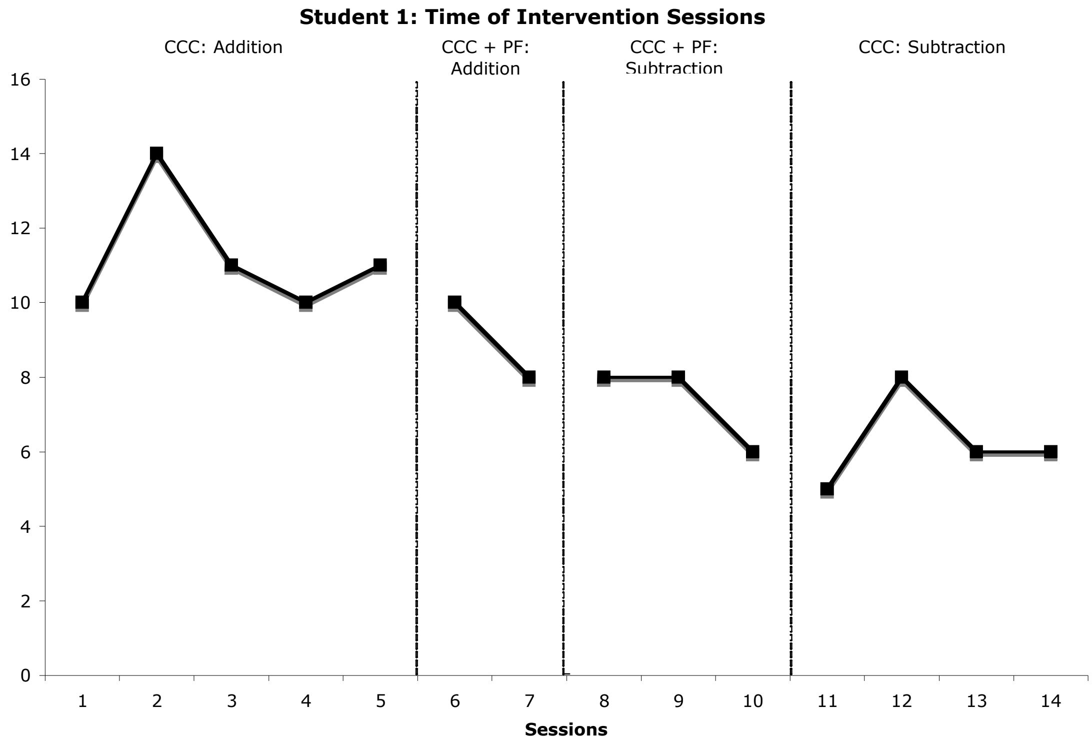

Figure 17

Time spent for each intervention session for Student 1, across interventions and addition and subtraction conditions.

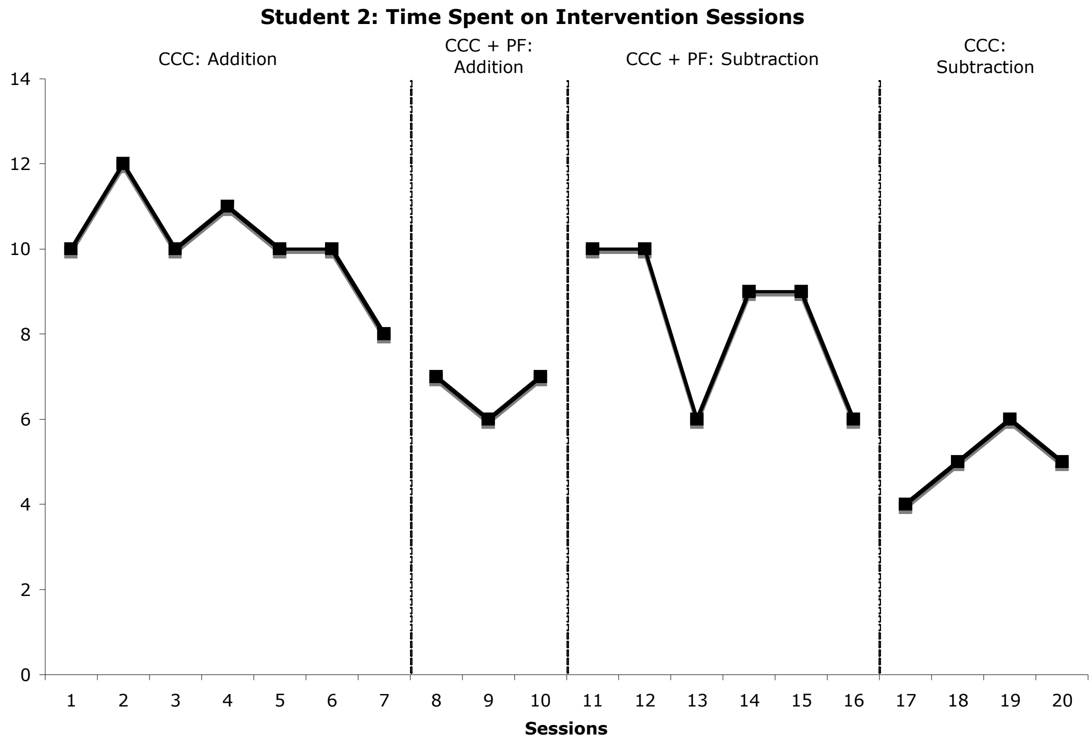

Figure 18

Time spent for each intervention session for Student 2, across interventions and addition and subtraction conditions.

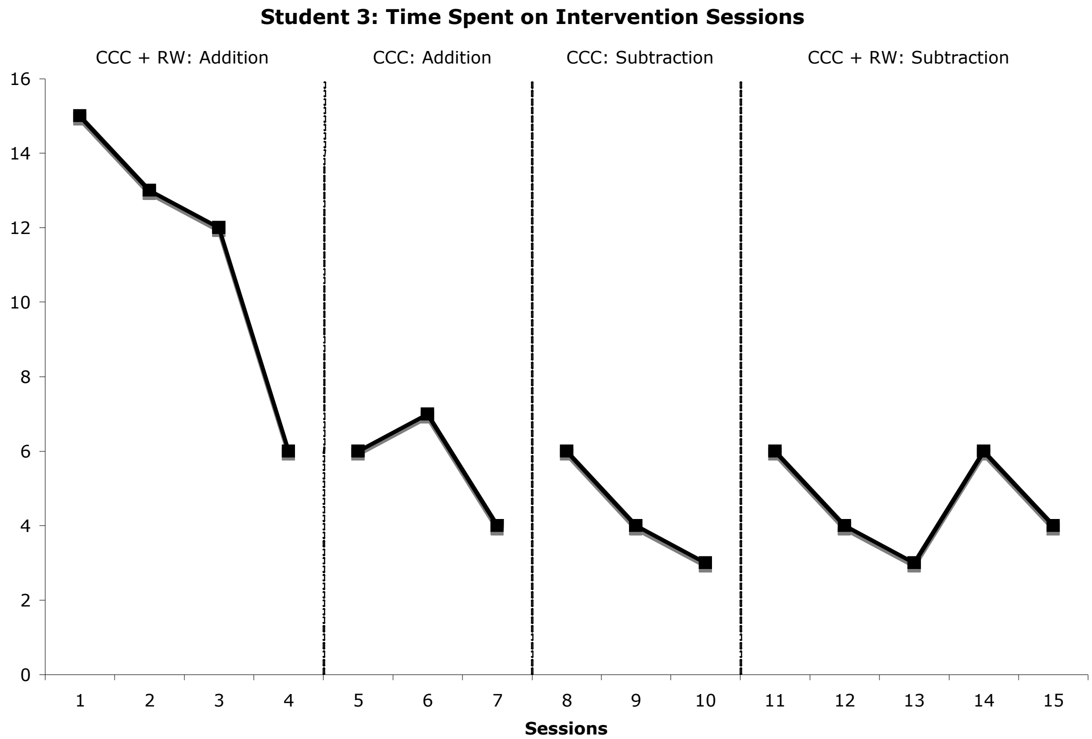

Figure 19

Time spent for each intervention session for Student 3, across interventions and addition and subtraction conditions.

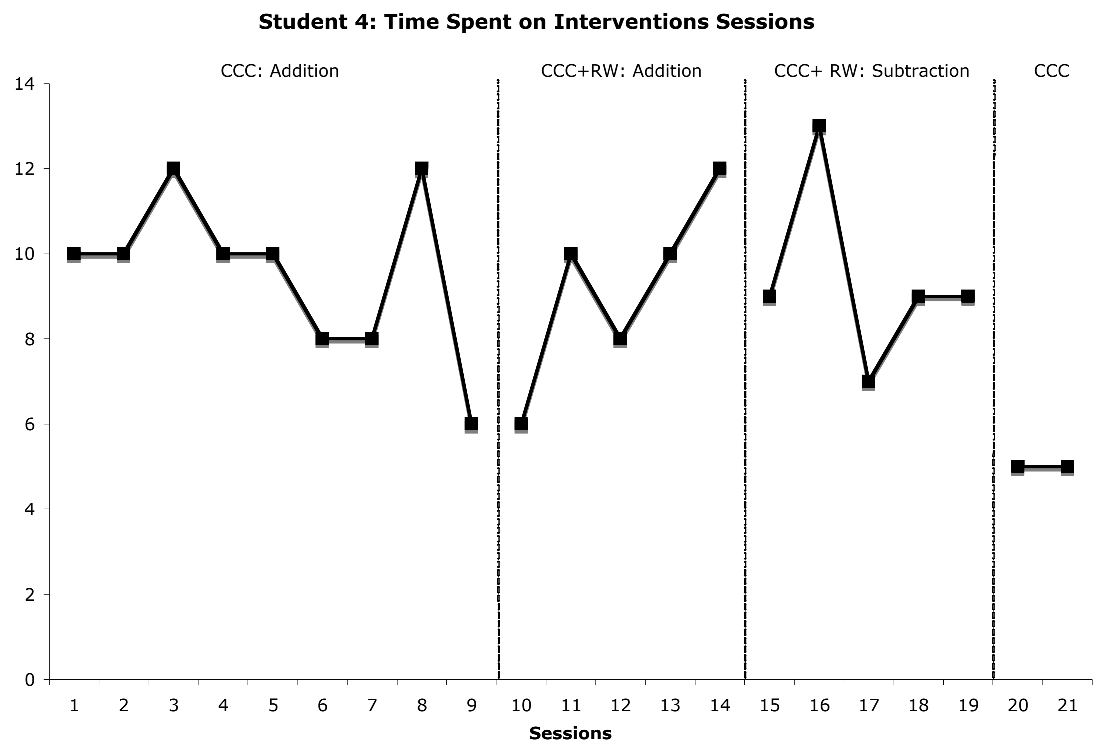

Figure 20

Time spent for each intervention session for Student 4, across interventions and addition and subtraction conditions.

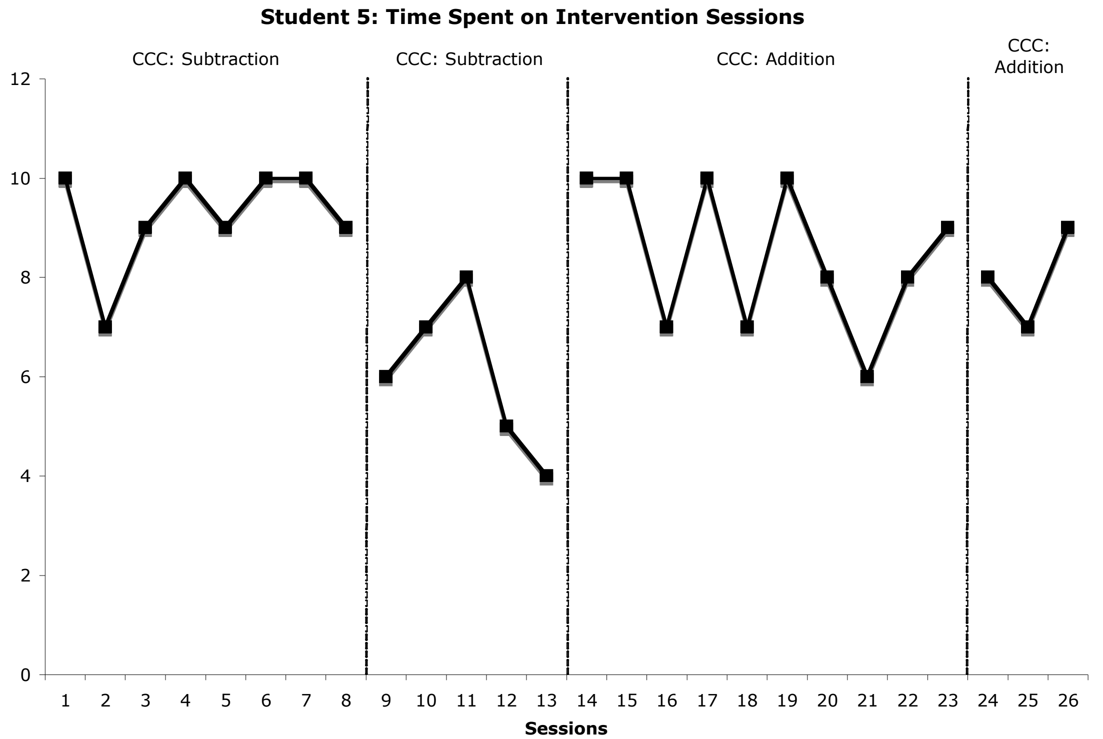

Figure 21

Time spent for each intervention session for Student 5, across interventions and addition and subtraction conditions.

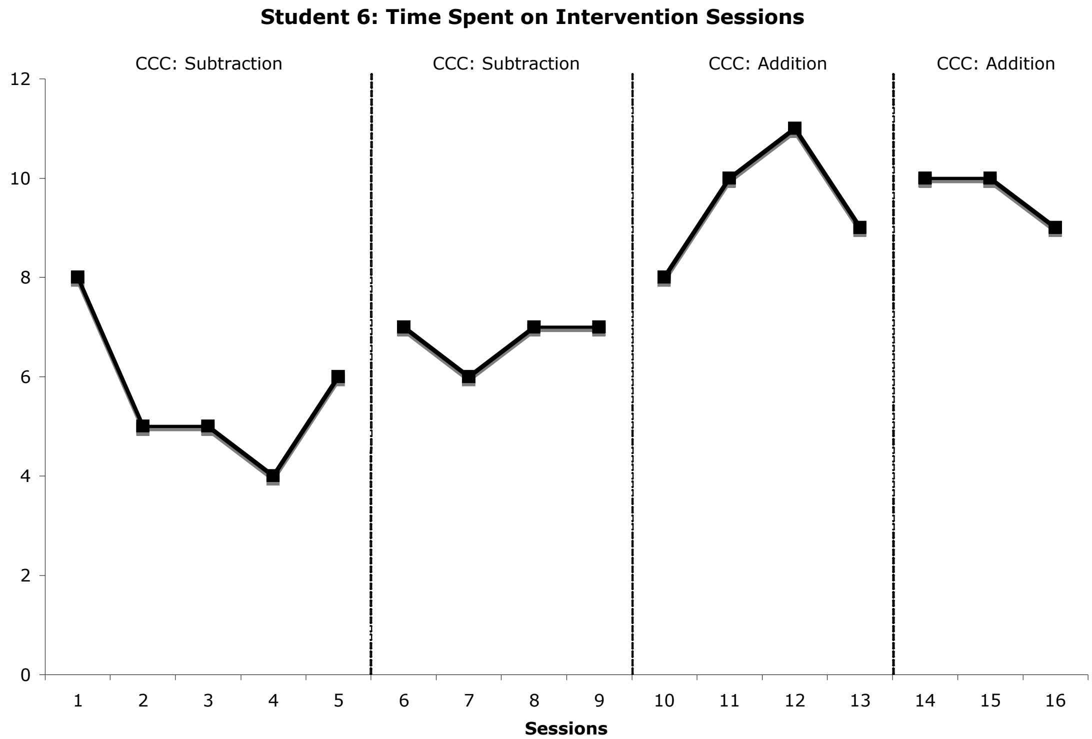

Figure 22

Time spent for each intervention session for Student 6, across interventions and addition and subtraction conditions.

Individual students. Students did not respond equally to each of the intervention conditions. For example, Student 1 enjoyed the CCC intervention, and was particularly happy when she received gold stars for correct responses in the CCC+PF condition. Although her performances were similar in both conditions, her affect became even more positive when given PF.

In contrast, Student 2 became noticeably upset when he did not initially receive gold stars and correct responses during the CCC+PF condition. His affect continued to become more distraught and during subsequent intervention sessions, he refused to complete several probes. When asked, he indicated that he wanted to continue participation in the study, and then was able to complete the probes in successive intervention sessions. His refusal to complete some probes, however, resulted in a gain in EPM during the CCC + PF during the subtraction phase and an average gain in EPM.

Student 3, who received CCC and CCC+RW, appeared to be highly motivated throughout the study, and the addition of a reward did not significantly improve her performance. Receiving a reward each time the intervention session ended did not appear to motivate Student 4, who also received CCC and CCC+RW, to work at a faster pace. Students 5 and 6, who received only CCC, displayed the similar affect throughout the interventions and demonstrated increases in DCPM and decreases in EPM, suggesting that the process of self-checking their work was sufficient to motivate them to perform well.

Summary of Results

The following table summarizes the results of testing the hypotheses in the current study.

Table 2

Results of Hypothesis Testing

Hypothesis	Results
HO1: Participants identified as at risk for MLD will display an increase in DCPM and decrease in EPM compared with their baseline performances when the intervention CCC is implemented.	Partially Supported
HO2: Participants in the CCC + PF condition will demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions relative to their own baseline performances and to the performance of participants who received CCC in isolation.	Not Supported
HO3: Participants in the CCC + PF condition will demonstrate significant increases in DCPM and significant decreases in EPM in both the addition and subtraction conditions relative to their own baseline performances and to the performance of participants who received CCC in isolation.	Not Supported
HO4: Participants in the CCC in isolation intervention condition, the CCC + PF intervention condition, and the CCC + RW intervention condition will all demonstrate significant increases in DCPM and significant decreases in EPM when compared with their own baseline performance and the baseline performance of the participants in the control condition.	Supported

The results of the study indicate partial support for Hypothesis 1, because all students who received CCC as an initial intervention showed decreases in EPM, and most showed gains in DCPM. Hypotheses 2 and 3 were not supported because adding PF or a RW to CCC did not result in greater gains in DCPM than did CCC alone, nor did it result in greater decreases in EPM. All three conditions (CCC, CCC+RW, and CCC+PF) were superior to the control condition, which supported Hypothesis 4.

CHAPTER V

Discussion

This chapter describes the main findings obtained from analyses in the present study, as well as educational implications of the findings, limitations of the study, and directions for future research.

Research on MLD is far less extensive than research on reading disorder (RD) (Mazzocco & Myers, 2003). A search of articles published between 1974 and 2003 resulted in 14 to 33 times as many citations for “dyslexia” than for “dyscalculia” (Mazzocco & Myers, 2003). Given the amount of research devoted to reading disorders, they are much better understood than are math disorders, and the research on math disorders is much less developed. Research on math disorders demonstrates cognitive differences between students with and without math disorders (Geary, Bow-Thomas, & Ya0, 1992; Geary, Hoard, & Hamson, 1999; Russel1 & Ginsburg, 1984). However, a significant gap in our knowledge of MD remains.

Gaps in research of MLD are translated into gaps in interventions within classrooms and schools. The prevalence of MLD is reported to be approximately 6%, and parallels the frequency of RD (Badian, 1983; Shalev, Auerbach, Manor, & Gross-Tsur 2000). There is a growing need not only to diagnose MLD, but also to identify students who present with a delay in math versus those with a disability. Through research on MLD, pre-referral interventions that are effective for students who are identified with a delay in math can be identified.

In this study, 43 students were given a pre-test to identify if they are having difficulty in math and present with a delay in math skills. Of the 43 students, 30 (69%) of the students qualified for the study. All the students who were administered a pre-test had received instruction in math calculation skills in both addition and subtraction. Performing below 70% on the pre-test, and qualifying for this study indicated that these students are at-risk for math difficulty. This finding emphasizes the lack of interventions in mathematics for students who are struggling to acquire basic math facts. Without interventions to address these delays, students will go on to higher grades without having acquired basic math facts and a further delay in math skills will arise.

Key Findings

The purpose of this study was to compare the isolated effects of CCC with the effects of CCC paired with performance feedback (PF) and CCC paired with a reward (RW) in a sample of first-grade students who were at risk for MLD. CCC appeared to be an effective intervention for increasing correct digits and decreasing the errors made during single-digit addition and subtraction facts among first-grade students identified as at-risk for MLD in computation skills. Overall, the study revealed that the CCC intervention produced modest gains in DCPM and more significant decreases in EPM.

When the CCC intervention was paired with performance feedback or paired with a reward, the number of DCPM and EPM were at similar levels as the isolated CCC intervention. Adding either performance feedback or a reward to CCC did not increase the power of CCC as an intervention. CCC functioned effectively in isolation, replicating past research that used CCC as an academic intervention (Codding et al., 2007, Codding et al., 2009; Skinner, 1997; Struthers et al., 1994). Further, the study extends the use of

CCC as an effective intervention for math difficulties to first-grade children at risk for MLD.

According to Greenwood et al. (1984), instructional procedures that increase student engagement and responding are likely to increase learning. However, simple engagement is not enough for proficient learning to occur and to avoid inaccurate responding (Grafman & Cates, 2009; Skinner, 1998). Students need to be active participants in their learning, and many academic interventions function by means of an antecedent-response-consequence chain (ARC). Students are given a stimulus, they provide a response, and then are given a consequence, such as feedback. CCC operates in the same ARC fashion; however, students are given instructions to learn to complete the ARC independently. Therefore, CCC is considered a self-managed strategy.

CCC in this study provided students with a stimulus (a math problem), and an opportunity to respond and evaluate their responses and to make corrections as necessary. CCC contains performance-based components and Skinner et al. (1993) suggested that accurate responding during CCC is self-provided reinforcement, and inaccurate responses are self-provided punishers. The self-reward and self-punishment characteristics of CCC may explain the lack of differences between CCC, CCC+PF, and CCC+RW conditions, because in all conditions, students were able to compare their responses to the correct response and thus provide feedback for themselves. Adding performance feedback (PF) from the examiner, who used gold stars placed on accurate responses after students completed each probe, did not further enhance students' performances, nor did providing students with a reward (RW) after they completed each probe facilitate their performances.

This study examined order effects of the interventions by counterbalancing the order of the interventions across the operations. No differences in student performance were evidenced related to the order in which the interventions were implemented. This adds strength to the findings of the study, since regardless of the order that the interventions were implemented, the outcome was comparable for the students within each treatment condition. Treatment interference was not evidenced based on the order of the interventions.

I had expected that increases in DCPM would occur at the same rate as decreases in EPM. Instead, however, DCPM increases were produced at a slower rate than the decreases in EPM. These results suggest that it was easier for the students to learn to stop responding incorrectly or guessing when they may not have been sure of the correct answers than it was for them to remember the correct answers. Typically, children with MLD have difficulty retrieving basic arithmetic facts from memory (Geary, 2004; Howell et al., 1987). Additional CCC intervention sessions might show greater increases in DCPM. Developmentally, first grade students, particularly those at risk for or with MLD, may need more than a brief intervention such as that in the present study to memorize and retrieve the correct responses. Readers should note that, despite the relative low rate of DCPM, all students achieved both addition and subtraction accuracy (i.e., 30 of a possible 36 digits correct within the 15-minute intervention time) within a few sessions.

The results suggest that the CCC interventions were effective in facilitating participants' long-term memory for the correct answers to the problems for the majority of participants. This assertion is based on the lack of return to initial EPM and DCPM baseline levels during the second and third baselines. Four of the six participants displayed second and third baseline performances that were very similar to their intervention performances, which suggests that they learned the answers to the problems and were able to retrieve them from long-term memory. Because long-term memory deficits are typical of students with MLD (Geary, 2004; Howell et al., 1987), current results suggest the CCC may help these students to commit arithmetic facts to memory and retrieve them.

In this study, the CCC intervention alone appeared to be sufficiently motivating to students to result in increased DCPM and decreased EPM. Students who received either PF or RW after they completed the probes performed similarly to students who did not receive these. The CCC+PF and the CCC+RW conditions did, however, result in faster performances. Because students knew that they would either receive gold stars for correct answers or a reward when they finished each probe, they may have gone faster through the probes to get to their stars and rewards.

Results suggest that external feedback may not be needed to facilitate students' performances. In fact, in some cases, external feedback may inhibit student responses, as was the case for Student 2, who quit responding when presented with the CCC+PF condition. Thus, researchers and practitioners may need to tailor CCC interventions to particular students in order to achieve optimal results.

In this study, participants were given the interventions individually. The participants were aged 6-7, and prior to this study, it was unknown how children of this age and developmental and academic level would respond to a self-managed intervention and how long it would take them to learn the steps to carry out the intervention independently. This study was designed to give each participant individual attention for this reason. The participants learned the steps of the intervention rather quickly and were easily able to self-manage the intervention. In previous research, CCC has been applied using a class-wide model (Codding et al., 2009; Poncy, McCallum, & Schmitt, 2010), and following the results of this study, CCC could be applied to groups of students even as young as 6 years of age to improve math skills. This makes CCC a useful intervention for teachers who are constrained by time and cannot apply interventions individually. Furthermore, adding a reward or performance feedback to the CCC intervention also increased response speed, displaying that young students respond well to both feedback and to a reinforcer such as a reward.

In summary, given the immediacy of the effect of CCC and the results of past research that support CCC as a powerful academic intervention (Codding et al., 2007, Codding et al., 2009; Skinner, 1997; Struthers, et al. 1994), CCC can be used within mathematics instructional practices for the early childhood grades to improve basic mathematics skills for students who lack the foundations of higher level math skills. This study extended previous research by applying CCC to first grade students. The decrease in time spent on intervention sessions for several of the participants in this study demonstrated that students need minimal explanation of the procedures, making CCC able to be incorporated into a group lesson, rather than individual instruction as it was implemented within this study.

Limitations of Study and Directions for Future Research

The study used an ABA design. Two intervention phases, one in each treatment condition that each participant received, were administered for both addition and subtraction operations. Although the study was implemented within the guidelines that are recommended for single-case designs (Kratochwill et al., 2010, What Works Clearinghouse, 2011), and four baseline (A) and treatment (B) phases were implemented, a repetition phase may have allowed for a clearer relationship between the independent variable and outcome variable to be demonstrated, as well as allow for possible differentiations between treatment conditions. Because small gains in DCPM were evidenced within the intervention phases, repetition would also allow more time for the students to display larger gains.

The reversal baseline conditions were each only one session long. These baseline sessions should have been continued until response stability was reached. Future researchers should have longer baselines during reversal that continue until participants' performances revert to original baseline levels. A reversal to original levels would have allowed for a stronger functional relationship between the independent and dependent variables to be demonstrated.

Following each treatment condition, assessment of a baseline level was conducted using a CBM. If a CBM was administered to each student following each intervention session, the rate of learning would have been more easily assessed and reversal to baseline might have been more evident.

Student 2's reaction to the CCC + PF condition could indicate that the type of performance feedback provided in this study (i.e., gold stars placed next to correct problems) was not rewarding to the student, but rather drew attention to his difficulty with the calculations. CCC as an intervention includes self-provided feedback when the students look at their response and compare them to the answers. For Student 2, the addition of gold stars for correct answers at the end of each probe appeared to call attention to his incorrect responses rather than reinforce his correct responses, causing him distress. According to Bandura (1997), self-efficacy is a person's belief that he or she can act in ways that will produce desired outcomes, and it is a significant factor in shaping the goals an individual sets for himself and his level of persistence in working towards those goals. Self-managed academic strategies attempt to increase a student's motivation by involving the student in the evaluation of his or her responses. If Student 2's self-efficacy beliefs suffered as a result of his attention to his incorrect responses, it may have caused him to work at a lower than desired level of motivation. Given the limited number of subjects in each condition, it is difficult to know if the performance feedback selected in this study was the cause of the distress and lowered self-efficacy beliefs, or if Student 2 had a reaction to feedback for other reasons, because treatment acceptability was not assessed prior to implementation of the treatment conditions.

Adverse reactions to interventions can be avoided if the acceptability of an intervention is evaluated before it is used. This can be done with an acceptability checklist. Treatment acceptability is a judgment by laypersons, clients, and others of whether treatment procedures are appropriate, fair, and reasonable for the problem or client (Arra & Bahr, 2005; Kazdin, 1981). Students can answer questions after receiving a session of an intervention, such as on the Children's Intervention Rating Profile (CIRP; Witt & Elliott, 1985), to determine each child's acceptability of a teaching approach and its perceived appropriateness and effectiveness (Arra & Bahr, 2005). Using an acceptability checklist can prevent emotional reactions, such as what was observed for Student 2, and help researchers determine how to modify interventions prior to their implementation. This study should have used an acceptability checklist.

Treatment integrity is the extent to which the treatment was implemented as planned (Arra & Bahr, 2005; Noelle, Gresham, & Gansle, 2002). Evidence suggests that the treatment integrity of school-based interventions is related to behavior outcomes (Fiske, 2008). If student behavior or academic skills improve after an intervention is implemented, it is assumed that the intervention is effective. In order for response to intervention to be a true measure of intervention effectiveness, the intervention must be implemented as intended (Fiske, 2008). Treatment integrity was not measured in this study and is therefore a limitation of the results of the study.

The CCC+RW condition provided students with a reward after each probe regardless of the students' performances. Students completed the probes faster under this condition, and also under the CCC+PF condition, than they completed the probes under the CCC alone condition. Thus, the rewards were contingent on finishing within the time limit rather than on getting digits correct or reducing errors. To determine if CCC+RW can result in better performances than CCC alone, future researchers should make rewards contingent on achievement of a particular DCPM criterion.

Future research may extend this study by using an alternating treatments design in which each participant would receive all the intervention conditions. Inclusions of more subjects would also be possible, because this study demonstrated that students as young as 6 years old were able to learn and master the CCC intervention procedures easily and quickly. Providing the intervention to students in a group rather than individually would also be possible given the ease with which the students in this study carried out the intervention procedures. A group design would allow for future research to be more generalizable, because more participants could be incorporated.

Implications for Practice

School Psychologists understand that certain types of instructional practices and methods work because they involve components that are associated with effective instruction techniques. Active student responding is a large part of effective instruction (Berliner, 1984; Greenwood, et al., 1984). Cover, copy, and compare is an instructional technique that not only involves active student responding, but also uses self-management to produce responses.

Despite the limitations of this study, all students increased DCPM within only a few sessions of intervention, and EPM were significantly reduced for all students as well. Thus, there is evidence that CCC could be used as part of a pre-referral intervention to help students who are at-risk for failure in mathematics to improve their fluency and performance in mathematical calculation skills at the first-grade level. CCC is a research-based intervention, which may be used in the current RTI framework advocated by IDEA when evaluating students for learning disabilities.

CCC is also brief intervention with an immediacy of the effect, and does not require many materials or staff to administer. Once the procedure has been taught to students, less support is required from a teacher or service provider, making a self-managed strategy such as CCC effective both in implementation and outcome. CCC procedures have a quick and significant impact on student performance (Grafman & Cates, 2010).

An intervention such as CCC can also be used for progress monitoring of students’ mathematics skills (Grafman & Cates, 2010). The CBM used in the baseline phase gives indication of whether the student has mastered items in that skill set (i.e., single-digit subtraction). The CCC worksheets could be used to improve the students' skills in a particular mathematical operation and may be incorporated into math lessons, or into small group instruction for those students who are having difficulty reaching fluency in basic math skills. CCC can be applied to different mathematical operations, such as multiplication, as well.

Student 2's adverse reaction to receiving PF following his responses suggests that teachers should be cautioned to assess the type of PF they provide to students with lower levels of motivation or self-efficacy beliefs. When working with any student, but particularly students such as Student 2, careful attention must be given to monitoring progress and provide support when the student is struggling to make progress. CBM could be used to continuously monitor such students' progress in math calculation skills and tailor the intervention used to the students' emotional, as well as academic, needs.

When incorporating rewards into classroom instruction using CCC and CBM, setting a criterion level for receiving a reward may help to motivate students who do not have a high level of motivation, such as Student 4. A criterion level would also assist teachers in implementing an intervention such as CCC + RW, so that the reward would not have to be given to students as frequently. Teachers may not have the time or resources to implement rewards in the classroom after each math lesson. The present results suggest that CCC alone is a powerful intervention that teachers can use to facilitate student progress without having to spend extra time providing external feedback or rewards.

Conclusion

In sum, CCC was found to increase the DCPM and to decrease the EPM for students in the first grade identified as at-risk for MLD. Adding either performance feedback or a reward to the CCC intervention did not increase the DCPM or decrease the EPM significantly to differentiate between the treatment conditions. This study involved students with Math Difficulty within the first grade, which is a population that has not been studied frequently within the literature on math difficulty. Future research may expand upon this study to provide a better understanding of how CCC can be implemented within a classroom setting and used as a method to improve the math calculation skills of students with learning disabilities in the early childhood grades.

Appendix A

List of Acronyms

Acronym	**Meaning**
ATD	Alternating Treatments Design
ARC	Antecedent-Response-Consequence
CBA	Curriculum Based Assessment
CBA-M	Curriculum Based Assessment-Math
CBM	Curriculum Based Measurement
CCC	Cover, Copy and Compare
CCC + PF	Cover, Copy and Compare and Performance Feedback
CCC + RW	Cover, Copy and Compare and Rewards
CRA	Concrete Representational Abstract
DC	Digits Correct
DCPM	Digits Correct Per Minute
DIPM	Digits Incorrect Per Minute
EPM	Errors Per Minute
IDEA	Individuals with Disabilities Education Act
IEP	Individualized Education Plan
IF AT	Immediate Feedback Assessment Technique
IR	Incremental Rehearsal
MCBM	Curriculum Based Measurement-Math
MLD	Mathematics Learning Disability
MLD/RD	Mathematics Learning Disability and Co-Morbid Reading Disorder
NCTM	National Council of Teachers of Mathematics
OTR	Opportunities to Respond
RD	Reading Disorder
RTI	Response to Treatment Intervention
SLD	Specific Learning Disability

Appendix B

Principal Recruitment Letter

Dear Principal,

My name is Geetal Benson, and I am a student in the Ph.D. Program in Educational Psychology at the Graduate School and University Center of the City University of New York (CUNY). I am also the principal investigator of this project entitled “The Effects of Cover, Copy and Compare, Performance Feedback and Rewards on the Mathematical Computation Skills in First Grade Students.” This is a research study using an academic intervention that was created to increase elementary school students’ fluency in addition and subtraction skills.

Purpose:

- This study is expected to provide students who are having difficulty with mathematics at the 1st grade level additional instruction in computation skills through an intervention that previous research has shown to improve academic fluency.
- I would like permission to work with several students within your 1st grade General Education classes and include them in my research study after obtaining parental consent and student assent.

Procedures:

- Participation within the research project is strictly voluntary.
- I will not access the student’s permanent school records. All information on the students’ current academic functioning in mathematics will be obtained from his/her classroom teacher. Data collected by the teachers will not be provided to me, or used in this project.
- Once given permission by you to conduct my research project within your school, I will recruit the participation of 1st grade teachers with a flyer placed in their mailbox. This recruitment instrument will explain the definitions and procedures of my research project. I will ask the teachers to contact me if they wish to participate in the study.
- Since I do not wish to interfere with the students’ academic instruction time, the intervention will be conducted at a time when the classroom teacher and principal determine academic instruction is not occurring.
- The research project will involve working with each student for at least 20, 15-minute sessions over the course of one school semester, or approximately 12 weeks.
- All information about the students’ performance will be confidential, and will be stored in a locked file cabinet, to which only I, and my advisors, have access.
- There will be 8 children taking part in this study in total, and may be from several schools.
- Once parental consent is obtained, I will ask the students’ if they want to receive extra help in math and obtain their assent to participate before selecting them for the study. I will explain to each student that they may withdraw at any time with no negative consequences.
- Data collection will take place in a private area, such as the guidance suite so that the student feels that their performance is confidential.

Possible Benefits of Participation:

- There is a possibility that the intervention will improve the students' mathematical computation skills, which are pre-cursors to higher-level mathematical skills. Since mathematical computations have been difficult for some students in the 1st grade, participation in this study may provide the students with the help that he/she needs and make them feel more confident about their academic skills.
- The findings from my study may add to the research on academic interventions for students with academic difficulties in math.

Possible Risks:

- This study may involve frustration during mathematical calculations, though the calculations that the students will do can be completed in a brief period (5 minutes) during each session. If the students should express frustration or fatigue during the session, he/she may take a break during the session.
- The students will also be reassured that he/she can withdraw from participation in the study at any time.

I may publish results of this study, but names of people, or any identifying characteristics, will not be used in any of the publications. I will send you a copy of the completed research study and results at the conclusion of the research project.

If you have any questions regarding this research, you can contact me at (917) 757-5307 or at gmanglani@gc.cuny.edu, or my advisor, Dr I. Jeltova, at (212) 817 – 8288 IJeltova@gc.cuny.edu.

Thank you for consideration and participation in this study. I will give you a copy of the signed consent forms for your records. I will also give you a copy of the final approval letter to conduct my research project from the New York City Department of Education Institutional Review Board.

Sincerely,

Geetal Benson
Principal Investigator

I have read and AGREE that students in my school can participate in the research project.

____________________________ ________ _______________________ ____
Principal's Signature Date Investigator's Signature Date

I have read and I DO NOT wish for students in my school to participate in the research study.

___________________________ ________ ____________________ ______
Principal's Signature Date Investigator's Signature Date

Appendix C

Principal Consent Form

To Whom It May Concern,

The research project "The Effects of Cover Copy Compare, Performance Feedback and Rewards on the Mathematics Calculation Skills of First Grade Students Identified with Mathematical Difficulty" will be conducted by Geetal Benson. In this intervention, 8 students will be involved. These students will be selected from general education first grade classes. The Principal Investigator, Ms. Benson, will not access the students' permanent school records or any data collected by teachers. Ms. Benson will recruit the participation of first grade teachers through a teacher recruitment flier. The teachers who consent to participate in the study will administer a pre-test to their class. Those students performing below a criterion will be given parental consent forms in an attempt to recruit them for the study.

The students selected will give assent after parental consent is obtained. The students' involvement will be participating in 20 sessions of a math intervention, with each session lasting no more than 15 minutes. The students will attend 2 sessions of the intervention per week. The students will participate in the research during non-academic instruction or during an after-school program. Participating in the intervention will not interfere with academic instruction time.

I give permission for the students' to be involved in this research study. I am aware that parental consent must be and will be obtained, and students' assent will also be obtained the final consent for students' participation. I feel the students' involved will benefit from additional instruction in mathematical calculation skills. I hope that the study will increase our knowledge as educators in how to help children with mathematical learning disabilities and delays in mathematical skills.

Sincerely,

Principal

Appendix D

Teacher Recruitment Letter

Dear Teacher,

My name is Geetal Benson, and I am a student in the Ph.D. Program in Educational Psychology at the Graduate School and University Center of the City University of New York (CUNY). I am also the principal investigator of this project entitled “The Effects of Cover, Copy and Compare, Performance Feedback and Rewards on the Mathematical Computation Skills in First Grade Students.” This is a research study using an academic intervention that was created to increase elementary school students’ fluency in addition and subtraction skills.

Purpose:

- This is a study to provide students who are having difficulty obtaining basic mathematical calculation skills, in both addition and subtraction, additional instruction in computation skills through an intervention that previous research has shown to improve academic fluency.
- I would like to work with several students within your 1st grade General Education class and include them in my research study, after obtaining parental consent and student assent.
- The students will be identified by a pre-test administered by you, the classroom teacher. I will score the pre-test and follow-up with parental consent forms given out to those students who meet the operational definition of at-risk for math difficulty.

Operational Definition:

- At-risk is defined as receiving 1’s or 2’s on their report card in mathematics. If assessed by a Curriculum Based Measure in Mathematics, the student would score lower than 70% out of 100%.
- Students who are identified as possible participants should not be receiving any mandated Special Education programs, or have an Individualized Education Program (IEP) for academic remediation.

Procedures:

- Your decision to identify students, and participate in this study is strictly voluntary.
- I will not receive any student data from you, just your consent to participate in the study and administer a pre-test to your class.
- I will not use any identifying information in my study, even though I may publish the results.
- Your participation is only required administering a pre-test to your class. I do not need any written documentation from you, nor any questionnaires or surveys. I will ask you only to give the consent forms to students who are selected as possible participants and the parent will return the consent form to me in a self-addressed stamped envelope that will be provided to them by the principal researcher.
- Since I do not wish to interfere with academic instruction time, the intervention sessions will only occur at a time that you state academic instruction is not occurring, and you permit the student to leave the classroom. The sessions will last approximately 15-minutes each, over the course of 12 weeks.

Possible Benefits of Participation:

- There is a possibility that the intervention will improve your students' mathematical computation skills, which are pre-cursors to higher-level mathematical skills.
- The findings from my study may add to the research on academic interventions for students with academic difficulties in math.

Possible Risks:

- This study may involve frustration during mathematical calculations, though the calculations that the students' will do can be completed in a brief period (5 minutes) during each session. If the students should express frustration or fatigue during the session, he/she may take a break during the session.
- The student will also be reassured that he/she can withdraw from participation in the study at any time.
- Arrangements will be made for the students to meet with the school guidance counselor should they become stressed or overly frustrated from the study.

I may publish results of this study, but names of people, or any identifying characteristics, will not be used in any of the publications. If you would like a copy of the results of this study, please provide me with your name and address on the bottom of this form, and I will happily send you a copy in the future.

Please contact me, Geetal Benson, at (917) 757-5307 or gmanglani@gc.cuny.edu if you have a student you would like to recommend for this research project. You may also contact me if you have any further questions.

Thank you for your time and consideration and your participation in this study, should you decide to participate.

Sincerely,

Geetal Benson
Principal Investigator

Appendix E

Parent/Guardian Permission Form

Parent/Guardian Permission Form

My name is Geetal Benson, and I am a student in the Ph.D. Program in Educational Psychology at the Graduate School and University Center of the City University of New York (CUNY). I am also the principal investigator of this project entitled "The Effects of Cover, Copy and Compare, Performance Feedback and Rewards on the Mathematical Computation Skills of First Grade Students Identified with Math Difficulty" This is a study using an academic intervention that was created to increase elementary school students' fluency in addition and subtraction skills.

Purpose:

- This study is expected to provide students who are having difficulty with mathematics at the 1st grade level additional instruction in computation skills through an intervention that previous research has shown to improve academic fluency.
- I would like permission to work with your son/daughter to assess the ability of this intervention to improve your child's mathematical computation skills in addition and subtraction of single-digit numbers.

Procedures:

- Participation within the research project is strictly voluntary.
- I will not access your child's permanent school records. All information on your child's current academic functioning in mathematics will be obtained from his/her classroom teacher. Data collected by the teachers will not be provided to me, or used in this project.
- Since I do not wish to interfere with your son/daughter's academic instruction time, the intervention will be conducted at a time at a time when the classroom teacher determines academic instruction is not occurring.
- The study will involve working with your child for at least 20, 15-minute sessions over the course of one school semester.
- All information about your child's performance will be confidential, and will be stored in a locked file cabinet, to which only I, and my advisors, have access.
- At any time, you may withdraw consent for your child's participation and any information collected will be destroyed.
- There will be 8 children taking part in this study.
- I will ask your child if they want to receive extra help in math and obtain their assent to participate before selecting them for the study.

Possible Benefits of Participation:

- There is a possibility that the intervention will improve your child's mathematical computation skills, which are pre-cursors to higher-level mathematical skills. Since mathematical computations have been difficult for your child, participation in this study may provide your child with the help that he/she needs and make them feel more confident about their academic skills.
- The findings from my study may add to the research on academic interventions for students with academic difficulties in math.

Possible Risks:

- This study may involve frustration during mathematical calculations, though the calculations that your child will do can be completed in a brief period (5 minutes) during each session. If your child should express frustration or fatigue during the session, he/she may take a break during the session.
- Your child will also be reassured that he/she can withdraw from participation in the study at any time.
- Arrangements will be made for your child to meet with the school guidance counselor should they become stressed or overly frustrated from the study.

I may publish results of this study, but names of people, or any identifying characteristics, will not be used in any of the publications. If you would like a copy of the results of this study, please provide me with your name and address on the bottom of this form, and I will happily send you a copy in the future.

If you have any questions regarding this research, you can contact me at (917) 757-5307 or at gmanglani@gc.cuny.edu, or my advisor, Dr I. Jeltova, at (212) 817 – 8288 or at IJeltova@gc.cuny.edu. If you have any questions regarding your rights to having your child participate in this study, you can contact Kay Powell, IRB Administrator, The Graduate Center/City University of New York, (212) 817-7525 or at kpowell@gc.cuny.edu

Thank you for your child's participation in this study. I will give you a copy of this consent form for your records.

I have read and AGREE that my child can participate in the study.

____
Child's Name

____________________	_____	____________________	____
Participant's Guardian's Signature	Date	Investigator's Signature	Date

I have read and I DO NOT wish for my child to participate in the study.

____________________	_______	____________________	_____
Participant's Guardian's Signature	Date	Investigator's Signature	Date

Appendix F

Student Assent Form

Assent Form

Read to student:
"I would like to help you with your math calculations. Your teacher has told me that you have difficulty with your addition and subtraction skills. I would like to meet with you for 15 minutes every other day to practice your addition in a different way than in class to help you learn your addition and subtraction facts. Would you like to work with me? You don't have to do this; it's your choice. You can decide at any time that you don't want to work with me anymore, and there will be no consequences. This means if you start working with me, and don't' want to do it anymore, it's up to you and you will not get in trouble of any kind."

I have been asked if I would like to participate in the intervention:

______________________________ ____________

Student Name, Printed Date

______________________________ ____________

Geetal Benson Date

Principal Investigator

Appendix G

Data Collection Sheet – CCC Condition, Addition

Student #

Session-Addition	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Addition	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Addition	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Addition	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute
16					
17					
18					
19					
20					
21					
22					

Appendix H

Data Collection Sheet – CCC + PF Condition, Addition

Student # Session-Addition	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Addition	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Addition	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Addition	Total Time	Total # Digits Correct CCC + PF	Total # Digits Incorrect CCC + PF	Digits Correct Per Minute CCC + PF	Digits Incorrect Per Minute CCC + PF
16					
17					
18					
19					
20					
21					
22					

Appendix I

Data Collection Sheet – CCC + RW Condition, Addition

Student #

Session-Addition	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Addition	Total Time	Total # Digits Correct CCC + RW	Total # Digits Incorrect CCC + RW	Digits Correct Per Minute CCC + RW	Digits Incorrect Per Minute CCC + RW
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Addition	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Addition	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
16					
17					
18					
19					
20					
21					
22					

Appendix J

Data Collection Sheet – Control Condition, Addition

Student #

Session-Addition	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Addition	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
6					

Appendix K

Data Collection Sheet – CCC Condition, Subtraction

Student #

Session-Subtraction	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Subtraction	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Subtraction	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Subtraction	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute
16					
17					
18					
19					
20					
21					
22					

Appendix L

Data Collection Sheet - CCC + PF, Subtraction

Student #

Session-Subtraction	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Subtraction	Total Time	Total # Digits Correct CCC + PF	Total # Digits Incorrect CCC + PF	Digits Correct Per Minute CCC + PF	Digits Incorrect Per Minute CCC + PF
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Subtraction	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Subtraction	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
16					
17					
18					
19					
20					
21					
22					

Appendix M

Data Collection Sheet – CCC + RW Condition, Subtraction

Student #

Session-Subtraction	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Subtraction	Total Time	Total # Digits Correct CCC	Total # Digits Incorrect CCC	Digits Correct Per Minute CCC	Digits Incorrect Per Minute CCC
6					
7					
8					
9					
10					
11					
12					
13					
14					

Session-Subtraction	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
15					

Session-Subtraction	Total Time	Total # Digits Correct CCC + RW	Total # Digits Incorrect CCC + RW	Digits Correct Per Minute CCC + RW	Digits Incorrect Per Minute CCC + RW
16					
17					
18					
19					
20					
21					
22					

Appendix N

Data Collection Sheet – Control Condition, Subtraction

Student #

Session-Subtraction	Total Time	Total # Digits Correct Baseline	Total # Digits Incorrect Baseline	Digits Correct Per Minute Baseline	Digits Incorrect Per Minute Baseline
1					
2					
3					
4					
5					

Session-Subtraction	Total Time	Total # Digits Correct Baseline #2	Total # Digits Incorrect Baseline #2	Digits Correct Per Minute Baseline #2	Digits Incorrect Per Minute Baseline #2
6					

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