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THE STATIC AND DYNAMIC MEAN SCATTERING CROSS-SECTIONS FROM ROUGH SURFACES

by

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Abstract

The static and dynamic mean scattering

cross-sections from rough surfaces

by

Seung Kack Cha

Advisor: Professor George Eichmann

This dissertation investigates the phenomena of scattering from rough surfaces. Three scattering problems are considered; backscattering from an isotropic rough surface, scattering near grazing angle from a moving ocean surface and Doppler scattering from a stationary surface by moving source-observer. The rough surface is assumed to be a real, isotropic Gaussian surface height random process with Gaussian surface height and Gaussian-travelling wave-like surface height correlation function for the static and dynamic rough surfaces, respectively. The surface is considered to be an ideal conductor. The static and dynamic ensemble averaged mean scattering cross sections and ensemble and time averaged temporal frequency spectra are evaluated. The ensemble averages are obtained for arbitrary Gaussian surface height realizations. The results show good agreement with measred data.

CHAPTER I INTRODUCTION

1.1 Background

The problem of scattering from rough surfaces arises in the many areas; electromagnetic and acoustic wave propagations, optical scattering, radar and communication etc. Literature on experimental studies on rough surface scattering is abundantly available. Data has been collected on various rough surfaces i.e. land, sea, planetaries and man-made laboratory rough surfaces etc.[1,2,3,4]. The data are usually presented in terms of the scattering cross section. The scattering cross section gives information on the nature of the scattering surface and it is a function of the angles of wave incidence and the wave scattered.

The first theoretical investigation on the scattering from static rough surfaces was introduced by Rayleigh[5] followed by Rice[6]. Their approach is based on the perturbations of the rough surface and is now known as small surface perturbation method. Shortly after Rice, Isacovitch[7] introduced a new approach called the physucal optics approximation. In this approximation the scattering field is obtained from an image current that is induced by the incident field. The result of small surface perturbation analysis is generally regarded as a better interpretation of the physical data for relatively low degree of surface roughness while the result of the physical optics approximation is generally regarded as a better interpretation of the scattering data for fairly rough surfaces. The surface roughness depends on the wavelength of the source. For large frequency, a small change of the surface dimension will cause a large phase shift of the incident field yielding a large effect on the scattered field. Further, the physical optics results agree with the data for near normal angle of wave incidence while the perturbation approach predicts the data for near grazing angle of incidence even when the surface is very rough[2]. Neither theories explain the data for all angle of wave incidence.

A number of modifications, especially to the physical optics approximation, has been introduced in order to improve the validity of the model. These are non-Gaussian correlation functions[8,9], composite statistical surface[10,11], shadowing correction to the effective scattering surface[12,13,14], non-Gaussian joint distribution functions[15,16] and multiple scattering by the use of the ray double bounce[17] etc.

For the dynamic rough surface scattering, and the scattering by static rough surface due to a moving source-observer, both the Doppler power spectrum and dynamic scattering cross section is measured[19-24]. The Doppler power spectrum yeilds information on either the relative rough surface movements or the relative source movements. The theoretical investigation on the Doppler power spectrum are presented by various authors using physical optics[23,24] and perturbation-geometrical optics[21,25-29].

In this thesis the physical optics approximation together with ensemble averaging over a number of surface realizations is used to predict the mean square fields. In the next two sections, the pertinent facts on the physical optics approximation and the statistical averaging is discussed.

1.2 Physical Optics Approximation(PO)

This approach is based on approximating boundary conditions for the scattered wave equation by a magnetic image current. This approach has various names i.e. Kirchhoff boundary condition, image current theory, local tangent approximation and physical optics approximation. PO assumes that the field is negligible in the shadow surface region and the scattered field on the illuminated surface is twice induced current. By this approximation, the scalar scattered far field can be written, by the use of the Kirchhoff-Helmholtz integral[1] as

$$E = \frac{je^{jkR}}{2\pi R} \int_{S_o} (R_F \underline{P}_1 - \underline{P}_2)\cdot \underline{a}_n e^{j\underline{P}_1 \cdot \underline{r}} \, dS_o \qquad (1.2\text{-}1)$$

where R_F are Fresnel reflection coefficients which depend on the polarization and angles of wave incidence and electrical properties of the illuminating surface and

$$\underline{P}_1 = \underline{k}_i - \underline{k}_s \qquad (1.2\text{-}2)$$

$$\underline{P}_2 = \underline{k}_i + \underline{k}_s \qquad (1.2\text{-}3)$$

with $\underline{k}_i$ and $\underline{k}_s$ are incident and scattered wave propagation vectors. R is a distance from origin to the observation point, $\underline{r}$ is the local surface radius vector

$$\underline{r} = x\underline{a}_x + y\underline{a}_y + \zeta(x,y)\underline{a}_z \qquad (1.2\text{-}4)$$

where $\zeta(x,y)$ is a local surface height measured from x-y plane which is the mean plane and $\underline{a}_n$ is the local surface unit normal vector and S_o is the illuminated scattering area. The scattered scalar far field E is a spherical wave with an amplitude that is proportional to the transform of the effective optical surface current distribution on the scattering surface. The vector form of the Kirchhoff-Helmholtz integral is known as Stratton-Chu integral[30]. However modified Stratton-Chu integral, the Silver integral[31], is frequently used. The vector scattered field due to the Silver integral has a cross-polarized field term[32]. For backscattering from ideal conductor this term disappears. In fact all integral expressions, the Kirchhoff-Helmholtz, the Silver and Kodis[33] reduce to an same form for backscattering from an ideal conducting surface. Parkins[23] also derived identical form for backscattering from time dependent scalar Kirchhoff integral for his acoustic wave scattering investigation from rigid rough boundary.

1.3 Statistical Averaging of the Scattered Fields

To evaluate the PO intgral the functional representation of the rough surface and its directional partial derivatives must be available. The rough surfaces are described by their statistical properties. In this dissertation the surface is assumed to be a Gaussian stochastic surface height process. The scattering field is also a stochastic process since the field is a function of the random surface and its partial derivatives. Since the data is given in terms of the scattering cross section which is proportional to the first two statistical moments of the scattered field it is necessary to know the detailed statistics of the surface. Hoffman[34] has evaluated the moments of the field, which are moments of Wiener integrals, using the Karhunen-Loeve expansion theorem. This theorem states that the random process $\zeta(r)$ has an orthogonal expansion in the interval L

$$\zeta(r) = \sum_m \lambda_m z_m q_m(r) \qquad |r| \in L \tag{1.2-5}$$

where z_m's are uncorrelated, orthogonal random variables

$$\langle z_m z_n^* \rangle = \delta_{mn} \tag{1.2-6}$$

and

$$\int_{|r| \in L} q_m(r) q_n^*(r)\, dr = \delta_{mn} \tag{1.2-7}$$

if and only if $|\lambda_m|^2$ and $q_m(r)$ are the eigenvalues and eigenfunctions of the integral equation

$$\int_{|r| \in L} R(r_1, r_2) q_m(r_2)\, dr_2 = |\lambda_m|^2 q_m(r_1) \tag{1.2-8}$$

where $R(r_1, r_2)$ is the autocorrelation function of $\zeta(r)$. This theorem decomposes the random process into an infinite set of random variables Z_m and an infinite set of deterministic function $q_m(r)$. The various partial derivatives as well as integral are now specified in terms of known, i.e. deterministic functions. The indicated integrations and differentiations can now be formally performed. For a Gaussian process, with arbitrary correlation function, the resulting integrals can then formally be evaluated. The results of this manipulations are available in Hoffman's original paper.

1.4 Scope of the Dissertation

Based on the conditions of the source radiation and the rough surface with respect to the temporal variation, the three scattering models are formulated. These are

A. The scattering observation by a fixed source-observer from a static rough surface.

B. The scattering observation by a fixed source and observer from a dynamic rough surface.

C. The scattering observation by a moving source-observer from a static rough surface.

Thefield scattering is based on a physical optics approximation and the rough surface is assumed to be a real, isotropic Gaussian process with normalized Gaussian correlation function. These assumptions permit the complete evaluations of the physical optics intergral.

In the first model, the topic of Chapter II, the mean backscattering cross section is evaluated. The main contribution of this investigation is the numerical evaluations of the diffuse, non-coherent mean backscattering cross section valid for all angles of wave incidence, for both smooth and rough surface and for wide range of the source frequency.

In the second model, the topic of Chapter III, the temporal frequency spectrum is evaluated in terms of partial Doppler frequency spectrum. The scattered field is calculated based on an extended Parkins[32] model while the surface is an isotropic, Gaussian random process with correlated function suggested by Medwin and Clay[24]. The results give complete information of the partial mean scattering cross section, partial Doppler spectral components with Doppler frequency shifts and spreads which are equivalent to the higher order Bragg resonant components. The partial components are a function of the angles of wave incidence near grazing angle of indicence and scattering the source frequency, surface roughness and the velocity and direction of the moving surface. Finally, a second order surface correlation function is proposed and analyzed. This second order model better predicts the measured data if the source wavelength is shorter than 20 m.

In the last model, the topic of Chapter IV, the mean Doppler spectrum is evaluated. The return field from the rough surface, which is assumed to be a real, isotropic Gaussian process, becomes not only temporal non-stationary but also non-isotropic character due to the source velocity. The non-stationary and non-isotropic characters of the PO integrand are main contributions to the mean Doppler spectral spread, while the stationary character determines the magnitude coefficients of the spectrum. The Doppler spectrum spread is due to the surface roughness and the direction cosine of the source velocity.

In the final Chapter, the results are summerized and some additional work is suggested.

CHAPTER II MEAN BACKSCATTERING CROSS SECTION OF AN IDEALLY CONDUCTING ROUGH SURFACE FOR ARBITRARY ANGLE OF INCIDENCE

2.1 Introduction

When a plane wave impinges on an ideal, flat conductor the wave reflects according to Snell's law in a single direction. Backscattering is therefore only possible for normal angle of incidence. However, when the reflecting surface is rough the wave reflects in various directions and backscattering is possible for all angles of incidence. The terms specular and non-specular scatterings have been coined to denote those angular scattering components that are due to reflection from the mean surface which is assumed to be flat, according to geometrical optics and angular directions away from geometrical optical direction. The subject of this paper is the computation of the non-specular angular scattering.

There are a number of statistical parameters that characterize a rough surface. Two such parameters are length parameters; the height variance and the height correlation length. The variance describes the degree of the surface height roughness while the correlation length yields information on horizontal "periodicity". Since the measured data, on the reflection of electromagnetic waves from rough surface, is usually presented in terms of normalized mean backscattering cross section (MBC), the evaluation of the statistical second order moment of the field which is proportional to the MBC will be performed. For the purpose of physical interpretation it is convenient to divide the MBC near normal angle of incidence into two component; a quasi-coherent and a diffuse component. The quasi-coherent component is dominant normalized geometrical optic MBC and the diffuse component is the additional contribution to the dominant quasi-coherent MBC. Near grazing angle of incidence it is convenient to decompose the MBC into a coherent and an incoherent component. The coherent component is due to the coherent field contribution while the incoherent component is the additional effect on the MBC.

Data on MBC's are available for a variety of physical surface, such as backscattering from terrains[5-7], reflection from planetary surface[9-11], such as Mars and Venus, and reflections from optical surfaces[12,13]. There are two electromagnetic models that allow the calculation of the MBC; a physical optics and a surface perturbation model. Further each model requires additional assumptions on the nature of the surface statistics for the MBC calculation. In the physical optics model the current on the surface of the scatterer is approximated by an image current abtained using a local infinite tangent plane approximation. The early statistical models assume a Gaussian statistical surface height distribution with height correlation function that is also of Gaussian form. This approach, first introduced by Isacovitch[2], has been reviewed in early[3] as well as a number of recent publications[4,33]. Although this model correctly predicts specular backscattering for diffuse scattering there are discrepancies between the measured data and the theoretical predictions[4,7,33]. A Number of modification's have been introduced to the height correlation function in order to be able to predict the diffuse scattering. One approach uses an exponential correlation function. The improvement here is due to the longer correlation lengh of exponential damping as compared to the Gaussian correlation function[24,25]. It has been pointed out, however, that an exponential surface height correlation function can not represent a physical surface[7,32]. To correct this deficiency a number of authors suggested to use a combination of correlation functions, a Gaussian correlation function for short correlation distances and an exponential correlation function for long correlation distances[7,24,36]. In the same spirit, composite statistical surface model has been discussed for a single physical surface[14,15]. In this model two or more independent statistical surface height variables are specified. A large-scale surface height variable produces a strong quasi-coherent backscattering with small additional diffuse backscattering while small-scale surface height variable produces a diffuse backscattering that adds to the diffuse scattering produced by the large-scale surface height variable. Physically, the large-scale surface height variable represents gross feature of the surface like mountains or large ocean waves, while small-scale surface height variable represents detailed features of surfaces, like gravel, stones or ocean ripples. It is possible to postulate two independent statistical variables one which is the surface height and the other is the surface slope[25]. Since the slope and the height of the surface represent the same physical surface, correction terms, called shadowing functions that reduce the effective scattering area for a wave with a given oblique angle of incidence, are needed. For a normal angle of incidence the effective area is the physical scattering area, however, for oblique angle of incidence the area is rediced. This area reduction increases the effective scattering cross section leading to an increased diffuse scattering component. A number of shadowing functions are available[16-20]. It is also possible to introduce non-Gaussian joint distribution functions[25,26]. However, such distribution function lead to mathematical difficulties and complicate physical interpretations of the results. It is also possible to improve the physical optics approximation. These improvements depend on the introduction of either an edge effect[1,3] or the inclusion of additional slope dependent terms[21,22].

The surface perturbation model analyzed by Rice[8] was originally developed by Rayleigh[29] for an acoustical surface scattering. In this method the equation representing the surface is expanded in a spatial harmonic Fourier series with ramdom amplitude Fourier coefficients. These coefficients characterize the roughness of the surface. In the Rayleigh method, the scattering field is decomposed into a spatial harmonic series with unknown reflection coefficients for each rectangular component of the field. Using the divergence equation and boundary conditions and the spatial harmonic series for statistical surface in a small slope approximation, the averaged reflection coefficients are evaluated.

In this paper, the electromagnetic backscattered field is evaluated using the physical optics approximation with statistical surface realizations that are sample functions of a Gaussian process. A Gaussian random height process is equivalent to a very large number of joint height probability distribution function. The resulting integral expression for backscattered field is a Wiener integral. The mean and mean square field have been formally evaluated by Hoffman[1]. Using some of expressions derived by Hoffman, we evaluated the MBC for isotropic surface realizations for all angles of wave incidence,

In section two, we define of electromagnetic scattering model and the parameters of the rough surface. The evaluation of the MBC for all angles of wave incidence is done in section three. Finally, in section four we summerize the results.

2.2 The Rough Surface Backscattering Model

It is known that the backscattering electromagnetic field due to a plane wave incident on an arbitrary condcting surface can be cast in an integral form. This integral can be obtained either by proceeding from the Kirchhoff-Helmholtz integral[3], which is a mathematical statement of the Huygens' principle, or by postulating an optical surface current induced by the incident field[37], or by starting diretly from the free space dyadic Green function[28]. The result of any these caculations for the backscattered electric far-field amplitude due an unit amplitude plane wave incidence is

$$E(P) = \frac{je^{jkR}}{2\pi R} \int_{S_o} (\underline{k}_i \cdot \underline{a}_n)\, e^{-j2\underline{k}_i \cdot \underline{r}} dS_o \qquad (2.2\text{-}1)$$

where $k = \omega/c$ is the free space wave number with ω as the radian frequency of the source and c is the velocity of the light in free space. In Eq.(2.2-1), R is a distance from the origin to the point of observation P, $\underline{r}$ is the local surface radius vector

$$\underline{r} = x\,\underline{a}_x + y\,\underline{a}_y + \zeta(x,y)\,\underline{a}_z \qquad (2.2\text{-}2)$$

$\underline{a}_n$ is the local surface normal unit vector

$$\underline{a}_n = -\left\{ \frac{\frac{\partial \zeta(x,y)}{\partial x}\underline{a}_x + \frac{\partial \zeta(x,y)}{\partial y}\underline{a}_y - \underline{a}_z}{\left[1 + \left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2\right]^{\frac{1}{2}}} \right\} \qquad (2.2\text{-}3)$$

$\underline{k}_i$ is the incident wave number vector

$$\underline{k}_i = k \sin u \cos v \underline{a}_x + \sin u \sin v \, \underline{a}_y - \cos u \, \underline{a}_z \qquad (2.2\text{-}4)$$

with u and v as the angles of elevation and azimuth of the incident plane wave, and S_o is the effective illuminated area (See Fig.2-1). It is assumed the field is time harmonic of the form $\exp(-j\omega t)$ where $j=(-1)^{\frac{1}{2}}$. By projecting the optical differential surface area dS_o on the x-y plane, the backscattered field Eq.(2.2-1) is

$$E(P) = \frac{-jke^{jkR}}{2\pi R} \int_S F \, e^{-j2\underline{k}_i \cdot \underline{r}} dx \, dy \qquad (2.2\text{-}5)$$

with

$$-F = (\underline{k}_i \cdot \underline{a}_n)/k(\underline{a}_z \cdot \underline{a}_n) = \sin u \cos v \frac{\partial \zeta}{\partial x} + \sin u \sin v \frac{\partial \zeta}{\partial y} + \cos u \quad (2.2\text{-}6)$$

and S the effective illuminated area on the x-y plane. Eq.(2.2-5) is the final form of the backscattered electric field suitable for statistical computations.

The backscattered EM field is a random process since it is dependent on the surface height $\zeta(x,y)$ which is itself a random process, The random surface height $\zeta(x,y)$ is assumed to be a real Gaussian random process of two independent variables x and y. The process $\zeta(x,y)$ is characterized by its mean value $\langle \zeta(x,y) \rangle = o$, that is the z = o is the mean plane, variance $\sigma^2 = \langle \zeta^2(x,y) \rangle$, independent of the location of the surface, and surface height correlation function $C(x,x',y,y') = \langle \zeta(x,y) \, \zeta(x',y') \rangle / \sigma^2$ where $\langle \cdot\cdot \rangle$ is the ensemble average of the process. The surface height correlation function $C(x,x',y,y')$ is assumed to be radially isotropic, that is, $\zeta(x,y)$ is a wide-sense stationary

Gaussian process with correlation function dependent on the radial variable $t_1=[(x-x')^2+ (y-y')^2]^{\frac{1}{2}}$ only. Our interest is in the mean magnitude square of the electric field since it is the quantity that is proportional to the MBC. Eq.(2.2-6), the mean magnitude square of the electric field is

$$<EE^*> = A\int_S \int_{S'} I\, e^{-j\,[k_x(x-x')+k_y(y-y')]} ds\, ds' \qquad (2.2\text{-}7)$$

where

$$I = \cos^2 u\, M_1 + \sin u \cos u(\cos v\, M_2 + \sin v\, M_3) + \sin^2 u \sin v \cos v$$
$$(M_4 + M_5) + \sin^2 u(\cos^2 v\, M_6 + \sin^2 v\, M_7) \qquad (2.2\text{-}8$$

$$A = \left(\frac{k}{2\pi R}\right)^2 \qquad (2.2\text{-}9)$$

and

$$M_1 = < e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}10)$$

$$M_2 = < \left(\frac{\partial\xi}{\partial x} + \frac{\partial\xi}{\partial x'}\right) e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}11)$$

$$M_3 = < \left(\frac{\partial\xi}{\partial y} + \frac{\partial\xi}{\partial y'}\right) e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}12)$$

$$M_4 = < \frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial y'} e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}13)$$

$$M_5 = < \frac{\partial\xi}{\partial y}\frac{\partial\xi}{\partial x'} e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}14)$$

$$M_6 = < \frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial x'} e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}> \qquad (2.2\text{-}15)$$

$$M_7 = < \frac{\partial\xi}{\partial y}\frac{\partial\xi}{\partial y'} e^{j2k \cos u[\xi(x,y) - \xi(x',y')]}>. \qquad (2.2\text{-}16)$$

The M functions are known from the work of Hoffman[1] on scattering of EM waves from a rough surface. For a random, stationary Gaussian surface, Hoffman was able to formally evaluate the M functions using the Karhunen-Loeve expansion theorem. The result of this calculation is

$$M_1 = \exp\{-p^2(1-C)\} \qquad (2.2\text{-}17)$$

$$M_2 = -j\sigma p\left(\frac{\partial C}{\partial x} - \frac{\partial C}{\partial x'}\right) M_1 \qquad (2.2\text{-}18)$$

$$M_3 = -j\sigma p\left(\frac{\partial C}{\partial y} - \frac{\partial C}{\partial y'}\right) M_1 \qquad (2.2\text{-}19)$$

$$M_4 = \sigma^2\left(\frac{\partial^2 C}{\partial x \partial y'} + p^2 \frac{\partial C}{\partial x}\frac{\partial C}{\partial y'}\right) M_1 \qquad (2.2\text{-}20)$$

$$M_5 = \sigma^2\left(\frac{\partial^2 C}{\partial x' \partial y} + p^2 \frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}\right) M_1 \qquad (2.2\text{-}21)$$

$$M_6 = \sigma^2\left(\frac{\partial^2 C}{\partial x \partial x'} + p^2 \frac{\partial C}{\partial x}\frac{\partial C}{\partial x'}\right) M_1 \qquad (2.2\text{-}22)$$

$$M_7 = \sigma^2\left(\frac{\partial^2 C}{\partial y \partial y'} + p^2 \frac{\partial C}{\partial y}\frac{\partial C}{\partial y'}\right) M_1 \qquad (2.2\text{-}23)$$

where

$$p = 2k\sigma\cos u \qquad (2.2\text{-}24)$$

For additional details the calculation can be found in the Ref.1. Knowing that integrand of Eq.(2.2-7) is function of (x-x') and (y-y') only, the integral can be simplified by the coordinate transformation

$$\bar{x} = x - x' \qquad (2.2\text{-}25)$$

$$\bar{y} = y - y' \qquad (2.2\text{-}26)$$

The result of this transformation is

$$\langle EE^* \rangle = AS\int\int_{-\infty}^{\infty} T_1 T_2 I(\bar{x},\bar{y}) e^{-j2(k_x\bar{x}+k_y\bar{y})} d\bar{x}d\bar{y} \qquad (2.2\text{-}27)$$

where T_1 and T_2 are the aperture functions which for rectangular effective are of illumination are

$$T_1 = \begin{cases} 1-|\bar{x}|/2X & |\bar{x}| = 2X \\ 0 & |\bar{x}| = 2X \end{cases} \qquad (2.2\text{-}28)$$

$$T_2 = \begin{cases} 1-|\bar{y}|/2Y & |\bar{y}| = 2Y \\ 0 & |\bar{y}| = 2Y \end{cases} \qquad (2.2\text{-}29)$$

where 2X and 2Y are the lengths and S is the effective area as 4XY. The function $I(\bar{x},\bar{y})$ takes on significant values near the origin of $\bar{x}$ and $\bar{y}$ due to the exponential damping of the correlation coefficient $C(\bar{x},\bar{y})$. Thus area effects are negligible when the aperture dimension is large compared to the correlation distance L, and therefore we approximate the integral

$$\langle EE^*\rangle = AS\int\int_{-\infty}^{\infty} I(\bar{x},\bar{y})e^{-j2(k_x\bar{x}+k_y\bar{y})}d\bar{x}d\bar{y} \qquad (2.2\text{-}30)$$

Because the surface statistics is isotropic, the correlation function depends on the radial distance t only, where t and w are the cylindrical coordinate variables

$$t^2 = \bar{x}^2 + \bar{y}^2 \qquad (2.2\text{-}31)$$

$$w = \tan^{-1}\bar{x}/\bar{y} \qquad (2.2\text{-}32)$$

We next transform the integral Eq.(2.2-30) into the new coordinate system. The phase term in the integrand is

$$J_w = e^{-jqt\sin(w+v)} \qquad (2.2\text{-}33)$$

where

$$q = 2k\sin u \qquad (2.2\text{-}34)$$

In Appendix A, the M functions are transformed to radial coordinate system resulting the following transformed M functions

$$M_1 = \exp\{-p^2[1-C(t)]\} \qquad (2.2\text{-}35)$$

$$M_2 = -j2\sigma p \sin w \frac{dC}{dt} M_1 \quad (2.2\text{-}36)$$

$$M_3 = -j2\sigma p \cos w \frac{dC}{dt} M_1 \quad (2.2\text{-}37)$$

$$M_4 = M_5 = -\tfrac{1}{2}\sigma^2 \sin 2w \; L_1 M_1 \quad (2.2\text{-}38)$$

$$M_6 = \tfrac{1}{2}\sigma^2 \{L_1 \cos 2w - L_2\} \; M_1 \quad (2.2\text{-}39)$$

$$M_7 = \tfrac{1}{2}\sigma^2 \{-L_1 \sin 2w - L_2\} \; M_1 \quad (2.2\text{-}40)$$

where

$$L_{1,2} = \frac{d^2C}{dt^2} \mp \frac{1}{t}\frac{dC}{dt} + p^2\left(\frac{dC}{dt}\right)^2 \quad (2.2\text{-}41)$$

The integral Eq.(2.2-30) can be now written in cylindrical coordinates as

$$\langle EE^*\rangle = AS\int_0^\infty \int_0^{2\pi} I(t,w)\; e^{-jqt\sin(w+v)} t\,dw\,dt \quad (2.2\text{-}42)$$

The quantity of interest is the MBC

$$\bar{\sigma} = \lim_{R\to\infty}[4\pi R^2 \langle EE^*\rangle / |E_i|^2] \quad (2.2\text{-}43)$$

where $|E_i|^2$ is magnitude square of the amplitude of the incident electric field. In the next section the mean magnitude square of the scattered field will be evaluated for an surface whose correlation function of the height is isotropic Gaussian function with correlation distance L

$$C(t) = \exp(-t^2/L^2) \quad (2.2\text{-}44)$$

It is known that the Gaussian correlation function leads to a reasonable representation of variety of physical surfaces. The random surface is characterized by two lengths, the vertical and horizontal roughness parameter σ and L respectively. It will be convenient to deal with normalized parameters

$$\sigma_n = 2\sigma/L \tag{2.2-45}$$

$$\ell = k\sigma = 2\pi\sigma/\lambda \tag{2.2-46}$$

where σ_n is the normalized roughness called rms slope and ℓ is the Rayleigh parameter and λ is the wavelength of the source. We also find it convenient to introduce a normalized MBC

$$\sigma^{o} = \bar{\sigma}/S \tag{2.2-47}$$

where S is the scattering area. The degree of the surface roughness is characterized by the Rayleigh parameter which for a rough surface is

$$\ell > 1$$

and for a smooth surface it is

$$\ell < 1$$

Obviously λ should be small for physical optics approximations to be valid

2.3 The Evaluation of the Mean Backscattering Cross Section(MBC)

To evaluate the MBC we note, from Eqs.(2.2-42)-(2.2-47), that the angle and radial integrations separate. Separating the t and w integrals, we have

$$\sigma^o = \frac{k^2}{2\pi}\int_0^\infty dt\ t\ \exp^{-p^2(1-C)}\Big[(\cos^2u-\tfrac{1}{2}\sigma^2L_2\sin^2u)\int_0^{2\pi} J_w dw$$
$$- j\sigma p\sin 2u\frac{dC}{dt}\int_0^{2\pi}\sin(w+v)J_w dw \qquad (2.3\text{-}1)$$
$$- \tfrac{1}{2}\sigma^2L_1\{\sin^2u\sin 2v\int_0^{2\pi}\sin 2wJ_w dw - \cos^2u\cos 2v\int_0^{2\pi}\cos 2wJ_w dw\}\Big]$$

The w-integrations are now identified as special cases of the Richard-Wolf identities[35]

$$\int_0^{2\pi}\begin{matrix}\cos\\ \sin\end{matrix}(nw)e^{-jr\cos(w-\gamma)}dw=(-j)^n 2\pi\ J_n(r)\begin{matrix}\cos\\ \sin\end{matrix}(n\gamma) \qquad (2.3\text{-}2)$$

where $J_n(r)$ are the Bessel function of the first kind of order n. From Eq.(2.3-2) we find for the three cases of the w-integration in Eq.(2.3-1), noting J_w defined in Eq.(2.2-33)

$$\int_0^{2\pi} J_w dw = 2\pi\ J_o(qt) \qquad (2.3\text{-}3)$$

$$\int_0^{2\pi} \sin(v+w)J_w dw = -j2\pi\ J_1(qt) \qquad (2.3\text{-}4)$$

$$\int_0^{2\pi} \begin{matrix}\cos\\ \sin\end{matrix}(2w)\ J_w dw = \begin{matrix}-\sin\\ +\cos\end{matrix}(2v)\ 2\pi\ J_2(qt) \qquad (2.3\text{-}5)$$

Using Eqs.(2.3-3)-(2.3-5), Eq.(2.3-1) simplifies as

$$\sigma^o=k^2\int_0^\infty dt\ te^{-p^2(1-C)}\Big[(\cos^2u-\tfrac{1}{2}\sigma^2L_2\sin^2u)J_o(qt)-\sigma p\sin 2u\frac{dC}{dt}J_1(qt)$$
$$+ \tfrac{1}{2}\sigma^2L_1\sin^2u\, J_2(qt)\Big] \qquad (2.3\text{-}6)$$

As expected σ^o does not depend on the azimuthal angle of incidence. To complete the radial integration, It is necessary to approximate the term $\exp[-p^2(1-C)]$. There are two cases of major interest, the case when the direction of wave incidence is near normal and near grazing angles.

First we will consider the near normal angle of incidence. For a rough surface the correlation length is small and the correlation function has appreciable values only for t close zero. Therefore we expand the Gaussian surface height correlation function in a Taylor series about the origin and truncate this series after the first two terms

$$e^{-p^2(1-C)} \approx e^{-(pt/L)^2} \tag{2.3-7}$$

$$L_1 \approx (1+p^2)(2t/L^2)^2-(1+2p^2)(2t^2/L^3)^2+(1+3p^2)2(t^3/L^4)^2 \tag{2.3-8}$$

$$L_2 \approx -4/L^2+(2+p^2)(2t/L^2)^2-(3+4p^2)2(t^2/L^3)^2+(\tfrac{1}{3}+p^2)(2t^3/L^4)^2 \tag{2.3-9}$$

Substituting $L_{1,2}$ and the approximation for the exponential term into Eq.(2.3-6), σ^o becomes

$$\begin{aligned}\sigma^o = k^2[\cos^2u\ I_{00} &+ \sin 2u\{2\sigma pL^{-2}I_{11}-2\sigma pL^{-4}I_{31}+\sigma pL^{-6}I_{51}\} \quad (2.3\text{-}10)\\ &+ \sin^2u\{2\sigma^2L^{-2}I_{00}-2\sigma^2L^{-4}(2+p^2)I_{20}\ \sigma^2L^{-6}(3+4p^2)I_{40}\\ &+ 2\sigma^8L^{-8}I_{60}+2\sigma^2L^{-4}I_{22}-2\sigma^2L^{-6}(1+2p^2)I_{42}+\sigma^2L^{-8}(1+3p^2)I_{62}\}]\end{aligned}$$

where I_{mn} are

$$I_{mn} = \int_0^\infty t^{m+1}J_n(bt)\ e^{-at^2}dt \tag{2.3-11}$$

The integrals of Eq.(2.3-12) are Weber exponential integrals[31] of first kind and derivatives with respect to a. The Weber integrals are

$$I_{mm} = \int_0^\infty t^{m+1} J_m(bt) e^{-at^2} dt = b^m (2a)^{-(m+1)} e^{-(b^2/4a)} \qquad (2.3\text{-}12)$$

Substituting Eq.(2.3-12) and appropriate derivatives in Eq.(2.3-10), we have

$$\sigma^o = \frac{L^2}{4\sigma^2}\left[g_o(u) + \frac{1}{k^2\sigma^2} g_1(u) + \frac{1}{k^4\sigma^4} g_2(u) + \frac{1}{k^6\sigma^6} g_3(u) + \cdots \right] \qquad (2.3\text{-}13)$$

where

$$g_o(u) = G_o(u)[1+2\tan^2 u+\tan^4 u+\cdots] \qquad (2.3\text{-}14)$$

$$g_1(u) = G_o(u)\tan^2 u\sec^2 u[a_0'+a_1'\tan^2 u+a_2'\tan^4 u+\cdots] \qquad (2.3\text{-}15)$$

$$g_2(u) = G_o(u)\tan^2 u\sec^4 u[b_0'+b_1'\tan^2 u+b_2'\tan^4 u+\cdots] \qquad (2.3\text{-}16)$$

$$g_3(u) = G_o(u)\tan^2 u\sec^6 u[c_0'+c_1'\tan^2 u+c_2'\tan^4 u+\cdots] \qquad (2.3\text{-}17)$$

and

$$G_o(u) = \exp\left(-\frac{L^2}{4\sigma^2}\tan^2 u\right) \qquad (2.3\text{-}18)$$

The coefficients in Eqs.(2.3-14)-(2.3-17) are

$$a_o' = \sigma^2/L^2 - 1, \qquad a_1' = L^2/8\sigma^2 - 7/8, \qquad a_2' = L^2/8\sigma^2 \qquad (2.3\text{-}19)$$

$$b_o' = 15/2^6 + 3L^2/2^6\sigma^2, \quad b_1' = 3L^4/2^6\sigma^4 + L^2/2^6\sigma^2; \quad b_3' = L^6/2^6\sigma^6 - 19L^4/2^{11}\sigma^4 \qquad (2.3\text{-}20)$$

$$c_o' = \sigma^2/2^4L^2 - 2^{-7}, \qquad c_1' = L^2/2^6\sigma^2 + 5/3\cdot 2^4, \quad c_2' = L^2/2^{10}\sigma^2 \qquad (2.3\text{-}21)$$

Using the normalized length parameters defined in Eqs.(2.2-45) and (2.2-46), we have final form for MBC

$$\sigma^o = \sigma_g^o + \ell^{-2}\sigma_1^o + \ell^{-4}\sigma_2^o + \ell^{-6}\sigma_3^o + \cdots \qquad (2.3\text{-}22)$$

where

$$\sigma_g^o = \sigma_n^{-2}\, e^{-\delta^2}[1+2\tan^2 u+\tan^4 u+\cdots] \qquad (2.3\text{-}23)$$

$$\sigma_1^o = B\sec^2 u[a_0+a_1\tan^2 u+a_2\tan^4 u+\cdots] \quad (2.3\text{-}24)$$

$$\sigma_2^o = B\sec^4 u[b_0+b_1\tan^2 u+b_2\tan^4 u+\ldots] \quad (2.3\text{-}25)$$

$$\sigma_3^o = B\sec^6 u[c_0+c_1\tan^2 u+c_2\tan^4 u+\ldots] \quad (2.3\text{-}26)$$

and

$$\delta = \frac{\tan u}{\sigma_n} \quad (2.3\text{-}27)$$

$$B = \delta^2 e^{-\delta^2} \quad (2.3\text{-}28)$$

with normalized coefficients as

$$a_0=\tfrac{1}{4}\sigma_n^2-1 \qquad a_1=1/2\sigma_n^2+1/2^4\sigma_n^2 \qquad a_2=1/2\sigma_n^2 \quad (2.3\text{-}29)$$

$$b_0=3/2^4\sigma_n^2+15/2^6 \qquad b_1=3/2^2\sigma_n^4+1/2^4\sigma_n^2 \qquad b_2=1/\sigma_n^2 \quad (2.3\text{-}30)$$

$$c_0=\sigma_n^2/2^6-1/2^7 \qquad c_1=1/2^4\sigma_n^2+5/3\cdot 2^9 \qquad c_2=1/2^8\sigma_n^4 \quad (2.3\text{-}31)$$

The MBC near normal angle of incidence is an asymptotic series in term of the Rayleigh parameter ℓ. The first term σ_g^o is the geometric MBC while the additional terms are the high frequency correction to the geometric MBC. The geometric MBC can be separated into two terms

$$\sigma_g^o = \sigma_{go}^o + \sigma_{g1}^o \quad (2.3\text{-}32)$$

with

$$\sigma_{go}^o = \frac{1}{\sigma_n^2\cos^2 u} e^{-\delta^2} \quad (2.3\text{-}33)$$

and

$$\sigma_{g1}^o = \frac{1}{\sigma_n^2} e^{-\delta^2}[\tan^2 u + \tan^4 u + \ldots] \quad (2.3\text{-}34)$$

where we have been used the identity

$$1+\tan^2 u = \sec^2 u \quad (2.3\text{-}35)$$

The first term is the quasi-coherent MBC[21,22,30] while the second term represents the additional geometrical backscattering. The additional geometrical terms and high frequency corrections are the diffuse backscattering terms(DBS). In Figure 2.2-2.6 we plot the DBS contribution to the MBC vor ℓ equal to 1.5 and various values of σ_n. Of course, the DBS is zero for normal angle of incidence.

We now consider scattering near grazing angle of incidence. For grazing angle of incidence the approximation of Eq.(2.3-7) is not effective because $p = 2\ell \cos u$ is nearly zero. For small values of p, the approximation is

$$e^{-p^2(1-C)} = e^{-p^2}(1 + p^2 C) \qquad (2.3\text{-}36)$$

Eq.(2.3-36) is reasonable approximation for all angles of wave incidence for a smooth surface because p is small since the Rayleigh parameter ℓ is small. We find it convenient to separate to the mean square field into a coherent and an incoherent scattering component[3]

$$\langle EE^* \rangle = \langle E \rangle \langle E^* \rangle + \mathrm{covar}\{E\} \qquad (2.3\text{-}37)$$

where $\langle E \rangle \langle E^* \rangle$ is the coherent and the covariance of E. is the incoherent part of the mean square field.

From Eq.(2.2-5), we have for the coherent normalized MBC

$$\sigma^o_{coh} = \cos^2 u \; M_o^2 k^2 |E_c|^2 \qquad (2.3\text{-}38)$$

where

$$M_o = \langle e^{\pm j2k\cos u\, \zeta(x,y)} \rangle = e^{-\frac{1}{2}p^2} \qquad (2.3\text{-}39)$$

and $|E_c|^2$ is the coherent pattern function for a flat surface. The coherent pattern function for a rectangular illuminated area is the square of the sampling function. The sampling function argument depends on the patch length. M_o is a coherence factor (coherent reflectance) that smears the coherent pattern function[12]. The coherent MBC is large near normal angle of wave incidence and small near grazing angle of wave incidence.

The incoherent normalized MBC is obtained by subtracting from the total MBC the coherent MBC. Performing the subtraction, the result in cylindrical coordinates is

$$\sigma^o_{inc} = \frac{k^2}{2\pi} \int_0^{\infty}\int_0^{2\pi} (I - \cos^2 u e^{-p^2}) J_w t \, dt dw \qquad (2.3\text{-}40)$$

The angular integrals are identical to the angular integrals obtained for near normal angle of incidence(See Eq.(2.3-6)) and therefore

$$\sigma^o_{inc} = k^2 \int_0^{\infty} e^{-p^2} t [p^2 C\cos^2 u J_o(qt) + (1+p^2C)\{-\tfrac{1}{2}\sigma^2 L_2 \sin^2 u J_o(qt)$$
$$-\sigma p \sin 2u \frac{dC}{dt} J_1(qt) + \tfrac{1}{2}\sigma^2 \sin^2 u\, L_1 J_2(qt)\}] dt \qquad (2.3\text{-}41)$$

Using the exact expressions for $L_{1,2}$

$$L_1 = \frac{4t^2}{L^4}(1+p^2C)C \qquad (2.3\text{-}42)$$

$$L_2 = -\frac{4}{L^2} + \frac{4t^2}{L^4}(1+p^2C)C \qquad (2.3\text{-}42)$$

the incoherent MBC becomes

$$\sigma^o_{inc} = k^2 e^{-p^2} [2\sigma p L^{-2} \sin 2u I_{211} + 2\sigma^2 L^{-2} \sin^2 u I_{110} + 2\sigma^2 L^{-4} \sin^2 u I_{310}$$
$$+ p^2 \{\cos^2 u I_{110} + 2\sigma p L^{-2} \sin 2u I_{221} + 2\sigma^2 L^{-2} \sin^2 u I_{120}$$

$$+4\sigma^2L^{-4}\sin^2u(-I_{320}+ I_{322})\}+ p^4 2\sigma^2L^{-4}\sin^2u(-I_{330}+I_{332})] \qquad (2.3\text{-}43)$$

where $I_{mn\ell}$ are the integrals

$$I_{mn\ell}=\int_0^\infty t^m\, e^{-(nt^2/L^2)}\, J_\ell(qt)\, dt \qquad (2.3\text{-}44)$$

The integral of Eq.(2.3-44) can be evaluated using the Weber integrals of Eq.(2.3-12). Substituting Weber integrals and its derivatives into Eq.(2.3-43), we obtained the incoherent MBC as

$$\sigma^o_{inc}=2L^2k^4\sigma^2e^{-p^2}[(d'_{11}+d'_{12})\,\bar{W}_4+ \sigma^2k^2d'_2\bar{W}_8+ \sigma^4k^4d'_3\bar{W}_{12}] \qquad (2.3\text{-}45)$$

where

$$\bar{W}_n = \exp[-(2kL\sin u)^2/n] \qquad (2.3\text{-}46)$$

and the coefficients are

$$d'_{11}= \cos^4u \qquad (2.3\text{-}47)$$

$$d'_{12}= 2\sin^2u\cos^2u + \sin^4u \qquad (2.3\text{-}48)$$

$$d'_2 = 2\sin^2u\cos^4u + \sin^4u\cos^2u \qquad (2.3\text{-}49)$$

$$d'_3 = 2^4\sin^4u\cos^4u/3^3 \qquad (2.3\text{-}50)$$

Using the normalized length parameters defined in Eqs.(2.2-46) and (2.2-47), the final result for the incoherent MBC becomes

$$\sigma^o_{inc}= \frac{8}{\sigma_n^2}[\ell^4\cos^4u(d_{11}+d_{12})W_1+\ell^6\cos^6u d_2W_2+\ell^8\cos^8u\cos^8u d_3W_3+\ldots] \qquad (2.3\text{-}51)$$

where

$$W_i =\bar{W}_n e^{-p^2} = e^{-4\ell^2(\cos^2u+\sin^2u/i\sigma_n^2)} \qquad (2.3\text{-}51)$$

$$d_{11}= 1 \qquad (2.3\text{-}52)$$

$$d_{12} = \tan^2 u(2 + \tan^2 u) \tag{2.3-53}$$

$$d_2 = \tan^2 u \cos^2 u(2 + \tan^2 u) \tag{2.3-54}$$

$$d_3 = 2^4 \tan^4 u/3^3 \tag{2.3-55}$$

The incoherent MBC near grazing angle of incidence is a power series in the Rayleigh parameter ℓ. The dominant term in the series is proportional to ℓ^4 which is Rayleigh scattereing. A term similar to the dominant term of Eq.(2.3-51) has been obtained by both surface perturbation[23] and physical optics calculation[30]. In Fig.2.2-2.5, we plot the incoherent MBC(IMBC) with identical values of parameters of the MBC plots in order to compare with. For the convenience we plot $\sigma_n^2 \sigma^o$ in the decibel scale. We determine the transition region graphically. In Fig. 2.6, we plot the IMBC with respect to the Rayleigh parameter. These results show that physical optics does predict correctly the shape of MBC for all values of wave incidence.

2.4 Summary

Based on a physical optics current approximation, the MBC for an ideally conducting real, stationary, isotrppic Gaussian surface height process has been evaluated. In the physical optics approximation the backscattered field is a function of both the surface height and its derivative, the surface slope process. The results of the calculations show that for near normal angle of incidence the MBC can be expressed as an asymptotic series while near grazing angle of incidence as a power series in the Rayleigh parameter ℓ. The leading term of the asymptotic series is the quasi-coherent MBC that has been obtained by both simplified physical optics and geometric optical computation[2,3,4,21]. The additional terms in the asymptotic series, denoted as DBS, are due to the slope dependency for the backscattering integral. The DBS is zero for normal angle of incidence. The leading term near grazing angle of incidence is proportional to ℓ^4 that is typical for Rayleigh scattering. This result has been obtained by surface perturbational calculations[23,27,34]. Again, the higher order terms are due to the slope dependency for backscattering integral. As expected, because of the assimed isotropic surface statistic, the MBC does not depend on the azimuthal angle of wave incidence.

CHAPTER III SCATTERING FROM DYNAMIC ROUGH SURFACE

3.1 Introduction

A great many experiments have been performed in the investigation of acoustic and electromagnetic scattering from both static and dynamic rough surfaces. A great wealth data for a wide variety of surfaces are available in both early[1] and in recent[2,3,4] publications. Theoretical approaches for the interpretation of the mean scattering cross section, the usual result of the measured data, use either a physical[1,5] or geometrical[7] optics or a small surface slope perturbation theory.[6] Crombie[9,10] has measured the temporal frequency characteristics of scattering from an ocean surface using a very long wavelength radar of 20 m. The observed Doppler spectrum was a nominal Doppler frequency of .5 Hz shifted away from the source frequency and Doppler spectral spread of .03 Hz. Since then a large number of Dopller spectral measurements have been reported using different radar frequency and illumination[11-18].

The measured Doppler frequency shift can be predicted by assuming the sea wave trains contributing the most to the frequency shift are those that are travelling toward or away from a stationary observer. The Doppler frequency shift can be then calculated from a gravity wave dispersion equation. The result for this computation can be identified as an equivalent Bragg condition. However at a high frequency the measured data of the shift deviated from the one predicted by first order Bragg scattering. A correction to the Doppler shift in terms of the ratio of surface tension to water density was suggested by Wright.[11]

Bass[12] introduced a second correction term that is due to wind drift velocity and the angle between wind direction and wave incidence vector. Instead of predicting the Doppler shift alone Barrick[19] and Bass[12] used the geometrical optics-perturbation analysis to predict the magnitude of the Doppler spectrum. This analysis is called first order theory since it predicts the first order spectral peaks only. To predict higher order spectral components, which appeared in the measured data above 4 MHz, Hasselman[14,17] introduced a second order theory. Here it is assumed that an interaction between incident wave and two water waves occurs. As a result of this interaction, four spectral components are produced. The two additional components satisfy a second order Bragg condition. Further multiple interactions are needed if higher order Bragg conditions are necessary. The geometrical optics-perturbation analysis requires separate consideration for calculation of the amplitude and Doppler shift of the spectrum. A physical optics approach, using the Pierson-Neumann's simplified ocean surface directional spectrum[22] was introduced by Parkins[25]. However Parkins considered the coherent Doppler spectrum only. The coherent Doppler spectra yield good results for near normal angle of wave incidence. However, near grazing angle of wave incidence, non-coherent scattering is important leading to Plateau phenomena[2,3].

In this paper the autocorrelation function and the Doppler power spectrum is calculated for a time-varying rough surface. The scattered field is calculated by a physical optics current approximation. The scattering surface is modeled as a three dimensional, two spatial and one temporal, a Gaussian, stationary height process with zero mean and variance. The surface height correlation function is a Gaussian envelope. travelling wave function in the temporal and spatial variables. This type of correlation function was suggested by Clay-Medwin[21] as a good experimental fit to his as well as Kinsman's measured data[23].

In section two, the scattered field from a time-varying Gaussian surface height process is developed. In section three, the normalized incoherent autocorrelation function is derived. The results are presented in terms of normalized surface roughness parameters. In section four, the normalized temporal frequency spectrum is obtained. The first order spectrum predicts third order Doppler frequency shifts. In addition, by correcting the surface height correlation, second order Doppler frequency shifts and spreads are predicted. In the last section a summary is presented.

3.2 Scattering by a Time-Varying Rough Surface

It is known that the solution of a time dependent scalar wave equation can be cast an integral form

$$\mathcal{E}(t,R)=\frac{1}{4\pi}\int_{S''}\{\frac{1}{R}[\frac{\partial\mathcal{E}^t}{\partial n}]-\frac{\partial}{\partial n}[\mathcal{E}^t]\frac{1}{R}+\frac{1}{Rc}\frac{\partial}{\partial n}R[\frac{\partial\mathcal{E}^t}{\partial t}]\}dS'' \qquad (3.2\text{-}1)$$

where $\mathcal{E}$ is the scattered field at the observation point P(R) far from the surface at a distance R, n is the local surface normal, c is the velocity of wave propagation, $\mathcal{E}^t=\mathcal{E}^i+\mathcal{E}$ is the total field at the surface and the square bracket denotes time retardation as $[\mathcal{E}]=$ (x,y,z,t-R/c) and S" is the scattering area. Using the frozen surface hypothesis and the local tangent approximation, Parkins[20] derived the scattered field from rough surface for a time harmonic plane wave incidence withradian frequency ω_c and the wave number $k=\omega_c/c$

$$(t,R)=\frac{jke^{-jkR+j\omega_c t}}{2\pi R}\int_S F\ e^{-j\underline{K}\cdot\underline{r}(t)}dx\,dy \qquad (3.2\text{-}2)$$

where S" projected to the mean plane so that S is the effective scattering area on the mean plane and $\underline{r}$ is a surface radial vector

$$\underline{r}=x\underline{a}_x+y\underline{a}_y+\zeta(x,y,t)\underline{a}_z \qquad (3.2\text{-}3)$$

The function F and $\underline{K}$ are

$$F=\cos u+\sin u\cos v\frac{\partial\zeta}{\partial x}+\sin u\sin v\frac{\partial\zeta}{\partial y} \qquad (3.2\text{-}4)$$

$$\underline{K}=\underline{k}_i-\underline{k}_s \qquad (3.2\text{-}5)$$

with $\underline{k}_i$ and $\underline{k}_s$ are the incident and scattered wave number vectors as

$$\binom{\underline{k}_i}{\underline{k}_s}=k\{\sin\binom{u}{u_s}\cos\binom{v}{v_s}\underline{a}_x+\sin\binom{u}{u_s}\sin\binom{v}{v_s}\underline{a}_y\mp\cos\binom{u}{u_s}\underline{a}_z\} \qquad (3.2\text{-}6)$$

where u, v, u_s and v_s are the elevation and azimuth angles of wave incidence and scattering, respectively. We note that F is only function of the angles of wave incidence and directional surface slopes. All geometrical parameters are depicted in Fig.3.1. The field $\mathcal{E}(t,R)$ can be interpreted either as a acoustic field i.e. pressure, sonic and ultrasonic field or as a linearly polarized electromagnetic field.

The scattering field is a random process since it depends on the surface height $z = \xi(x,y,t)$ which is itself a random process. The random surface height is assumed to be a real Gaussian stationary process of three independent variables x,y and t. The process $\xi(x,t,y)$ is characterized by its mean value $<\xi(x,y,t)>=0$, which is the x-y plane, the variance $\sigma^2=<\xi^2(x,y,t)>$ which is independent of the temporal and spatial location of the surface due to the assumed stationarity, and the surface height normalized correlation function $C(\bar{x},\bar{y}, \;)=\sigma^{-2}<\xi(x,y,t)\xi(x' \; y',t'>$ where <...> stands for an ensemble average. The normalized correlation function C is a function of temporal and spatial separations as $\bar{x}=x-x'$, $\bar{y}=y-y'$ and $\tau=t-t'$. For wind driven sea surface C can be expressed as a modulated travelling wave as proposed by Clay-Medwin[21] based on their as well as Kinsmans[23] experimental investigations

$$C = \exp\{-\frac{\tau^2}{T} - \frac{\bar{x}^2+\bar{y}^2}{L^2}\} \cos(f_x\bar{x} + f_y\bar{y} - \omega_d\tau) \qquad (3.2\text{-}7)$$

where T is temporal correlation duration and L is spatial correlation distance, and wave is travelling in an arbitrary direction with phase velocity $\omega_d/\sqrt{f_x^2+f_y^2}$ where f_x and f_y are directional ocean wave numbers as $f_y=f\cos\alpha$, $f_x=f\sin\alpha$.

Since the most of the experimental data are given in terms of the mean scattering cross section and/or the power spectrum, the second order statistical moment, the autocorrelation function of the scattered field is of interest. The auto-correlation function of the scattered field is

$$Q(\tau) = < \mathcal{E}(t,R)\, \mathcal{E}^*(t+\tau,R)> \qquad (3.2\text{-}8)$$

where the asterisk denotes the complex conjugate of the field. This function is also known as self-coherence function[24] of the field at point P(R). Substituting Eq.(3.2-2) into Eq.(3.2-8) and interchanging the integrations and averaging, we have

$$Q(\tau)=Ae^{-j\omega_c\tau}\int_S\int_{S'} I\, e^{-j[K_x(x-x')+K_y(y-y')]}dxdx'dydy' \qquad (3.2\text{-}9)$$

where $A=k^2/(2\pi R)^2$ and K_x and K_y are the scalar coefficients of the rectangular components of the vector $\underline{K}$ and

$$I=\cos^2 u M_1+\sin u\cos u\{\cos v M_2+\sin v M_3\} +\sin^2 u\{\sin v\cos v(M_4+M_5)+\cos^2 v M_6+\sin^2 v M_7\} \qquad (3.2\text{-}10)$$

The M's, defined below have been evaluated by Hoffman[8] for real, stationary Gaussian surface heightprocess

$$M_1= < X > = \exp\{-p^2(1-C)\} \qquad (3.2\text{-}11)$$

$$M_2= <(\frac{\partial\zeta}{\partial x} + \frac{\partial\zeta}{\partial x'})X>=-j\sigma p(\frac{\partial C}{\partial x}-\frac{\partial C}{\partial x'})\, M_1 \qquad (3.2\text{-}12)$$

$$M_3= <(\frac{\partial\zeta}{\partial y} + \frac{\partial\zeta}{\partial y'})X>=-j\sigma p(\frac{\partial C}{\partial y}-\frac{\partial C}{\partial y'})\, M_1 \qquad (3.2\text{-}13)$$

$$M_4= <\frac{\partial\zeta}{\partial x}\frac{\partial\zeta}{\partial y'}X> = \sigma^2(\frac{\partial^2 C}{\partial x\partial y'} + p^2 \frac{\partial C}{\partial x}\frac{\partial C}{\partial y'})M_1 \qquad (3.2\text{-}14)$$

$$M_5= <\frac{\partial\zeta}{\partial x'}\frac{\partial\zeta}{\partial y}X> = \sigma^2(\frac{\partial^2 C}{\partial x'\partial y} + p^2 \frac{\partial C}{\partial x'}\frac{\partial C}{\partial y})\, M_1 \qquad (3.2\text{-}15)$$

$$M_6= <\frac{\partial\zeta}{\partial x}\frac{\partial\zeta}{\partial x'}X> = \sigma^2(\frac{\partial^2 C}{\partial x\partial x'} + p^2 \frac{\partial C}{\partial x}\frac{\partial C}{\partial x'})\, M_1 \qquad (3.2\text{-}16)$$

$$M_7= <\frac{\partial\zeta}{\partial y}\frac{\partial\zeta}{\partial y'}X> = \sigma^2(\frac{\partial^2 C}{\partial y\partial y'} + p^2 \frac{\partial C}{\partial y}\frac{\partial C}{\partial y'})\, M_1 \qquad (3.2\text{-}17)$$

where

$$X = e^{-jK_z[\zeta(x,y,t)-\zeta(x',y',t')]} \qquad (3.2\text{-}18)$$

$$p = K_z\sigma \qquad (3.2\text{-}19)$$

We note that the autocorrelation function $Q(\tau)$ is time-stationary if the surface height correlation function is temporal stationary. For the evaluation of the autocorrelation function, we separate coherent and incoherent components as

$$Q(\tau) = Q_c(\tau) + Q_I(\tau) \qquad (3.2\text{-}20)$$

where the coherent component is

$$Q_c(\tau) = <\mathcal{E}><\mathcal{E}^*> \qquad (3.2\text{-}21)$$

and the incoherent component is

$$Q_I(\tau) = \mathrm{covar}[\mathcal{E}] \qquad (3.2\text{-}21)$$

From Eq.(3.2-8), the coherent autocorrelation function is

$$Q_c(\tau) = Ae^{-j\omega_c'\tau}\cos^2 u\, M_o^2\, |E_c^2| \qquad (3.2\text{-}22)$$

where

$$M_o = < e^{-jK_z\zeta(x,y,t)}> = \exp\{-p^2/2\} \qquad (3.2\text{-}23)$$

is the characteristic function and

$$E_c = \int_S e^{-j(K_x x+K_y y)}dx\,dy \qquad (3.2\text{-}24)$$

The magnitude square of E_c is the coherent pattern function for a flat surface. This functin is dependent on illumination. For a rectangular patch it is a squared sampling function while for a circular patch it is an Airy Disc pattern. The term M_o^2 is a surface roughness factor(or coherent power factor) which smears the coherent pattern function. It is unity for specular scattering and it is near zero for backscattering. A great deal of measured data has obtained from source radiating near grazing angle of incidence[13,15,17,18]. Near grazing angle of incidence, thethe coherent power reflectance is near zero and therefore the incoherent component is dominant term in the total autocorrelation function.

3.3 The Incoherent Autocorrelation Function

The incoherent component of the autocorrelation function can be obtained by subtracting the coherent component from the total autocorrelation function

$$Q_I(\tau)=Ae^{-j}\ c \int_S \int_{S'} [I-\cos^2 u e^{-p^2}] e^{-j[K_x(x-x')+K_y(y-y')]} dxdx'dydy' \quad (3.3\text{-}1)$$

where S and S' are identical illumination areas. This integral is two dimensional Fourier transform with respect to (x-x') and (y-y'). Since the integrand is a function of (x-x') and (y-y') only, the integral can be simplified by using reasonable approximation as in the Chapter II

$$Q_I(\tau)=ASe^{-j\omega_c\tau}\int_0^{\infty}\int_0^{2\pi} [I-\cos^2 u e^{-p^2}]\ e^{-jzK_t\cos(\theta-\phi_K)} z\,d\theta\,dz \quad (3.3\text{-}2)$$

where

$$K_t^2 = K_x^2 + K_y^2, \qquad \phi_K = \tan^{-1}(K_x / K_y) \quad (3.3\text{-}3)$$

and

$$z^2 = (x-x')^2 + (y-y')^2, \ \theta = \tan^{-1} \frac{(x-x')}{(y-y')} \quad (3.3\text{-}4)$$

The quantity, I, in the integrand, using Eq.(3.3-4), is in the new cylindrical coordinates

$$I = e^{-p^2(1-C)}\{\cos^2 u - \frac{\sigma^2}{2}\sin^2 u\ L_2 - j\sigma p\sin 2u[\sin(v+\theta)\frac{\partial C}{\partial z}+\cos(v+\theta)\frac{1}{z}\frac{\partial C}{\partial\theta}] + \frac{\sigma^2}{2}\sin^2 u[\sin 2v(-\sin 2\theta L_1 - 2\cos 2\theta L_3)+\cos 2v(\cos 2\theta L_1 - 2\sin 2\theta L_3)]\} \quad (3.3\text{-}5)$$

where

$$L_{1,2} = \frac{\partial^2 C}{\partial z^2} \mp \frac{1}{z^2}\frac{\partial^2 C}{\partial\theta^2} \mp \frac{1}{z}\frac{\partial C}{\partial z} + p^2\{(\frac{\partial C}{\partial z})^2 \mp \frac{1}{z^2}(\frac{\partial C}{\partial\theta})^2\} \quad (3.3\text{-}6)$$

$$L_3 = \frac{1}{z}\frac{\partial^2 C}{\partial\theta\partial z} - \frac{1}{z^2}\frac{\partial C}{\partial\theta} + \frac{p^2}{z}\frac{\partial C}{\partial z}\frac{\partial C}{\partial\theta} \quad (3.3\text{-}7)$$

Since p^2 is small near grazing angle of incidence the term

$\exp\{-p^2(1-C)\}$ can be approximated as

$$\exp\{-p^2(1-C)\} \cong \exp\{-p^2\}(1+p^2C) \qquad (3.3\text{-}8)$$

Using this approximation in the integrand leads to a cancellation of the term $\exp\{-p^2\}\cos^2u$, and the integrand containes term up to C^3. Letting

$$J\ \exp\{-p^2\} = I - \cos^2u\exp\{-p^2\} \qquad (3.3\text{-}9)$$

we have for J(See Appendix B)

$$J = CN_{10} + \bar{C}N_{01} + C^2N_{20} + \bar{C}^2N_{02} + C\bar{C}N_{11} + C^3N_{30} + C^2\bar{C}N_{21} + C\bar{C}^2N_{12} \qquad (3.3\text{-}10)$$

with

$$N_{10} = p^2\cos^2u + j\sigma p\sin 2u\,\sin(v+\theta)\frac{2z}{L^2} + \frac{\sigma^2}{2}\sin^2u\{\frac{4}{L^2} + f_1 - \frac{4z^2}{L^4}[1-\cos 2(v+\theta)]\} \qquad (3.3\text{-}11)$$

$$N_{01} = j\sigma p\sin 2u(f_x\sin v + f_y\cos v) - \frac{\sigma^2}{2}\sin^2u\frac{4z}{L^2}\ f_2 \qquad (3.3\text{-}12)$$

$$N_{11} = j\sigma p^3\sin 2u(f_x\cos v + f_y\sin v) - \frac{\sigma^2p^2 8z}{2L^2}\sin^2u\ f_2 \qquad (3.3\text{-}13)$$

$$N_{20} = p^2(N_{10} - \cos^2u) \qquad (3.3\text{-}14)$$

$$N_{02} = -p^2\sigma^2\sin^2u f_1/2 \qquad (3.3\text{-}15)$$

$$N_{21} = -p^2\sigma^2 2z^2\sin^2u\ f_2/L^4 \qquad (3.3\text{-}16)$$

$$N_{12} = -p^4\sigma^2\sin^2u\ f_1/2 \qquad (3.3\text{-}17)$$

$$N_{30} = -p^4\sigma^2 2z^2\sin^2u[1-\cos 2(v+\theta)]/\ L^4 \qquad (3.3\text{-}18)$$

where

$$f_1 = (f_x^2 + f_y^2) + 2f_xf_y\sin 2v + (f_x^2 - f_y^2)\cos 2v \qquad (3.3\text{-}19)$$

$$f_2 = \sin\theta(f_x - f_y\sin 2v - f_x\cos 2v) + \cos\theta(f_y - f_x\sin 2v + f_y\cos 2v)$$
$$= \sin\theta\ V_1 + \cos\theta\ V_2 \qquad (3.3\text{-}20)$$

and

$$\bar{C} = \exp\{-\frac{\tau^2}{T^2} - \frac{z^2}{L^2}\}\ \sin\{z(f_x\sin\theta + f_y\cos\theta) - \omega_d\tau\} \qquad (3.3\text{-}21)$$

The integral for the incoherent autocorrelation function is

$$Q_I(\tau)=ASe^{-p^2-j\omega_c\tau}\int_0^{\infty}\int_0^{2\pi}Je^{-jzK_t\cos(\theta-\phi_K)}\,zd\theta dz \qquad (3.3\text{-}22)$$

For algebraic convenience, we decompose C and $\bar{C}$ into forward and backward travelling waves as

$$\binom{\bar{C}}{C} = \binom{-j}{1}(C^{+1}\mp C^{-1})\frac{C_o^1}{2} \qquad (3.3\text{-}23)$$

where we have used the identities

$$\frac{\sin}{\cos}(\alpha) = \tfrac{1}{2}(e^{j\alpha}\mp e^{-j\alpha})\binom{-j}{1} \qquad (3.3\text{-}24)$$

and C_o^1 is spatial and temporal envelope function as

$$C_o^n = \exp\{-n(\frac{\tau^2}{T^2}+\frac{z^2}{L^2})\} \qquad (3.3\text{-}25)$$

and $C^{\pm 1}$ is phasors

$$C^{\pm m}=\exp\{-jm[\pm\omega_d\tau \mp z(f_x\sin\theta+f_y\cos\theta)]\} \qquad (3.3\text{-}26)$$

Similarly, we decompose the set $\{C^m C^{\pm n}\}$ into the travelling waves

$$C^2= \tfrac{1}{2}C_o^2 + \tfrac{1}{4}C_o^2(C^{+2}+ C^{-2}) \qquad (3.3\text{-}27)$$

$$\bar{C}^2= \tfrac{1}{2}C_o^2 - \tfrac{1}{4}C_o^2(C^{+2}+ C^{-2}) \qquad (3.3\text{-}28)$$

$$C\bar{C}=-j\tfrac{1}{4}C_o^2(C^{+2}- C^{-2}) \qquad (3.3\text{-}29)$$

$$C^3= \frac{1}{8}C_o^3(C^{+3}- C^{-3}+ 3C^{+1}+ 3C^{-1}) \qquad (3.3\text{-}30)$$

$$C^2\bar{C}=-j\frac{1}{8}C_o^3(C^{+3}- C^{-3}+ C^{+1}- C^{-1}) \qquad (3.3\text{-}31)$$

$$C\bar{C}^2=\frac{1}{8}C_o^3(-C^{+3}- C^{-3}+ C^{+1}+ C^{-1}) \qquad (3.3\text{-}32)$$

and substituting Eqs.(3.3-23) through (3.3-32) into J and factoring the same order of C_o^n and $C^{\pm m}$, we have

$$J=\tfrac{1}{2}C_o^2(N_{20}+N_{02})+\tfrac{1}{2}C^{+1}C_o^1(N_{10}\mp jN_{01})+\tfrac{1}{4}C^{\pm 2}C_o^2(N_{20}-N_{02}\mp jN_{11})$$
$$+\tfrac{1}{8}C_o^3[C^{\pm 1}(3N_{30}+N_{12}\mp jN_{21})+C^{\pm 3}(N_{30}-N_{12}\mp jN_{21})] \qquad (3.3\text{-}33)$$

Separating $Q_I(\tau)$ as

$$Q_I(\tau)=\sum_{mn} Q_{mn}^{\pm}(\tau) \qquad (3.3\text{-}34)$$

where

$$Q_{mn}^{\pm}=ASe^{-p^2-j\omega_c\tau}\int_0^{\infty}\int_0^{2\pi}J_{mn}^{\pm}e^{-jzK_t\cos(\theta-\phi_K)}z\,dzd\theta \qquad (3.3\text{-}35)$$

with

$$J_{02}=\tfrac{1}{2}C_o^2(N_{20}+N_{02}) \qquad (3.3\text{-}36)$$

$$J_{11}^{\pm}=\tfrac{1}{2}C^{\pm 1}C_o^1(N_{10}\mp jN_{01}) \qquad (3.3\text{-}36)$$

$$J_{22}^{\pm}=\tfrac{1}{8}C^{\pm 2}C_o^2(N_{20}-N_{02}\pm jN_{11}) \qquad (3.3\text{-}37)$$

$$J_{13}^{\pm}=\tfrac{1}{8}C^{\pm 1}C_o^3(3N_{30}+N_{12}\pm jN_{21}) \qquad (3.3\text{-}38)$$

$$J_{33}^{\pm}=\tfrac{1}{8}C^{\pm 3}C_o^3(N_{30}-N_{12}\mp jN_{21}) \qquad (3.3\text{-}39)$$

The term J_{02} has no phasor and therefore Q_{02} is the component of the incoherent autocorrelation function due to a quasi-static surface. The integrations $Q_{mn}^{\pm}$ with respect to z and θ are similar. Next the integration Q_{02} is carried out in detail. The evaluations of the rests of $Q_{mn}^{\pm}$ are done in Appendix C. We will first integrate the angular variable and then the radial variable. Defining the angular integrals

$$\binom{\Phi_m}{\Psi_m} = \int_0^{2\pi}\frac{\cos}{\sin}(m\theta)\ e^{-jzK_t\cos(\theta-\phi_K)}d\theta \qquad (3.3\text{-}40)$$

we rewrite $Q_{02}(\tau)$

$$Q_{02}(\tau)=ASe^{-p^2-j\omega_c\tau}\int_0^\infty dz\{z\,(\sigma pC_0 \sin u/L)^2(1-2z^2/L^2)\Phi_0$$

$$-j\sigma p(pC_0 z/L)^2 \sin 2u(\sin v\Phi_1+\cos v\Psi_1)+2z^3(\sigma pC_0 \sin u/L^2)^2\cdot$$

$$(-\sin 2v\Psi_2+\cos 2v\Phi_2)\} \qquad (3.3\text{-}41)$$

The angular integrals are Richard and Wolf integrals[15]

$$\binom{\Phi}{\Psi}_m=(-j)2\pi\ J_m(K_t z)\ {\cos \atop \sin} m(\phi_K) \qquad (3.3\text{-}42)$$

where $J_m(x)$ is the Bessel function of the first kind of order m. In particular

$$\Phi_0= 2\pi\, J_0(K_t z) \qquad (3.3\text{-}43)$$

$$\binom{\Phi}{\Psi}_1=2\pi(-j)\ J_1(K_t z){\cos \atop \sin}(\phi_K) \qquad (3.3\text{-}44)$$

$$\binom{\Phi}{\Psi}_2=-2\pi\ J_2(K_t z){\cos \atop \sin}2(\phi_K) \qquad (3.3\text{-}45)$$

Substituting Eqs.(3.3-43)-(3.3-45) into Q_{02}, we have

$$Q_{02}=2\pi ASe^{-p^2-j\omega_c\tau-\frac{2\tau^2}{T^2}}\{(\sigma p\sin u/L)^2(\aleph_0^2-L^{-2}\aleph_0^{2\prime})+\sigma p^3L^{-2}\sin 2u\sin(v+\phi_K)$$

$$\cdot\aleph_1^2\ -2(\sigma p\sin u/L^2)^2\cos 2(v+\phi_K)\aleph_2^2\} \qquad (3.3\text{-}46)$$

where $\aleph_m^n$ are the radial integrals as

$$\aleph_m^n=\int_0^\infty z^{m+1}J_m(K_t z)\exp\{-nz^2/L^2\}\,dz \qquad (3.3\text{-}47)$$

and $\aleph_m^{n\prime}$ are the derivatives with respective to $(-n/L^2)$. The integrand of the radial integral is real as expected with real K_t and ϕ_K defined by Eq.(3.3-3). The integral $\aleph_m^n$ are of the first kind Weber exponential integrals[26] as

$$\mathcal{N}_m^n = K_t^m[2(nL^{-2})]^{-(m+1)}\exp[-L^2K_t^2(2n)^{-2}] \qquad (3.3\text{-}48)$$

In particular

$$\mathcal{N}_o^2 = L^2/4\cdot\exp[-L^2K_t^2/8] \qquad (3.3\text{-}49)$$

$$\mathcal{N}_1^2 = L^4K_t/16\cdot\exp[-L^2K_t^2/8] \qquad (3.3\text{-}50)$$

$$\mathcal{N}_2^2 = L^6K_t^2 4^{-3}\cdot\exp[-L^2K_t^2/8] \qquad (3.3\text{-}51)$$

and

$$\mathcal{N}_o^{2\prime} = L^4[1-L^2K_t^2/8]/8\cdot\exp[-L^2K_t^2/8] \qquad (3.3\text{-}52)$$

Substituting $\mathcal{N}_m^n$ into Q_{02}, we have

$$Q_{02}(\tau) = 2\pi AS\ \exp\{-p^2-j\omega_c\tau - \frac{2\tau^2}{T^2} - \frac{L^2K_t^2}{8}\}\{(\sigma pLK_t\sin u)^2/2^5$$
$$+\sigma p^3L^2K_t\sin 2u\sin(v+\phi_K)/2^4-(\sigma pLK_t\sin u)^2\cos 2(v+\phi_K)/2^5\} \qquad (3.3\text{-}53)$$

This step completes the evaluation of $Q_{02}(\tau)$. As expected, Q_{02} is independent of the surface wave number f and the radial frequency ω_d.

From Appendix C

$$Q_{mn}^{\pm}(\tau) = 2\pi AS\ \exp\{-p^2-n\tau^2T^{-2}-j\omega_c\tau \mp jm\omega_d\tau\}\cdot G_{mn}^{\pm} \qquad (3.3\text{-}54)$$

where

$$G_{11}^{\pm}=\exp\{-L^2K_{1t}^{\pm 2}/4\}\{(pL\cos u)^2/4\pm\sigma pL^2\sin 2u(f_y\sin v+f_x\cos v)/2$$
$$+f_1(\sigma L\sin u)^2/8+(\sigma LK_{1t}^{\pm}\sin u)^2/2^4+\sigma pL^2K_{1t}^{\pm}\sin 2u\sin(v+\phi_{1K}^{\pm})/4$$
$$\pm \tfrac{1}{4}(\sigma L\sin u)^2K_{1t}^{\pm}(V_1\sin\phi_{1K}^{\pm}+V_2\cos\phi_{1K}^{\pm})-(\sigma LK_{1t}^{\pm})^2\cos 2(v+\phi_{1K}^{\pm})\} \qquad (3.3\text{-}55)$$

$$G_{22}^{\pm}=\exp\{-L^2K_{2t}^{\pm 2}/8\}\{(L\sigma p\sin u)^2(K_{2t}^{\pm 2}+\left[{0 \atop 1}\right]f_1)/8\pm\sigma p^3L^2\sin 2u(f_x\sin v+f_y\cos v)/4$$
$$+\sigma p^3L^2K_{2t}^{\pm}\sin 2u\sin(v+\phi_{2K}^{\pm})\pm(\sigma pL)^2K_{2t}^{\pm}\sin^2u(V_1\sin\phi_{2K}^{\pm}+V_2\cos\phi_{2K}^{\pm})/16$$
$$-(\sigma pLK_{2t}^{\pm}\sin u 2^{-3})^2\cos 2(v+\phi_{2K}^{\pm})\} \qquad (3.3\text{-}56)$$

$$G_{13}^{\pm} = \exp\{-L^2K_{1t}^{\pm 2}/12\}\{-(\sigma p^2 Lf_1 \sin u)^2/3\cdot 2^5 + (\sigma p^2 LK_{1t}^{\pm} \sin u)^2/3^2 2^5$$
$$\pm(\sigma P^2 L \sin u)^2 K_{1t}^{\pm}/3^2 2^4 (V_1 \sin\phi_{1K}^{\pm} + V_2 \cos\phi_{1K}^{\pm})$$
$$-(\sigma p^2 LK_{1t}^{\pm} \sin u)^2 \cos 2(v \pm \phi_{1K}^{\pm})/3^2 2^5\} \qquad (3.3\text{-}57)$$

and

$$G_{33}^{\pm} = \exp\{-L^2K_{3t}^{\pm 2}/12\}\{(\sigma p^2 L \sin u)^2 f_1/3\cdot 2^5 + (\sigma p^2 LK_{3t}^{\pm} \sin u)^2/3^3 2^5$$
$$\pm(\sigma p^2 LK_{3t}^{\pm} \sin u)^2 (V_1 \sin\phi_{3K}^{\pm} + V_2 \cos\phi_{3K}^{\pm}) - (\sigma p^2 K_{3t}^{\pm} \sin uL)^2 \cos 2(v + \phi_{3K}^{\pm})/3^2 2^5$$
$$(3.3\text{-}58)$$

where

$$K_{mt}^{\pm 2} = (K_x \mp mf_x)^2 + (K_y \mp mf_y)^2 \qquad (3.3\text{-}59)$$

$$\phi_{mK}^{\pm} = \tan^{-1}\{(K_x \mp mf_x)/(K_y \mp mf_y)\} \qquad (3.3\text{-}60)$$

The results will now be expressed in terms of normalized quantities which are functions of normalized parameters. Let the normalized autocorrelation function(NAC), $Q(\tau)$ be defined as

$$\overline{Q(\tau)} = \frac{4\pi R^2}{S} Q(\tau) \qquad (3.3\text{-}61)$$

For reasonable surface realizations, NAC is related to the mean scattering cross section(MSC) as

$$\overline{Q(\tau)} = W(\tau)\cdot\bar{\sigma} \qquad (3.3\text{-}62)$$

where $W(\tau)$ is a function of the temporal variable and $\bar{\sigma}$ is the MSC. The MSC is a function of the following three normalized parameters

$$\ell = k\sigma \qquad (3.3\text{-}63)$$

$$\sigma_n = \sigma/L \qquad (3.3\text{-}64)$$

$$\bar{f} = f/k = (f_x^2 + f_y^2)^{\frac{1}{2}}/k \qquad (3.3\text{-}65)$$

where ℓ is the Rayleigh, σ_n is the rms surface slope or normalized roughness parameters and $\bar{f}$ is the normalized sea wave number parameter. Using the normalization condition defined by Eq.(3.3-61) and the dimensionless surface parameters, Eqs(3.3-63)-(3.3-65), we have for the total NAC

$$\overline{Q(\tau)} = e^{-j\omega_c\tau}\ \bar{\sigma}_c + \sum_{mn} \exp(-j\omega_c\tau \mp jm\omega_d\tau - n\tau^2/T^2)\cdot\bar{\sigma}_{mn}^{\pm} \qquad (3.3\text{-}66)$$

where $\bar{\sigma}_c$ and $\bar{\sigma}_{mn}$ are coherent and partial incoherent MSC respectively. The normalized coherent MSC is

$$\bar{\sigma}_c = \cos^2 u\ \exp(-p^2)\,|E_c|^2 \qquad (3.3\text{-}67)$$

and the normalized partial incoherent MSCs are

$$\bar{\sigma}_{mn}^{\pm} = k^2 \exp(-p^2)\ G_{mn}^{\pm} \qquad (3.3\text{-}68)$$

where

$$\bar{\sigma}_{02} = g_1(2)\{\ell^4\bar{p}^2\bar{K}_t^2\sin^2 u/2^5 + \ell^6[\bar{p}^3\bar{K}_t\sin 2u\sin(v+\phi_K)/2^4 - 2^{-5}\bar{p}^2\bar{K}_t^2\sin^2 u\cos 2(v+\phi_K)]\} \qquad (3.3\text{-}69)$$

$$\bar{\sigma}_{11}^{\pm} = g_1^{\pm}(1)\,\ell^4\{\tfrac{1}{4}\bar{p}^2\cos u + \tfrac{1}{4}\bar{p}^2\bar{K}_{1t}^{\pm}\sin 2u\sin(v+\phi_{1K}^{\pm}) + \tfrac{1}{4}\bar{f}_1\sin^2 u + \bar{K}_{1t}^{\pm}\sin^2 u/8 \pm \tfrac{1}{2}\bar{p}\bar{f}\sin 2u\cos(v-\alpha) \pm \tfrac{1}{4}\bar{f}\bar{K}_{1t}^{\pm}\sin^2 u(\bar{V}_1\sin\phi_{1K}^{\pm} + \bar{V}_2\cos\phi_{1K}^{\pm}) - \bar{K}_{1t}^{\pm 2}\sin^2 u\cos 2(v+\phi_{1K}^{\pm})\} \qquad (3.3\text{-}70)$$

$$\bar{\sigma}_{22}^{\pm} = g_2^{\pm}(2)\,\ell^6\{2^{-7}\bar{p}^2\bar{K}_{2t}^{\pm 2}\sin^2 u + 2^{-5}p^{-3}\bar{K}_{2t}^{\pm}\sin 2u\sin(v+\phi_{1K}^{\pm}) \pm 2^{-2}\bar{p}^3\bar{f}\ \sin 2u\cos(v-\alpha) + 2^{-4}\bar{p}^2\bar{f}_1\sin^2 u - 2^{-6}\bar{p}^2\bar{K}_{2t}^{\pm 2}\sin^2 u\cos 2(v+\phi_{1K}^{\pm}) \pm 2^{-4}\bar{p}^2\bar{f}\bar{K}_{2t}^{\pm}\sin^2 u(\bar{V}_1\sin\phi_{2K}^{\pm} + \bar{V}_2\cos\phi_{2K}^{\pm})\} \qquad (3.3\text{-}71)$$

$$\bar{\sigma}_{13}^{\pm}=g_1^{\pm}(3)\ell^8\{2^{-5}3^{-2}\bar{p}^{-4}\bar{K}_{1t}^{\pm 2}\sin^2u\pm 2^{-4}3^{-2}\bar{p}^{-4}\bar{f}\bar{K}_{1t}^{\pm}\sin^2u(\bar{V}_1\sin\phi_{1K}^{\pm}+\bar{V}_2\cos\phi_{1K}^{\pm})$$
$$-2^{-5}3^{-1}\bar{p}^{-4}\bar{f}_1\sin^2u-2^{-5}3^{-2}\bar{p}^{-4}\bar{K}_{1t}^{\pm 2}\sin^2u\cos 2(v+\phi_{1K}^{\pm})\} \qquad (3.3\text{-}72)$$

$$\bar{\sigma}_{33}^{\pm}=g_3^{\pm}(3)\ell^8\{2^{-4}3^{-1}\bar{p}^{-4}\bar{f}_1\sin^2u - 2^{-5}3^{-3}\bar{p}^{-4}\bar{K}_{3t}^{\pm 2}\sin^2u\cos 2(v+\phi_{3K}^{\pm})$$
$$+2^{-5}3^{-2}\bar{p}^{-4}\bar{K}_{3t}^{\pm}\sin^2u \pm 2^{-4}3^{-3}\bar{p}^{-4}\bar{f}\bar{K}_{3t}^{\pm}\sin^2u(\bar{V}_1\sin\phi_{3K}^{\pm}+\bar{V}_2\cos\phi_{3K}^{\pm})\} \qquad (3.3\text{-}73)$$

with

$$\bar{K}_{mt}^{\pm 2}=(\sin u\cos v-\sin u_s\cos v_s\mp m\sin\alpha)^2 +(\sin u\sin v-\sin u_s\sin v_s\mp m\cos\alpha)^2 \qquad (3.3\text{-}74)$$

$$\phi_{mK}^{\pm}=\tan^{-1}\frac{K_x\mp mf_x}{K_y\mp mf_y} \qquad (3.3\text{-}75)$$

$$\bar{p}=\cos u+\cos u_s \qquad (3.3\text{-}76)$$

$$\bar{f}_1=1+\cos 2(v-\alpha) \qquad (3.3\text{-}77)$$

$$\binom{\bar{V}_1}{\bar{V}_2}=\genfrac{}{}{0pt}{}{\sin}{\cos}(\alpha)\mp\genfrac{}{}{0pt}{}{\sin}{\cos}(\alpha+2v) \qquad (3.3\text{-}78)$$

$$g_m^{\pm}(n)=\sigma_n^{-2}\exp\{-\ell^2(\bar{p}^2+\bar{K}_{mt}^{\pm 2}/4n\sigma_n^2)\} \qquad (3.3\text{-}79)$$

We note that $\bar{K}_t=\bar{K}_{0t}$. This step completes the evaluation of the NAC. We will next discuss a few special cases of our general MSC results.

For quasi-specular scattering,

$$u=u_s, \qquad v\neq v_s \qquad (3.3\text{-}80)$$

In this case we assume that the transmitter and receiver are nearly identical in heights. Therefore, angularly dependent parameters are

$$\bar{p} = 2\cos u \qquad (3.3\text{-}81)$$

with

$$\bar{K}_{mt}^{\pm 2} = \{\sin u(\cos v - \cos v_s) \mp m\sin\alpha\}^2 + \{\sin u(\sin v - \sin v_s) \mp m\cos\alpha\}^2 \qquad (3.3\text{-}82)$$

$$\phi_{mK}^{\pm} = \tan^{-1}\frac{\sin u(\cos v - \cos v_s) \mp m\sin\alpha}{\sin u(\sin v - \sin v_s) \mp m\cos\alpha} \qquad (3.3\text{-}83)$$

$$\bar{K}_t^2 = \bar{K}_{0t}^2 = 2\sin^2 u[1 - \cos(v - v_s)] \qquad (3.3\text{-}84)$$

The coherent component $\bar{\sigma}_c$ is

$$\bar{\sigma}_c = \cos^2 u \exp\{-(\ell\bar{p})^2\} \qquad (3.3\text{-}85)$$

independent of the azimuth angle. The partial MSC's for quasi specular scattering are listed in Table 3.1.

Quasi-specular scattering is identical to specular scattering if the azimuthal angle of incident and scattered wave vectors are identical,i.e.

$$v = v_s \qquad (3.3\text{-}86)$$

with $u=u_s$. The parameters in this case are

$$\bar{p} = 2\cos u \qquad (3.3\text{-}87)$$

$$\bar{K}_{mt}^{\pm} = \bar{K}_{mt} = \begin{cases} 1 & m \neq 0 \\ 0 & m = 0 \end{cases} \qquad (3.3\text{-}88)$$

$$\phi_{mK}^{\pm} = \begin{cases} \alpha & m \neq 0 \\ 0 & m = 0 \end{cases} \qquad (3.3\text{-}89)$$

The coherent specular MSC is identical to the quasi-specular MSC since it is not a function of v. The results for the specular partial MSC's are listed in Table 3.1.

In the backscattering case

$$\underline{k}_i = \underline{k}_s \qquad (3.3\text{-}90)$$

This statement is equivalent to

$$u = u_s, \; v = v_s + \pi/2 \qquad (3.3\text{-}91)$$

The parameters now become

$$\bar{p} = 2\cos u \qquad (3.3\text{-}92)$$

$$\bar{K}_{mt}^{\pm 2} = (2\sin u\cos v \mp m\sin\alpha)^2 + (2\sin u\sin v \mp m\cos\alpha)^2 \qquad (3.3\text{-}93)$$

$$\phi_{mK}^{\pm} = \tan^{-1}\frac{2\sin u\cos v \mp m\sin\alpha}{2\sin u\sin v \mp m\cos\alpha} \qquad (3.3\text{-}94)$$

for $m \neq 0$, and

$$\bar{K}_t = \bar{K}_{ot} = 2\sin u \qquad (3.3\text{-}95)$$

$$\phi_K = \phi_{oK} = \pi/2 - v \qquad (3.3\text{-}96)$$

for $m=0$. Further we note that

$$\frac{\sin}{\cos}(v+\phi_K) = \left(\frac{1}{-1}\right) \qquad (3.3\text{-}97)$$

The coherent backscattering MSC for grazing angle of incidence is small since the coherent pattern function is small. The result for partial backscattering MSC's are listed in Table 3.1.

When the incident wave illuminates the moving ocean surface in the cross wind direction i.e. the incident wave vector $\underline{k}_i$ and the direction of the sea wave propagation are perpendicular, there are no resonance sea wave effects in the scattered field if observation is in the cross wind direction[6, 11,14,16]. In this case

$$v + \alpha = \pi/2 \qquad (3.3\text{-}98)$$

The MSC's for the cross wind direction are listed in Table 3.2

for quasi-specular, specular and backscattering angles of wave incidence. For quasi-specular wave incidence it is shown that the result depends on sea wave number f while for the specular and backscattering MSC the sea wave number f drops out.

In Fig.3.3 through Fig.3.7, the partial MSC's for various values of parameters are plotted.

3.4 Temporal Frequency Spectrum

The Fourier transform of the autocorrelation function for the scattering field is the temporal power spectrum

$$S(\omega) = \int_{-\infty}^{\infty} Q(\tau)\, e^{j\omega\tau} d\tau \tag{3.4-1}$$

Since the NAC is separable

$$Q(\tau) = \bar{\sigma}\, W(\tau) \tag{3.4-2}$$

where $W(\tau)$ is an envelope function. We define the normalized temporal power spectrum(TPS) as

$$\bar{S}(\omega) = \bar{\sigma}\, W(\omega) \tag{3.4-3}$$

where

$$W(\omega) = \frac{\epsilon}{\sqrt{\pi}} \int_{-\infty}^{\infty} W(\tau)\, e^{j\omega\tau}\, d\tau \tag{3.4-4}$$

where ϵ is the time correlation frequency. Defining the normalized radian frequency parameters as

$$\bar{\omega} = \omega/\epsilon \tag{3.4-5}$$

$$\bar{\omega}_d = \omega_d/\epsilon \tag{3.4-6}$$

$$\bar{\omega}_c = \omega_c/\epsilon \tag{3.4-7}$$

Where $\bar{\omega}$, $\bar{\omega}_d$ and $\bar{\omega}_c$ are normalized spectral, Doppler and source radian frequencies. We have as the final result for the TPS

$$\overline{S(\bar{\omega})} = \sum_{mn} S^{\pm}_{mn}(\bar{\omega}, \bar{\omega}_d, \bar{\omega}_c) \tag{3.4-8}$$

with

$$S^{\pm}_{mn}(\bar{\omega}) = \sigma^{\pm}_{mn}\, W^{\pm}_{mn}(\bar{\omega}, \bar{\omega}_d, \bar{\omega}_c) \tag{3.4-9}$$

where $W^{\pm}_{mn}$ sre partial spectral envelope functions

$$W^{\pm}_{mn} = \frac{\epsilon}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-n\epsilon^2\tau^2 - j(\omega - \omega_c \mp m\omega_d)\tau} d\tau = \frac{1}{\sqrt{n}} \exp[-(\bar{\omega} - \bar{\omega}_c \mp m\bar{\omega}_d)^2/n] \quad (3.4\text{-}10)$$

The index m and n represent the order of normalized Doppler frequency shifts and normalized Doppler frequency spreads, respectively. The coherent spectral component $W_c(\bar{\omega})$ can be found by letting the correlation time to be infinite. Using the identity

$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\epsilon\sqrt{\pi}} \exp(-x^2/\epsilon^2) \quad (3.4\text{-}11)$$

where $\delta(x)$ is the Dirac delta function, we have

$$W_c(\bar{\omega}) = \lim_{\epsilon \to 0} W_{00}(\bar{\omega}) = \sqrt{\pi}\, \delta(\bar{\omega} - \bar{\omega}_c) \quad (3.4\text{-}12)$$

Eqs.(3.4-8),(3.4-9) and (3.4-12) are the final results for the TPS. The TPS has seven spectral peaks corresponding to a surface with up to third order of Bragg scattering. The partial MSC's are the magnitude coeffients for the spectral components. The coherent spectral component is a discrete line [20]. For the quasi-static spectral component S_{02} has a spread but no Doppler shift. This component is due to the slope dependent terms in the scattering integral. The quasi-static component appears in the measured data of Crombie[9] and it has been described as an undesirable land echo. The partial terms $S^{\pm}_{11}$ are the first order spectral components and they have hightest spectral peaks and the narrowest spectral spreads. The partial MSC $\bar{\sigma}^{\pm}_{11}$ have been found previously[12,19]. The partial terms $S^{\pm}_{13}$ are correction factors to the first order spectral peaks and widths. The partial terms

$S_{22}^{\pm}$ and $S_{33}^{\pm}$ have second and third order spectral peaks and are equivalent to higher Bragg effects. In Fig. 3.8 and Fig.3.9, we plot TPS for various values of parameters.

The results thus obtained indicate that the spectral peaks of the echo are aligned with the Doppler frequencies $\pm m_d$ that include zero order shift m=0. Also the spectral peaks of the up-Doppler and the down-Doppler are asymmetrical. This result shows good agreement with early very low radar frequency sea echo measurements[9,10]. However, recent measurements, using higher radar frequencies[13,17,18] show that the peaks shift away from the nominal Doppler frequencies and the skewness of the spectra markedly increases. An explanation for this effect, refered to as a second order theory[12,14,19], considers a non-linear wave interaction between two water waves travelling in the same direction to form an additional Doppler shift in the TPS. The additional wave is an evanescent wave that can neither take energy nor can it propagate independently of the first order water wave.

To predict the additional Doppler shift from a physical optics point of view we must modify the surface height correlation function. Kinsmann's experimental record on sea surface height correlation function[23] (See Fig.3.2) shows a deviation from Eq.(3.2-7) in the second temporal cycle. To correct this deficiency we now introduce a second order sea surface height correlation function

$$C_2 = C_o \cos\{zf_s(\sin\alpha\sin\theta+\cos\alpha\cos\theta)-w_s\tau\}\cos\{zf_e(\sin\alpha\sin 0+\cos\alpha\cos 0)-w_e\tau\} \quad (3.4\text{-}12)$$

which gives a better fit to the experimental data for $f_e \ll f_s$ and $\omega_e < \omega_s$. Using the trigonometric identity

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta) \qquad (3.4\text{-}13)$$

the second order correlation function can be represented as two interacting surface waves

$$C_2 = C_o'(\cos a + \cos b) \qquad (3.4\text{-}14)$$

where

$$\binom{a}{b} = z(f_s \pm f_e)(\cos\alpha\cos 0 + \sin\alpha\sin 0) - (\omega_s \pm \omega_e) \qquad (3.4\text{-}15)$$

$$C_o' = C_o/2 \qquad (3.4\text{-}16)$$

The two waves have different phase velocities of propagations. Using the second order correlation function, we next evaluate the NAC and TPS. By expanding the correlation function into a series of travelling waves and identifying the appropriate surface number and surface wave Doppler frequency components, TPS and NAC can be cast in a series of partial TPS and NAC form. The result of the NAC(Details of calculation are in Appendix D) is

$$\bar{Q}(\tau) = e^{-j\omega_c\tau}\,\bar{\sigma}_c + \sum_{hln} e^{-j\omega_c\tau \mp (h\omega_s + l\omega_e) - n\tau^2/T^2}\,\bar{\sigma}^{\pm}_{hln} \qquad (3.4\text{-}17)$$

where

$$\bar{\sigma}_{002} = \tfrac{1}{2}\,\bar{\sigma}_{02}(\bar{f}=\bar{f}_s) \qquad (3.4\text{-}18)$$

$$\bar{\sigma}^{\pm}_{1\pm 11} = \tfrac{1}{2}\bar{\sigma}_{11}(\bar{f}=\bar{f}_s, \bar{K}^{\pm}_{1t}=\bar{K}^{\pm}_{1\pm 1}, \phi^{\pm}_{1K}=\phi^{\pm}_{1\pm 1}) \qquad (3.4\text{-}19)$$

$$\bar{\sigma}^{\pm}_{102} = \tfrac{1}{2}\bar{\sigma}^{\pm}_{22}(\bar{f}=\bar{f}_s, \bar{K}^{\pm}_{2t}=\bar{K}^{\pm}_{10}, \phi^{\pm}_{2K}=\phi^{\pm}_{10}) \qquad (3.4\text{-}20)$$

$$\bar{\sigma}^{\pm}_{2\pm 22} = \tfrac{1}{2}\bar{\sigma}^{\pm}_{22}(\bar{f}=\bar{f}_s, \bar{K}^{\pm}_{2t}=\bar{K}^{\pm}_{2\pm 2}, \phi^{\pm}_{2K}=\phi^{\pm}_{2\pm 2}) \qquad (3.4\text{-}21)$$

$$\bar{\sigma}^{\pm}_{0\pm12} = \frac{1}{4}\,\bar{\sigma}^{\pm}_{02}(\bar{f}=\bar{f}_s,\ \bar{K}_t = \bar{K}^{\pm}_{0\pm1},\ \phi_K = \phi^{\pm}_{0\pm1}) \qquad (3.4\text{-}22)$$

$$\bar{\sigma}^{\pm}_{1\pm13} = \frac{1}{16}\bar{\sigma}^{\pm}_{13}(\bar{f}=\bar{f}_s,\ \bar{K}^{\pm}_{1t} = \bar{K}^{\pm}_{1\pm1},\ \phi^{\pm}_{1K} = \phi^{\pm}_{1\pm1}) \qquad (3.4\text{-}23)$$

$$\bar{\sigma}^{\pm}_{3\pm13} = \frac{3}{16}\bar{\sigma}^{\pm}_{33}(\bar{f}=\bar{f}_s,\ \bar{K}^{\pm}_{3t} = \bar{K}^{\pm}_{3\pm1},\ \phi^{\pm}_{3K} = \phi^{\pm}_{3\pm1}) \qquad (3.4\text{-}24)$$

$$\bar{\sigma}^{\pm}_{3\pm33} = \frac{1}{16}\bar{\sigma}^{\pm}_{33}(\bar{f}=\bar{f}_s,\ \bar{K}^{\pm}_{3t} = \bar{K}^{\pm}_{3\pm3},\ \phi^{\pm}_{3K} = \phi^{\pm}_{3\pm3}) \qquad (3.4\text{-}25)$$

$$\bar{\sigma}^{\pm}_{1\pm33} = \frac{3}{16}\bar{\sigma}^{\pm}_{13}(\bar{f}=\bar{f}_s,\ \bar{K}^{\pm}_{3t} = \bar{K}^{\pm}_{3\pm3},\ \phi^{\pm}_{3K} = \phi^{\pm}_{1\pm3}) \qquad (3.4\text{-}26)$$

where

$$K^{\pm 2}_{h\pm l} = (K \mp hf_s \mp lf_e)^2_x + (K \mp hf_s \mp lf_e)^2_y \qquad (3.4\text{-}27)$$

$$\phi^{\pm}_{h\pm l} = \tan^{-1}(K \mp hf_s \mp lf_e)_x / (K \mp hf_s \mp lf_e)_y \qquad (3.4\text{-}28)$$

and $\bar{K} = K/k$ where subscript x and y denote components of the rectangular coordinate.

The normalized TPS is

$$S(\bar{\omega}) = S_c(\bar{\omega}) + \sum_{hln} \bar{\sigma}^{\pm}_{hln}\, W^{\pm}_{hln}(\bar{\omega}, \bar{\omega}_c, \bar{\omega}_d) \qquad (3.4\text{-}29)$$

where

$$W^{\pm}_{hln} = \frac{1}{\sqrt{n}} \exp\{-(\bar{\omega} - \bar{\omega}_c \mp h\bar{\omega}_s \mp l\bar{\omega}_e)^2/n\} \qquad (3.4\text{-}30)$$

The partial spectral envelope functions W_{hln} have the indices hln, h and l are nominal Doppler shifts and n is the Doppler spread index parameter. By choosing h and l for various values of ω_s and ω_e, the nominal Doppler shift moves away from the shift obtained in thefirst order model. The skewness of the Doppler spread is due to the interaction of the new additional spectral component. If we let m=h and l=0 then the second order TPS reduces to the first order TPS obtained previously.

3.5 Summary

A general scalar model of wave scattering from statistically rough spatial-temporal stationary surfaces has been shown to give good results for both the MSC and temporal frequency characteristics of the returns. Unlike previous models which require independent considerations of the Bragg scatter for the Doppler shift using geometrical optics and sea wave angular spectrum for the magnitude of the MSC using small surface slope perturbation[12,14,19], this model uses physical optics which requires the knowledge of a temporal-spatial surface correlation function. The temporal-spatial surface height correlation function is a gross resultant of the interaction of gravity, wind and ocean wave velocity and it is a accurately measured quantity. Because of nonlinear dependence of the scattered return on the surface height correlation function, the spectra contain higher order Bragg resonance terms. Also physical optics analysis not only predicts the correct partial side-spectral components but also small interaction components as well

At large sea wave number height order MSCs increase leading to increased non-coherent effects. These effects cause the observed Plateau[2] phenomena. The asymmetry in the equal order spectral magnitudes are due to magnitude and direction of the wind and the direction of the incident wave. The dynamic model also predicts a quasi-static spectral component, a component that has a spectral spread but no source radian frequency shift. This component appears in reported data[8,10,17,18] has been denoted as a land echo. In general higher order spectra has wider spreads with lower peak values.

To predict second order effects of the Doppler spectral data, which spectral peaks shift away from the nominal Doppler frequencies and Doppler peak skew from the symmetry, we introduced a second order surface height correlation function based on the reported data[21,23]. The second order surface correlation function is decomposed into two ocean surface waves and these waves produce a number of higher order spectral and corrected primary components which contribute each primary spectral component skewness. The quasi-static component, as compared with first order model, decreases to half of the first order peak. The second order model is applicable if illuminating source is higher than 4 MHz because high frequency forms typical Rayleigh parameter for small vertical roughness parameter. In Fig.3.5 we plot a typical first and second order TPS's together with identical values of the common parameters.

CHAPTER IV DOPPLER SPECTRA OF A MOVING SOURCE-OBSERVER TRAVELLING ABOVE A STATIONARY ROUGH SURFACE

4.1 Introduction

An accurate description of scattering mechanism from rough sutface due to a radiating moving source observer has practical interest. Aircraft-satellite altimetry and commnication, airborne radar Doppler velocity measuring system, terrain imaging and target surveillance radar etc. require extensive informations of Doppler frequency characteristics of the scattering from rough surfaces. Doppler return power frequency spectrum may be characterized by Doppler frequency shift and spread of the backscattered radiation due to a relative velocity between moving source and stationary rough surface. The first Doppler return power spectral calculation from rough surface[1,2,3] have been limited to a ray optics computation from fixed mean surfaces. The computations neglected the effect of the surface roughness. The effect of the Doppler spectral spread due to a surface roughness was considered recently[4,5]. Sohel[4] approaches the problem by using a physical optics approximation. However in his model the spectral spread was due to the intensity distribution of the incident beam while spectral spread broadens when surface becomes smooth. Berger's model[5] which is based on geometrical optics, assimes a Gaussian random delay due to a range fluctuation devated from equal range speckle points in the scattering ray tube. He then assumes that the Gaussian delay is due to a Gaussian surface height distribution.

For the Doppler return of our study we will use physical optics scattering model with source radiation that is due to a monochromatic i.e. linearly polarized time harmonic, plane wave. The surface is a height Gaussian random process independent of time. The static surface assumption allows us to consider the Doppler spectral characteristics of the moving source observer. The object of the study is to obtain the mean Doppler power spectrum as a function of the rough surface parameters, the direction and polarization of the incident radiation, the speed and direction of the source velocity.

In section two, the field scattered from a static Gaussian surface height process due to a moving source is developed.

In section three, the normalized mean Doppler spectrum(NMDS) is derived. The results are presented in terms of normalized surface roughness parameters. In the last section, the summary of the results and NMDS plots for various parameters are presented.

4.2 Model of the Doppler Return

It is known that the scattered linearly polarized electromagnetic field due to a time harmonic plane wave incident on an arbitrary surface can be obtained from the Kirchhoff-Helmholtz integral[6]

$$\mathcal{E}(t,R) = \frac{je^{j(kR-\omega_c t)}}{4\pi R}\int_{S_o} F\, e^{-j(\underline{P}_1\cdot\,\underline{r})} dS_o \qquad (4.2\text{-}1)$$

where $k=\omega_c/c$ is the free space wave number with ω_c as the radian frequency of the source and c is the velocity of the light in free space. In Eq.(4.2-1), R is the distance from origin to the observation point, P(R), $\underline{r}$ is the local radius vector

$$\underline{r} = x\underline{a}_x + y\underline{a}_y + \zeta(x,y)\underline{a}_z \qquad (4.2\text{-}2)$$

and

$$F = (R_F\underline{P}_1 - \underline{P}_2)\cdot\,\underline{a}_n \qquad (4.2\text{-}3)$$

where $\underline{a}_n$ is the local surface normal unit vector

$$\underline{a}_n = -\frac{\frac{\partial\zeta}{\partial x}\underline{a}_x + \frac{\partial\zeta}{\partial y}\underline{a}_y - \underline{a}_z}{\{(\frac{\partial\zeta}{\partial x})^2 + (\frac{\partial\zeta}{\partial y})^2 + 1\}^{\frac{1}{2}}} \qquad (4.2\text{-}4)$$

R_F is the Fresnel reflection coefficient which depends on the polarization and the material property of the surface, $\underline{P}_{1,2}$ are

$$\underline{P}_{1,2} = \underline{k}_i \mp \underline{k}_s \qquad (4.2\text{-}5)$$

where

$$\underline{k}_{i,s} = k\{\sin(\begin{smallmatrix}u_i\\u_s\end{smallmatrix})\cos(\begin{smallmatrix}v_i\\v_s\end{smallmatrix})\underline{a}_x + \sin(\begin{smallmatrix}u_i\\u_s\end{smallmatrix})\sin(\begin{smallmatrix}v_i\\v_s\end{smallmatrix})\underline{a}_y \mp \cos(\begin{smallmatrix}u_i\\u_s\end{smallmatrix})\underline{a}_z\} \qquad (4.2\text{-}6)$$

are incident and scattered wave vectors, respectively. The angles u_i,$_s$ and v_i,$_s$ are the angles of elevation and azimuth of incident and scattered waves, respectively. By projecting the differential surface area dS_o on the x-y plane, the scattered wave of Eq.(4.2-1)

$$\mathcal{E}(t,R) = \frac{je^{j(kR-\omega_c t)}}{4\pi R} \int_S F_n e^{-j\underline{P}_1 \cdot \underline{r}}\, dxdy \qquad (4.2\text{-}7)$$

where

$$F_n = (R_F \underline{P}_1 - \underline{P}_2) \cdot \left(-\frac{\partial \zeta}{\partial x}\underline{a}_x - \frac{\partial \zeta}{\partial y}\underline{a}_y + \underline{a}_z\right) \qquad (4.2\text{-}8)$$

We note that the field reradiated from a static rough surface due to a time harmonic plane wave incidence is a time harmonic spherical wave with amplitude by surface current transform

We are interested in that part of the scattered field that is scattered back to a moving source-observer. Let the velocity vector of the source

$$\underline{V} = V(\sin\alpha\cos\beta\,\underline{a}_x + \sin\alpha\sin\beta\,\underline{a}_y + \cos\alpha\,\underline{a}_z) \qquad (4.2\text{-}9)$$

where α and β are the elevation angle and azimuthal angle of the velocity vector. The time it takes the signal to propagate from the source to the observer at the surface and back to the moving source(See Fig.4.1)

$$\tau(t) = \frac{2R}{c} + \left(\frac{Vt}{2cR}\right)^2 - \frac{t}{2cR}\underline{k}_i \cdot \underline{V} \qquad (4.2\text{-}10)$$

where 2R is the approximate propagation distance. From Fig. 4.2 we note that the scattered wave vector $\underline{k}_s$ can be written

$$\underline{k}_s = -\underline{k}_i + \frac{kt}{R}\underline{V} \qquad (4.2\text{-}11)$$

The second term represents an effective bidirectional scattering. However when the source is stationary, i.e. V=0, the scattering is backscattering. Substituting Eqs.(4.2-10) and (4.2-11) into Eq.(4.2-7), we have

$$\mathcal{E}(t,R) = \frac{je^{j(kR-\phi)}}{4\pi R}\int_S F_n e^{-j(2\underline{k}_i-\frac{tk}{R}\underline{V})\cdot\underline{r}}dxdy \qquad (4.2\text{-}12)$$

where

$$\phi(t) = \omega_c t - \underline{k}_i\cdot\underline{V}t + \frac{2\pi V^2}{\lambda R}\tfrac{1}{2}t^2 \qquad (4.2\text{-}13)$$

The second and third terms of Eq.(4.2-13) are first and second order geometric optical Doppler shifts. The quadradic phase term $2\pi V^2/\lambda R$ is known as the linear FM rate[3,8,9] and it represents a complex modulation term. We will omit this term because it is independent of the surface roughness. Our final model of the Doppler scattered return field suitable for statistical computation is

$$\mathcal{E}(t,R) = \frac{je^{jkR-j\omega_o t}}{4\pi R}\int_S F_n e^{-j(2\underline{k}_i-\frac{tk}{R}\underline{V})\cdot\underline{r}}dxdy \qquad (4.2\text{-}14)$$

where ω_o is an apparent source radian frequency as

$$\omega_o = \omega_c - \underline{k}_i\cdot\underline{V} \qquad (4.2\text{-}15)$$

First, we will consider the case where the incident plane wave is horizontally polarized and the surface is an ideally conductor, then the Fresnel reflection coefficient reduces to $R_F = -1$ and F becomes

$$F_n^- = -2k(\cos u_i + \sin u_i \cos v_i\frac{\partial \zeta}{\partial x} + \sin u_i \sin v_i\frac{\partial \zeta}{\partial y}) \qquad (4.2\text{-}16)$$

since the factor $(R_F\underline{P}_1 - \underline{P}_2)$ is $-2\underline{k}_i$ and F_n^- denotes the case of the horizontal polarization. The scattered return for the horizontal polarization is therefore

$$\mathcal{E}(t,R) = Ae^{-j\omega_o t} \int_S F^- \, e^{-j(2\underline{k}_i - \frac{tk}{R}\underline{V})\cdot\underline{r}} dxdy \qquad (4.2\text{-}17)$$

where F^- and A are

$$F^- = -F_n^-/2k \qquad (4.2\text{-}18)$$

$$A = -\frac{jke^{jkR}}{2\pi R} \qquad (4.2\text{-}19)$$

When the incident plane wave is vertically polarized, the Fresnel reflection coefficient reduces for an ideally conductor $R_F = 1$, then the factor $(R_F\underline{P}_1 - \underline{P}_2)$ becomes $-2\underline{k}_s$. Using the Eq.(4.2-11), we have

$$\mathcal{E}^+(t,R) = \mathcal{E}(t,R) + \mathcal{E}^c(t,R) \qquad (4.2\text{-}20)$$

where

$$\mathcal{E}^c(t,R) = Ae^{-j\omega_o t}\int_S \frac{tk}{R}(\underline{V}\cdot\underline{a}_n)\, e^{-j(2\underline{k}_i \cdot \frac{tk}{R}\underline{V})\cdot\underline{r}} dxdy \qquad (4.2\text{-}21)$$

and $\mathcal{E}(t,R)$ is the horizontally polarized scattered return field. Thus $\mathcal{E}^c$ is a correction term to the horizontal scattering for the vertical polarization.

The field $\mathcal{E}(t,R)$ is a random process since it depends on the surface height $\xi(x,y)$ which is itself a random process. The random process $\xi(x,y)$ is assumed to be a real Gaissian random process of two independent variables x and y. The process is characterized by its mean value which is zero, and correlation function $\sigma^2 C$ where σ^2 is variance and C is normalized correlation function. The process is assumed to be isotropic, that is, it is a mean square stationary Gaussian process with contant variance and normalized correlation function dependent on the radial variable z only where

$$z^2 = (x-x')^2 + (y-y')^2 \tag{4.2-22}$$

We shall use a normalized Gaussian correlation function

$$C = e^{-z^2/L^2} \tag{4.2-23}$$

where L is the correlation distance.

Our ultimate interest is in the spectral representation of the field. For stationary process, the power spectrum is Fourier transform of the autocorrelation function of the return signal. However, in our case the second order moment of the scattered field $Q(t_1,t_2)$ is nonstationary where

$$Q(t_1,t_2) = \langle \mathcal{E}(t_1,R)\ \mathcal{E}^*(t_2,R) \rangle \tag{4.2-24}$$

This $Q(t_1,t_2)$ is also known as a self-coherence function of the field at point P(R)[17]. For a nonstationary process, the time averaged spectrum is appropriate[7], where averaged spectrum $W(\omega)$ is the Fourier transform of the time averaged second order moment

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} Q(t_1,t_2)\, dt_1 \tag{4.2-25}$$

$$W(\omega) = \int_{-\infty}^{\infty} R_\varepsilon(\tau) e^{-j\omega\tau}\, d\tau \tag{4.2-26}$$

where $t_2 = t_1 + \tau$. The averaged spectrum $W(\omega)$ can also be found from bifrequency spectrum of $Q(t_1,t_2)$ as[7]

$$W(\omega) = \Gamma(\omega_1,\omega_2)\Big|_{\omega_1=\omega_2=\omega} = \iint_{-\infty}^{\infty} Q(t_1,t_2) e^{-j\omega_1 t_1 + j\omega_2 t_2} dt_1 dt_2 \tag{4.2-27}$$

We will evaluate the mean spectrum $W(\omega)$ by the use of Eq.(4.2-27).

It is convenient to work with the following normalized variables and let the normalized mean Doppler spectrum(NMDS) be

$$W_n(\omega) \triangleq \frac{4V^2}{S} W(\omega) \qquad (4.2\text{-}28)$$

and let normalized surface roughness parameters be

$$\ell = k\sigma , \qquad \sigma_n = \sigma/L \qquad (4.2\text{-}29)$$

with normalized frequencies

$$\bar{\omega} = \frac{\omega}{\omega_f}, \qquad \bar{\omega}_c = \frac{\omega}{\omega_f}, \qquad \bar{\omega}_d = \frac{\omega}{\omega_f} \qquad (4.2\text{-}30)$$

where $\omega_f = V/R$ is a velocity-range radian frequency, and σ_n and ℓ are the rms surface slope and Rayleigh parameter, respectively while $\bar{\omega}$, $\bar{\omega}_c$ and $\bar{\omega}_d$ are the normalized transform, source and Doppler radian frequency parameters, respectively.

4.3 The Evaluation of the Mean Doppler Spectrum(MDS)

The ensemble averaged scattering intensity $Q(t_1,t_2)$, from Eq.(4.2-17) is

$$Q(t_1,t_2) = A^2 e^{-i\omega_o(t_1-t_2)} \int_S \int_{S'} H\, e^{-jq\{\cos v(x-x')+\sin v(y-y')\}} \cdot e^{-j\frac{k}{R}\{t_1(V_x x+V_y y)+t_2(V_x x'+V_y y')\}} dx dx' dy dy' \quad (4.3\text{-}1)$$

where we dropped the subscript i from the angles of wave incidence and

$$q=2k\sin u \quad (4.3\text{-}2)$$

$$H = \cos^2 u M_1+\sin u\cos u(\cos v M_2+\sin v M_3)+\sin^2 u\{\tfrac{1}{2}\sin 2v(M_4+M_5) +\cos^2 v M_6+\sin^2 v M_7\} \quad (4.3\text{-}3)$$

The ensemble averaged quantities, the M functions, for the Gauss Gaussian random process can be evaluated by using the method of the Karhunen expansion[15] as

$$M_1=\langle X\rangle=\exp\{-\tfrac{1}{2}(pq_v)^2(t_1^2+t_2^2-2Ct_1t_2)+p^2q_v(1-C)(t_1+t_2)-p^2(1-C)\} \quad (4.3\text{-}4)$$

$$M_2=\langle(\frac{\partial\xi}{\partial x}+\frac{\partial\xi}{\partial x'})X\rangle = -j\sigma p[\frac{\partial C}{\partial x}-\frac{\partial C}{\partial x'}-q_v(t_2\frac{\partial C}{\partial x}-\frac{\partial C}{\partial x'}t_1)]M_1 \quad (4.3\text{-}5)$$

$$M_3=\langle(\frac{\partial\xi}{\partial y}+\frac{\partial\xi}{\partial y'})X\rangle = -j\sigma p[\frac{\partial C}{\partial y}-\frac{\partial C}{\partial y'}-q_v(t_2\frac{\partial C}{\partial y}-t_1\frac{\partial C}{\partial y'}]M_1 \quad (4.3\text{-}6)$$

$$M_4=\langle\frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial y'}X\rangle = \sigma^2[\frac{\partial^2 C}{\partial x\partial y'}+p^2(1-t_1q_v)(1-t_2q_v)\frac{\partial C}{\partial x}\frac{\partial C}{\partial y'}]M_1 \quad (4.3\text{-}7)$$

$$M_5=\langle\frac{\partial\xi}{\partial x'}\frac{\partial\xi}{\partial y}X\rangle = \sigma^2[\frac{\partial^2 C}{\partial x'\partial y}+p^2(1-t_1q_v)(1-t_2q_v)\frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}]M_1 \quad (4.3\text{-}8)$$

$$M_6=\langle\frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial x'}X\rangle = \sigma^2[\frac{\partial^2 C}{\partial x\partial x'}+p^2(1-t_1q_v)(1-t_2q_v)\frac{\partial C}{\partial x}\frac{\partial C}{\partial x'}]M_1 \quad (4.3\text{-}9)$$

$$M_7=\langle\frac{\partial\xi}{\partial y}\frac{\partial\xi}{\partial y'}X\rangle = \sigma^2[\frac{\partial^2 C}{\partial y\partial y'}+p^2(1-t_1q_v)(1-t_2q_v)\frac{\partial C}{\partial y}\frac{\partial C}{\partial y'}]M_1 \quad (4.3\text{-}10)$$

where

$$X = e^{-j2k\cos u[(1-t_1 q_v)\zeta(x,y)-(1-t_2 q_v)\zeta(x',y')]} \qquad (4.3\text{-}11)$$

$$p = 2k\sigma\cos u \qquad (4.3\text{-}12)$$

$$q_v = \frac{V_z}{2R\cos u} = \frac{w_f \cos u}{2\cos u} \qquad (4.3\text{-}13)$$

and σ^2 and C are the variance and normalized correlation function of the surface height process $\zeta(x,y)$. Substituting these M functions into H and grouping them in terms of the order of t_1 and t_2, we have

$$H = M_1(H_0 + t_1H_1 + t_2H_2 + t_1t_2H_3) \qquad (4.3\text{-}14)$$

where

$$H_0=\cos^2u - j\tfrac{1}{2}\sigma p\sin 2u\left[\cos v\left(\frac{\partial C}{\partial x}-\frac{\partial C}{\partial x'}\right)+\sin v\left(\frac{\partial C}{\partial y}-\frac{\partial C}{\partial y'}\right)\right]+\sigma^2\sin^2u\left[\tfrac{1}{2}\sin 2v\,\cdot\right.$$

$$\cdot\left(\frac{\partial^2C}{\partial x\partial y'}+\frac{\partial^2C}{\partial x'\partial y}+p^2\frac{\partial C}{\partial x}\frac{\partial C}{\partial y}+p^2\frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}\right)+\cos^2v\left(\frac{\partial^2C}{\partial x\partial x'}+p^2\frac{\partial C}{\partial x}\frac{\partial C}{\partial x'}\right)$$

$$\left.+\sin^2v\left(\frac{\partial^2C}{\partial y\partial y'}+p^2\frac{\partial C\partial C}{\partial y\partial y'}\right)\right] \qquad (4.3\text{-}15)$$

$$H_1=-j\tfrac{1}{2}\sigma p\sin 2uq_v\left[\cos v\frac{\partial C}{\partial x'}+\sin v\frac{\partial C}{\partial y'}\right] \qquad (4.3\text{-}16)$$

$$-\sigma^2p^2\sin^2uq_v\left[\tfrac{1}{2}\sin 2v\left(\frac{\partial C}{\partial x\partial y'}+\frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}\right)+\cos^2v\frac{\partial C}{\partial x}\frac{\partial C}{\partial x'}+\sin^2v\frac{\partial C}{\partial y}\frac{\partial C}{\partial y'}\right]$$

$$H_2= -j\tfrac{1}{2}\sigma p\sin 2uq_v\left[\cos v\frac{\partial C}{\partial x}+\sin v\frac{\partial C}{\partial y}\right] \qquad (4.3\text{-}17)$$

$$-\sigma^2p^2\sin^2uq_v\left[\tfrac{1}{2}\sin 2v\left(\frac{\partial C}{\partial x}\frac{\partial C}{\partial y'}+\frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}\right)+\cos^2v\frac{\partial C}{\partial x\partial x'}+\sin^2v\frac{\partial C}{\partial y}\frac{\partial C}{\partial y'}\right]$$

and

$$H_3= {}^2p^2\sin^2uq_v^2\left[\tfrac{1}{2}\sin 2v\left(\frac{\partial C}{\partial x\partial y'}\frac{\partial C}{\partial x'}\frac{\partial C}{\partial y}\right)+\cos^2v\frac{\partial C}{\partial x}\frac{\partial C}{\partial x'}+\sin^2v\frac{\partial C}{\partial y}\frac{\partial C}{\partial y'}\right] \qquad (4.3\text{-}18)$$

Corresponding to the H_i, we will separate $Q(t_1,t_2)$ and the corresponding bifrequency spectrum into four terms as

$$Q(t_1,t_2)=Q_0(t_1,t_2)+t_1Q_1(t_1,t_2)+t_2Q(t_1,t_2)+t_1t_2Q_3(t_1,t_2) \qquad (4.3\text{-}19)$$

$$\Gamma(w_1,w_2)=\Gamma_0(w_1,w_2)+\Gamma_1(w_1,w_2)+\Gamma_2(w_1,w_2)+\Gamma_3(w_1,w_2) \qquad (4.3\text{-}20)$$

where

$$\Gamma_1(w_1,w_2)=-j\frac{\partial}{\partial w_1}\bar{\Gamma}_1(w_1,w_2) \qquad (4.3\text{-}21)$$

$$\Gamma_2(w_1,w_2)=j\frac{\partial}{\partial w_2}\bar{\Gamma}_2(w_1,w_2) \qquad (4.3\text{-}22)$$

$$\Gamma_3(w_1,w_2)=\frac{\partial^2}{\partial w_1\partial w_2}\bar{\Gamma}_3(w_1,w_2) \qquad (4.3\text{-}23)$$

with

$$\bar{\bar{\Gamma}}_i(w_1,w_2)=\mathfrak{F}_{w_1,w_2}[Q_i(t_1,t_2)] \qquad (4.3\text{-}24)$$

and Eq.(4.3-24) denotes double Fourier transform. Each partial bifrequency spectrum $\Gamma_i(w_1,w_2)$ has six integrations, four geometric and two transform integrations. When we perform any of six integrations, the new integral will have a singularity which then requires the integration of a singular integral in the next step. We remove this singularity by changing the transform integrals into a different form. The final result will be the MDS

$$W(w)=\Gamma(w,w)=\sum_{m=0}^{3}W_m(w) \qquad (4.3\text{-}25)$$

where W_m are the partial MDS. The partial bifrequency spectrum $\Gamma_i(w_1,w_2)$ transformed(See Appendix E) into

$$\Gamma_i=A^2\iint_{-\infty}^{\infty}P_i e^{-\frac{1}{2}(pq_v)^2(s_1^2+s_2^2)-j\{(w_1+w_o)(s_1-q_v^{-1})+(w_2+w_o)(s_2-q_v^{-1})\}}ds_1ds_2 \qquad (4.3\text{-}26)$$

where s_1 and s_2 are dummy integration variables and P_i are geometrical integrals

$$P_i = \iint_{SS'} H_i e^{-Cp^2 q_V^2 s_1 s_2 - j\{\frac{kV}{2R}x(s_1+s_2)(x+x') + \frac{kV}{2R}y(s_1+s_2)(y+y')\}} .$$
$$.e^{-j(y-y')\{\frac{kV}{R}y(q_V^{-1}+\frac{1}{2}s_1-\frac{1}{2}s_2)+q\sin v\}} .$$
$$.e^{-j(x-x')\{\frac{kV}{R}x(q_V^{-1}+\frac{1}{2}s_1-\frac{1}{2}s_2)+q\cos v\}} dx\,dx'\,dy\,dy' \quad (4.3\text{-}27)$$

The other partial spectra differ from $\Gamma_o(\omega_1, \omega_2)$ in the geometric integrals and derivatives with respect to radian transform frequencies. We will evaluate each of the partial spectra by first integrating geometrical integrals, P_i and then by integrating with respect to dummy variables.

The integrals P_i can be factored as(See AppendixE)

$$P_i = B.T_i \quad (4.3\text{-}28)$$

and changed to cylindrical coordinates as

$$B = \int_0^a \int_0^{2\pi} e^{-jh_2 z_2 \cos(\theta_2 - \phi_V)} z_2 d\theta_2 dz_2 \quad (4.3\text{-}29)$$

$$T_i = \int_0^\infty \int_0^{2\pi} H_i e^{-Cp^2 q_V^2 s_1 s_2 - jA_t z_1 \sin(\theta_1 + \phi_A) - jh_1 z_1 \cos(\theta_1 - \phi_V)} z_1 d\theta_1 dz_1 \quad (4.3\text{-}30)$$

by using cylindrical variables

$$z_{1,2}^2 = (x \mp x')^2 + (y \mp y')^2, \quad \theta_{1,2} = \tan^{-1}(x \mp x')/(y \mp y') \quad (4.3\text{-}31)$$

where index 1 is for the minus sign and 2 for the plus sign with

$$h_{1,2} = kV_t(s_1 \mp s_2)/2R \quad (4.3\text{-}32)$$

$$V_t^2 = V_x^2 + V_y^2 , \qquad \phi_V = \tan^{-1} V_x/V_y = \frac{1}{2}\pi - \beta \quad (4.3\text{-}33)$$

$$A_t^2 = A_x^2 + A_y^2, \qquad \phi_A = \tan^{-1} A_y/A_x \tag{4.3-34}$$

where

$$A_{x,y} = \frac{kV_{x,y}}{q_v R} + q_{\sin}^{\cos}(v) \tag{4.3-35}$$

The envelope functions of the integrand are also transformed in the new coordinates

$$H_0 = \cos^2 u - \tfrac{1}{2}\sigma^2 L_2 \sin^2 u - j\sigma p \sin 2u \frac{dC}{dz_1} \sin(\theta_1 + v) + \tfrac{1}{2}\sigma^2 L_1 \sin^2 u \cos 2(\theta_1 + v) \tag{4.3-36}$$

where $L_{1,2}$ are given Eq.(2.2-41) and

$$H_1 + H_2 = -\tfrac{1}{2}\sigma p \sin 2u q_v \frac{dC}{dz_1} \sin(\theta_1 + v) + \sigma^2 p^2 q_v \left(\frac{dC}{dz_1}\right)^2 \sin^2 u [1 - \cos 2(\theta_1 + v)] \tag{4.3-37}$$

$$\triangleq H_{12}$$

$$H_3 = -\tfrac{1}{2}\sigma^2 p^2 q_v^2 \left(\frac{dC}{dz_1}\right)^2 \sin^2 u [1 - \cos 2(\theta_1 + v)] \tag{4.3-38}$$

Since the envelope T_i are damping within the integration intervals of the integrations, the intervals are extended to infinity. The integral B is an Airy function

$$B = 2\pi \int_o^a z_2 \, J_o(h_2 z_2) dz_2 = S \, \frac{J_1(h_2 a)}{h_2 a} \tag{4.3-39}$$

where $S = 2\pi a^2$ is an effective scattering area.

We first consider the calculation of the dominant bifrequency spectrum $\Gamma_o(\omega_1, \omega_2)$ which in turn requires T_o. For the evaluation of T_o, we expand one of the phase term into infinite series

$$e^{-jh_1 z_1 \cos(\theta_1 - \phi_V)} = \sum_n (-j)^n \epsilon_n J_n(h_1 z_1) \cos n(\theta_1 - \phi_V) \tag{4.3-40}$$

$$\triangleq \bar{I} \cos n(\theta_1 - \phi_V)$$

where ϵ_n is the Neumann constant which is unity for n=0 and 2 for all non-zero n and $J_n(x)$ is the nth order Bessel function of the first kind. Substituting Eq.(4.3-40) into Eq.(4.3-30), T_o can be written

$$T_o = \int_o^\infty dz_1 z_1 e^{-Cp^2 q_v^2 s_1 s_2} \bar{I}[(\cos^2 u - \tfrac{1}{2}\sigma^2 L_2 \sin^2 u)\Phi_o - j\sigma p \sin 2u \frac{dC}{dz_1}\Phi_1 - \tfrac{1}{2}\sigma^2 L_1 \sin^2 u(\sin 2v \Phi_2 - \cos 2v \Phi_3)] \quad (4.3\text{-}41)$$

where $_i$ are the angular variable integrations

$$\Phi_{o,1,2,3} = \int_o^{2\pi} \begin{bmatrix} 1 \\ \sin(\theta_1+v) \\ \sin 2\theta_1 \\ \cos 2\theta_1 \end{bmatrix} \cos n(\theta_1+v) e^{-jA_t z_1 \sin(\theta_1+\phi_A)} d\theta \quad (4.3\text{-}42)$$

These angular integrals are evaluated by the Richard-Wolf[16] identities given by Eq.(2.3-2). The results for the $_i$ integrals are

$$\Phi_o = (-j)^n 2 2\pi J_n(A_t z_1)\cos n(\pi/2 - \phi_V - \phi_A) \quad (4.3\text{-}43)$$

$$\Phi_1 = \pi \cos v[\cos n\phi_V \{D_s(n+1) - D_s(n-1)\} + \sin n\phi_V\{-D_c(n+1) + D_c(n-1)\}]$$
$$\pi \sin v[\sin n\phi_V \{D_c(n+1) + D_c(n-1)\} + \sin n\phi_V\{D_s(n+1) + D_s(n-1)\}] \quad (4.3\text{-}44)$$

$$\Phi_{2,3} = \pi {\cos \atop \sin}(n\phi_V)[\pm D_c(n+2) + D_c(n-2)] + \pi {\sin \atop \cos}(n\phi_V)[D_s(n+2) \pm D_s(n-2)] \quad (4.3\text{-}45)$$

where

$$D_{c,s}(n \pm m) = (-j)^{n+m} J_{n+m}(A_t z_1) {\cos \atop \sin}(n+m)(\tfrac{1}{2}\pi - \phi_A) \quad (4.3\text{-}46)$$

We note that the integrand of T_o is an infinite series of $\bar{I}\cdot\Phi_i$. This infinite series can be reduced to finite form by the use of Graf's generalized Neumann addition theorem[10] which for $Z > z$

$$e^{jn\nu}J_n(r) = \sum_n J_{m+n}(Z)J_m(z)e^{jm\mu} \tag{4.3-47}$$

where

$$r^2 = Z^2+z^2-2Zz\cos\mu \tag{4.3-48}$$

and

$$\nu = \tan^{-1}\frac{z\sin\mu}{Z-z\cos\mu} \tag{4.3-49}$$

For example $\bar{I}\,\Phi_o$ becomes

$$\bar{I}\,\Phi_o=2\pi\sum_n(-j)^{2n}\epsilon_n J_n(h_1z_1)J_n(A_tz_1)\cos(\tfrac{1}{2}\pi-\phi_V-\phi_A)$$

$$=2\pi\ J_o(dz_1) \tag{4.3-50}$$

where

$$d^2 \doteq A_t^2 + h^2+2A_th_1\sin(\phi_V+\phi_A) \tag{4.3-51}$$

Similarly

$$\bar{I}\Phi_1=(-j)2\pi\cos(\phi_A-v-\nu)\ J_1(dz_1) \tag{4.3-52}$$

$$\bar{I}\Phi_{2,3}=\pm2\pi\cos2(\phi_A-v-\nu)\ J_2(dz_1) \tag{4.3-53}$$

with

$$\nu = \tan^{-1}[\ h_1\cos(\phi_A+\phi_V)/\{A_t-h_1\sin(\phi_A+\phi_V)\}] \tag{4.3-54}$$

This step completes the angular variable integration of T_o.
Substituting $\bar{I}_i$ into Eq.(4.3-41), we have

$$T_o=2\pi\int_o^\infty dz_1z_1e^{-Cp^2q_V^2s_1s_2}[(\cos^2u-\tfrac{1}{2}\sigma^2L_2\sin^2u)\ J_o(dz_1)$$
$$-\tfrac{1}{2}\sigma p\frac{dC}{dz_1}\sin2u\cos(\phi_A-v-\nu)J_1(dz_1)-\tfrac{1}{2}\sigma^2L_1\sin^2u\cos2(\phi_A-v-\nu)J_2(dz_1)] \tag{4.3-55}$$

The term $e^{-Cp^2q_V^2s_1s_2}$ is unity everywhere except near $z_1=0$.
therefore we approximate

$$e^{-Cp^2q_V^2s_1s_2} = 1-Cp^2q_V^2s_1s_2 \tag{4.3-56}$$

The Gaussian correlation function leads to $L_{1,2}$ as in the Eqs.(2.3-42) and (2.3-43). Substituting these $L_{1,2}$ into Eq.(4.3-55) we have

$$T_o = 2\pi(T_o^1 - p^2 q_V^2 s_1 s_2\ T_o^2) \tag{4.3-57}$$

where

$$\begin{aligned} T_o^1 = {} & \cos^2 u I_{100} - 2\sigma^2 L^{-2}\sin^2 u(-I_{110} + L^{-2} I_{310} + p^2 L^{-2} I_{320}) \\ & + \sigma p L^{-2} \sin 2u \cos(\phi_A - v - \nu) I_{211} \\ & - 2\sigma^2 L^{-4} \sin^2 u \cos 2(\phi_A - v - \nu)(I_{312} + p^2 I_{322}) \end{aligned} \tag{4.3-58}$$

$$\begin{aligned} T_o^2 = {} & \cos^2 u I_{110} - 2\sigma^2 L^{-2}\sin^2 u(-I_{120} + L^{-2} I_{320} + p^2 L^{-2} I_{330}) \\ & + \sigma p L^{-2} \sin 2u \cos(\phi_A - v - \nu) I_{221} \\ & - 2\sigma^2 L^{-4} \sin^2 u \cos 2(\phi_A - v - \nu)(I_{322} + p^2 I_{332}) \end{aligned} \tag{4.3-59}$$

with

$$I_{mni} = \int_0^\infty z_1^m e^{-nz_1^2/L^2} J_i(dz_1)dz_1 \tag{4.3-60}$$

The integrals I_{nmi} are identified as the Weber integrals[10] (See Eq.(2.3-12)). Substituting I_{nmi} into $T_o^{1,2}$, we have

$$\begin{aligned} T_o^1 = {} & \sqrt{\pi}\delta(d) + g_1 \tfrac{1}{2} L^2 \{\sigma^2 d^2 \sin^2 u(1-\cos 2\gamma) + \sigma p d \sin 2u \cos\gamma\} \\ & + g_2 2^{-5} \sigma^2 p^2 L^2 d^2 \sin^2 u(1-\cos 2\gamma) \end{aligned} \tag{4.3-61}$$

$$\begin{aligned} T_o^2 = {} & g_1 \tfrac{1}{2} L^2 \cos^2 u + g_2 2^{-5} L^2 \{12\sigma^2 \sin^2 u + \sigma^2 d^2 \sin^2 u(1-\cos 2\gamma) \\ & + 2\sigma p d \sin 2u \cos\gamma\} \end{aligned} \tag{4.3-62}$$

where

$$\gamma = \phi_A - v - \nu \tag{4.3-63}$$

This step completes the geometrical integrations.

The partial bifrequency spectrum $\Gamma_o(\omega_1,\omega_2)$ is

$$\Gamma_o=2\pi A^2 S e^{-j(\omega_1-\omega_2)/q_V}\iint_{-\infty}^{\infty}ds_1 ds_2\ \frac{J_1(ah_2)}{ah_2}\ (T_o^1-p^2q_V^2 s_1 s_2 T_o^2)\cdot$$
$$\cdot e^{-\frac{1}{2}p^2q_V^2(s_1^2+s_2^2)-j\frac{1}{2}[(\omega_1-\omega_2)(s_1-s_2)+(\omega_1+\omega_2+2\omega_o)(s_1+s_2)]} \qquad (4.3\text{-}64)$$

Since the integrand is function of $s_1 \pm s_2$, we now introduce new dummy variables

$$\binom{\tau}{t} = s_1 \pm s_2 \qquad (4.3\text{-}65)$$

Using Eq.(4.3-65), Γ_o is

$$\Gamma_o=2\pi A^2 S e^{-j(\omega_1-\omega_2)/q_V}\iint_{-\infty}^{\infty}dt d\tau\ \frac{J_1(\omega_u t)}{\omega_u t}[T_o^1+\frac{1}{2}p^2q_V^2(t^2+\tau^2)T_o^2]\cdot$$
$$\cdot e^{-\frac{1}{2}p^2q_V^2(t^2+\tau^2)-j\frac{1}{2}[(\omega_1-\omega_2)\ +(\omega_1+\omega_2+2\omega_o)t]} \qquad (4.3\text{-}66)$$

where we used the fact that

$$s_1^2 + s_2^2 = \tfrac{1}{2}(t^2 + \tau^2) \qquad (4.3\text{-}67)$$

$$s_1 s_2 = \tfrac{1}{2}(t^2 - \tau^2) \qquad (4.3\text{-}68)$$

with

$$\omega_u = kaV_t/2R \qquad (4.3\text{-}69)$$

We note that in the expression for Γ_o bifrequency ω_1 and ω_2 appear in the exponent only. Derivatives with respect to ω_1 and ω_2 yielding Γ_{12} and Γ_3, are equivalent to taking moments with respect to t_1 and t_2 as

$$\Gamma_{12}=A^2\iint_{-\infty}^{\infty}ds_1 ds_2 (P_1 j\frac{\partial}{\partial\omega_1}-P_2 j\frac{\partial}{\partial\omega_2})e^{-\frac{1}{2}p^2q_V^2(s_1^2+s_2^2)-j[(\omega_1+\omega_o)(s_1-q_V^{-1})}$$
$$^{+(\omega_2+\omega_o)(s_2-q_V^{-1})]} \qquad (4.3\text{-}70)$$

$$\Gamma_3 = A^2 \int\int_{-\infty}^{\infty} ds_1 ds_2 P_3 \frac{\partial^2}{\partial w_1 \partial w_2} e^{-\frac{1}{2}p^2 q_V^2 (s_1^2 + s_2^2) - j[(w_1 + w_o)(s_1 - \frac{1}{q_V})}$$

$$+ (w_2 + w_o)(s_2 - \frac{1}{q_V})] \qquad (4.3\text{-}71)$$

By letting $w = w_1 = w_2$ in the bifrequency spectrum, we have for the dominant partial mean Doppler spectrum(MDS)

$$W_o(w) = 2\pi A^2 S \int\int_{-\infty}^{\infty} \frac{J_1(w_u t)}{w_u t} [T_o - \tfrac{1}{2}p^2 q_V^2 (t^2 - \tau^2)] .$$

$$. e^{-\frac{1}{2}p^2 q_V^2 (t^2 + \tau^2) - j(w_o + w)t} dt d\tau \qquad (4.3\text{-}72)$$

We will first evaluate the t-integration and then we will perform τ-integration. Let the t-integral be denoted as

$$j_n = \int_{-\infty}^{\infty} t^n \frac{J_1(w_u t)}{w_u t} . e^{-\frac{1}{4}p^2 q_V^2 t^2 - j(w + w_o)t} dt \qquad (4.3\text{-}73)$$

we Have

$$W_o(w) = 2\ A^2 S \int\int_{-\infty}^{\infty} d\tau\ e^{-\frac{1}{4}p^2 q_V^2\ ^2} . [(T_o^1 + \tfrac{1}{2}p^2 q_V^2\ ^2 T_o^2) j_0 - \tfrac{1}{2}p^2 q_V^2 T_o^2\ j_2] \qquad (4.3\text{-}74)$$

The integral j_n can not be evaluated in a closed form. Therefore we first expand the circular sampling function $J_1(x)/x$ in an infinite series[10]

$$\frac{J_1(w_u t)}{w_u t} = \sum_m \frac{(-1)^m}{2 \cdot m! \cdot (m+1)!} (\tfrac{1}{2} w_u t)^{2m} \qquad (4.3\text{-}75)$$

By the use of Eq.(4.3-75), j_o is

$$j_o = \sum_m \frac{(\frac{1}{2} w_u)^{2m}}{2 \cdot m! (m+1)!} \int_{-\infty}^{\infty} (-jt)^{2m} e^{-\frac{1}{4}p^2 q_V^2 t^2 - j w_t t}\ dt$$

$$= \sum_m \frac{(\frac{1}{2} w_u)^{2m}}{2\ m! (m+1)!} \cdot \frac{d^{2m}}{d w_t^{2m}} (j_c) \qquad (4.3\text{-}76)$$

$$j_c = \int_{-\infty}^{\infty} e^{-\frac{1}{4}p^2 q_V^2 t^2 - j\omega_t t}\, dt = \frac{2\sqrt{\pi}}{pq_V} \exp\left[-\left(\frac{\omega_t}{pq_V}\right)^2\right] \tag{4.3-77}$$

and ω_t is shifted transform radian frequency variable as

$$\omega_t = \omega + \omega_o \tag{4.3-78}$$

Using the identity[11]

$$e^{-x^2} H_n(x) = (-1)^n \frac{d^n}{dx^n}(e^{-x^2}) \tag{4.3-79}$$

where $H_n(x)$ are the Hermite polynomials of order n, the final results of the j_o are

$$j_o = \tfrac{1}{2} j_c \sum_m \frac{1}{m!\cdot(m+1)!} \left(\frac{\omega_u}{2pq_V}\right)^{2m} H_{2m}(\bar{\omega}_t) \tag{4.3-80}$$

where

$$\bar{\omega}_t = \omega_t / pq_V \tag{4.3-81}$$

Since $H_n(\bar{\omega}_t)$ are even function of $\bar{\omega}_t$ for even n, therefore j_o is also symmetric with respect to $\bar{\omega}_t$. Similarly

$$j_2 = -\tfrac{1}{2} \frac{j_c}{p^2 q_V^2} \sum_m \frac{1}{m!\cdot(m+1)!} \left(\frac{\omega_u}{2pq_V}\right)^{2m} H_{2m} \tag{4.3-82}$$

Substituting j_o and j_2 into Eq.(4.3-74), we have

$$W_o(\omega) = \pi A^2 S j_c \sum_m \frac{\left(\frac{\omega_u}{2pq_V}\right)^{2m}}{m!\cdot(m+1)!} \left[H_{2m}\left(K_o^1(0) + \tfrac{1}{2}p^2 q_V^2 K_o^2(2)\right) + \tfrac{1}{2} H_{2m+2} K_o^2(0)\right] \tag{4.3-83}$$

where $K_i^{1,2}(n)$, the τ-integrations, are

$$K_i^{1,2}(n) = \int_{-\infty}^{\infty} \tau^n T_i^{1,2} \cdot e^{-\frac{1}{4}p^2 q_V^2 \tau^2}\, d\tau \tag{4.3-84}$$

The magnitude coefficients of the partial spectral components.

Using the similar calculations for the other partial MDS(See Appendix E), and the normalization condition defined by Eq.(4.2-28), we have for the NMDS

$$W_n(\bar{\omega}_t)=(A_o+A_1)\mathcal{W}_{n,2m}(\bar{\omega}_t)+A_2\mathcal{W}_{n,2m+2}(\bar{\omega}_t)+A_3\mathcal{W}_{n,2m+4}(\bar{\omega}_t) \qquad (4.3\text{-}85)$$

where $\mathcal{W}(\bar{\omega}_t)$ are spectral shape functions

$$\mathcal{W}_{n,2m+j}=e^{-\bar{\omega}_t^2/(\ell\sigma_n^2|\cos\alpha|)}\cdot\sum_m\frac{(\frac{\bar{a}}{4\ell}\tan\alpha)^{2m}}{m!(m+1)!}\cdot H_{2m+j} \qquad (4.3\text{-}86)$$

where

$$\bar{\omega}_t=(\bar{\omega}-\bar{\omega}_c-\bar{\omega}_d)/\ell\cos\alpha \qquad (4.3\text{-}87)$$

and magnitude coefficients of the spectra are listed in Table 4.1 in the order of the Rayleigh parameter ℓ. This step completes the evaluation of the NMDS.

The NMDS consists of two parts, a spectral shape function defined by Hermite polynomials and the magnitude of the spectrum defined by the coefficients A_j. The spectral center frequency shift is due to the angles of the wave incidence and source velocity vector. The spectral spread is due to the Rayleigh parameter and the vertical component of the source velocity. The higher components contribute the spectral skewness away from the dominant spectral shape. The higher order spectral components are generated by the effective patch length $\bar{a}$, which is associated with the elevation angle of the moving spurce, a result due to the expansion of the circular sampling function (See Eq.(4.3-75)). For a small value of $\bar{a}\sin\alpha$, the NMDS becomes

$$W_n=\frac{e^{-\bar{t}^2}}{\ell\sigma_n^2|\cos\alpha|}\cdot\{A_o+A_1+A_2H_2+A_3H_4\} \qquad (4.3\text{-}88)$$

because the circular sampling function is unity. When the elevation angle of the source velocity vector approaches $\pi/2$ and/or the Rayleigh parameter becomes zero, W_n reduces to a discrete spectrum. The first case represents a moving source travelling parallel to the mean surface while the second represents scattering from ideal flat surface. When the source moves in the vertical direction the patch effect disappears. When the source moves in the horizon al direction the full patch length contributes to the Doppler spectral spread. Actually for near horizontal direction of flight the circular sampling function series expansion of the NMDS is inappropriate. For large value of $\bar{a}$ with close to $\pi/2$ of angle α, the integral j_n can be approximated as

$$j_o = \int_{-\infty}^{\infty} \frac{J_1(\varpi_u t)}{\varpi_u t} e^{-j\varpi_u t} \, dt = \frac{1}{\varpi_u} \left[1 - (\varpi_t/\varpi_u)^2\right]^{\frac{1}{2}} \quad (4.3\text{-}89)$$

with j_2 and j_4 as zero. Therefore very large $\bar{a}$, corresponding to large value ϖ_u, the surface roughness contribution is negligible and the mean spectrum is discrete.

The magnitude coefficients A_j are dependent on the values u, v, α and β as well as on the normalized roughness parameters ℓ and σ_n. The expressions G_i and $\frac{n}{m}$ are exponential damping functions of the Rayleigh parameter ℓ. When the source illuminates the rough surface at the normal angle of incidence i.e. the case when u=0 all coefficient A_j are zero except

$$A_o = 2\sigma_n e^{-4\ell^2} \quad (4.3\text{-}90)$$

$$A_1 = (\tfrac{1}{2}+\tfrac{1}{2}\sigma_n^{-2})e^{-4\ell^2}\cdot[(4\sigma_n^2 \cos^2\alpha - \sin^2\alpha)/2^4\sigma_n^4\cos^2\alpha] \quad (4.3\text{-}91)$$

independent of v and β. First term is due to the coherent reflectance, which is proportional to the surface roughness, while other terms are flight correction terms. For an incident one dimensional slanted wave, the case where

$$v = \beta \tag{4.3-92}$$

the coefficient A_j can be simplified because many of the parameters reduce to simple forms such as $a_1=1$, $b_1=0$. $\Phi_m^h=0$, $\Lambda_m^n=1$ and ϕ_A becomes v. Consequently, the new parameters are

$$\bar{A}_t = 2(\tan\alpha\cos u + \sin u) \tag{4.3-93}$$

$$G_i = \exp\{-4\ell^2(\cos u + \sin u/\tan\alpha)^2\} \tag{4.3-94}$$

and all the factors $(1 - \cos\Phi_m^n \Lambda_m^n)$ are multiplied by zero. The coefficients A_j are

$$A_o = 2\sigma_n G_o \tag{4.3-95}$$

$$A_1 = \frac{\cos^2 u\cos^2\alpha}{2^3\ \bar{\delta}_1^2}\left(1+\frac{\sin^2\alpha\ \ \bar{A}_t^2}{2^5\sigma_n^4\ \bar{\delta}_1^2}\right)+\frac{\sin 2u\cos\alpha}{2^6\ \bar{\delta}_2^2}\left(\frac{\sin\alpha}{\bar{A}_t}+\frac{\cos^3 u\bar{A}_t^3}{2^7\sigma_n^4\ \bar{\delta}_2^2}\right) \tag{4.3-96}$$

The NMDS is zero for grazing angle of incidence. Finally we note that as expected, the NMDS is predominantly of Gaussian shape[1,3,13,14]. In Fig.4-3,4 we plot the NMDS for various values of scattering parameters.

Due the correction field ε^c of Eq,(4.2-21), the vertically polarized scattered field has more spectral spread because of the higher order moments generated by the t-dependencies in the integrand. However, the basic nature of the spectral spread is similar to the horizontal polarization except for small additional terms in the magnitude coefficients A_j.

4.4 Summary

A model of the mean Doppler frequency spectrum from rough surfaces by a moving source illumination has been formulated and analyzed using a physical optics approximation. The results provide a theorectical framework for studying experimental clutter data by moving source observer with regard to the surface roughness contribution to the mean Doppler spectral spread. The Doppler autocorrelation return function from rough surface is non-stationary. Mean Doppler spectrum has calculated for the horizontal polarization. The normalized mean Doppler spectrum (NMDS) is a function of Rayleigh, normalized surface roughness parameters and four angles, two elevations and two azimuthal angles of wave incident and source velocity vector.

The NMDS, consists of a spectral and magnitude coefficient parts, is characterized by a Doppler frequency shift from the source line spectrum. The spectral spread, generated by the nonstationary character of the scattered field, is due to the source motion and therefore a stationary source observer sees only line spectrum regardless surface roughness. The dominant NMDS is Gaussian like shape broadened from a line by the surface roughness and the directional cosine of the source velocity vector. The spectral broadening is proportional to the surface roughness and vertical velocity component. However, if the source moves parallel to the mean surface level, the spectrum is a line spectrum even though the surface is rough. The magnitude coefficients of the spectrum consists of two parts, a coherent and an incoherent reflectance. When surface is ideally flat, the coefficient due the coherent reflectance is zero, and the incoherent magnitude coefficient is the dominant term. This term is dependent on the all geometrical parameters. The all coefficients are zero for grazing angle of wave incidence. Finally we also noted that the NMDS depends on the scattering area. For wide beam, the peak of the spectrum is increasing while the spread is decreasing. This result correctly predicts the frequency characteristics of the Doppler return from rough surface by a moving source.

CHAPTER V RECOMMENDATION FOR FURTHER STUDIES

The direction of our investigations on the rough surface scattering has been being correct interpretations of the measured data, in terms of static and dynamic scattering cross section and Doppler temporal frequency spectra. However the obtained results can be used directly and indirectly to the various experimental and theoretical applications in various topics which suggested in recent studies. Some of these are; surface roughness determination by measuring scattering cross section[1-4], field coherence from rough surface[5,6], mobile radio reception[7], speckle pattern analysis, effect and application[8], reduction of speckle size[9], rough surface scattering channel and diversity[10,11], the improvements of holographic image construction by the use of diffusers[12,13], holographic coupler by use of random surface[14], hologram information capacity[15] and effect[16,17] by the film granular noises, guidance of surface wave by multilayer coating with irregularity[18], random mode locking[19], measurements of the moving surface velocity by the use of surface irregularities[20,21], the depolarization effects by the surface roughness[22,23] and the change of interaction between incident light and surface plasma oscillation by the roughness[24] etc. Most of these topics are recent experimental investigations and are analytically incomplete. To be completed additional investigations are necessary i.e. the scattering from dielectric rough surface, the forward and bidirectional scattering from layered medium with irregular interfaces and cross polarization effect by the rough surface. Further theoretical work could also be directed toward the temporal spatial physical optics approximation which carries more parameters than physical optics approximation. For an example, target detection in the clutter, clutter and temporal noise interaction, moving radar signal and noise ratio by the clutter troperspheric, ionospheric layer and mobile radio diversity channel etc. are of interest and temporal physical optics approximation analysis is appropriate, although toobtain the general random boundary condition or probability density functionals would be complecated task.

Quasi Specular Scattering	$\bar{\sigma}_c$	$g_0\ b$
	$\bar{\sigma}_{02}$	$g_2\ \bar{\sigma}_u^{-2}\ \ell^6\ [\frac{1}{8}a^2b + ab^2\sin(\upsilon+\phi_k) - \frac{1}{8}a^2b\cos 2(\upsilon+\phi_k)]$
	$\bar{\sigma}_{11}^{\pm}$	$g_1\ \bar{\sigma}_u^{-2}\ \ell^4\ [b^2 + \bar{K}_{1t}^{\pm}a^{\frac{1}{2}}b\sin(\upsilon+\phi_{1k}^{\pm}) + \frac{1}{4}\bar{f}_1 a \pm 2\bar{f}a^{\frac{1}{2}}b\cos(\alpha-\upsilon) \pm \frac{1}{4}\bar{f}a\bar{K}_{1t}^{\pm}(\bar{V}_1\sin\phi_{1k}^{\pm} + \bar{V}_2\cos\phi_{1k}^{\pm}) + \frac{1}{8}a\bar{K}_{1t}^{\pm 2}\{1-\cos 2(\upsilon+\phi_{1k}^{\pm})\}]$
	$\bar{\sigma}_{22}^{\pm}$	$g_2\ \bar{\sigma}_u^{-2}\ \ell^6\ [\frac{1}{16}ab\bar{K}_{2t}^{\pm 2}\{1-\cos 2(\upsilon+\phi_{2k}^{\pm})\} + \frac{1}{2}\bar{K}_{2t}^{\pm}a^{\frac{1}{2}}b\sin(\upsilon+\phi_{2k}^{\pm}) + \frac{1}{4}\bar{f}_1 ab \pm 2\bar{f}a^{\frac{1}{2}}b\cos(\alpha-\upsilon) \pm \frac{1}{4}\bar{f}\bar{K}_{2t}^{\pm}ab(\bar{V}_1\sin\phi_{2k}^{\pm} + \bar{V}_2\cos\phi_{2k}^{\pm})]$
	$\bar{\sigma}_{13}^{\pm}$	$g_3\ \bar{\sigma}_u^{-2}\ \ell^8\ [\frac{1}{18}\bar{K}_{1t}^{\pm 2} \pm \frac{1}{9}\bar{f}\bar{K}_{1t}^{\pm}(\bar{V}_1\sin\phi_{1k}^{\pm} + \bar{V}_2\cos\phi_{1k}^{\pm}) - \frac{1}{6}\bar{f}_1 - \frac{1}{18}\bar{K}_{1t}^{\pm 2}\cos 2(\upsilon+\phi_{1k}^{\pm})]\ ab^2$
	$\bar{\sigma}_{33}^{\pm}$	$g_3\ \bar{\sigma}_u^{-2}\ \ell^8\ [\frac{1}{3}\bar{f}_1 \pm \frac{1}{27}\bar{f}\bar{K}_{3t}^{\pm}(\bar{V}_1\sin\phi_{3k}^{\pm} + \bar{V}_2\cos\phi_{3k}^{\pm}) + \frac{1}{54}\bar{K}_{3t}^{\pm 2}\{1-\cos 2(\upsilon+\phi_{1k}^{\pm})\}]\ ab^2$
Specular Scattering	$\bar{\sigma}_c$	$g_0\ b$
	$\bar{\sigma}_{02}$	0
	$\bar{\sigma}_{11}^{\pm}$	$g_0\ \bar{\sigma}_u^{-2}\ \ell^4[b^2 + a^{\frac{1}{2}}b\sin(\upsilon+\alpha) + \binom{1}{0}\frac{b}{2}a\bar{f}\{1+\cos 2(\alpha-\upsilon)\} + \frac{1}{8}a \pm 2\bar{f}a^{\frac{1}{2}}b\cos(\alpha-\upsilon) - \frac{1}{16}ab\cos 2(\upsilon+\alpha)]$
	$\bar{\sigma}_{22}^{\pm}$	$g_0\ \bar{\sigma}_u^{-2}\ \ell^6[\frac{1}{16}ab + \frac{1}{2}a^{\frac{1}{2}}b\sin(\upsilon+\alpha) + \binom{1}{0}\frac{1}{2}ab\bar{f}\{1+\cos 2(\alpha-\upsilon)\} \pm 2\bar{f}a^{\frac{1}{2}}b\cos(\alpha-\upsilon) - \frac{1}{16}ab\cos 2(\upsilon+\alpha)]$
	$\bar{\sigma}_{13}^{\pm}$	$g_0\ \bar{\sigma}_u^{-2}\ \ell^8[\frac{1}{18}ab^2 - \binom{1}{5}\frac{1}{18}\bar{f}ab^2\{1+\cos 2(\alpha-\upsilon)\} - \frac{1}{18}ab^2\cos 2(\upsilon+\alpha)]$
	$\bar{\sigma}_{33}^{\pm}$	$g_0\ \bar{\sigma}_u^{-2}\ \ell^8[\frac{1}{18}ab^2 + \binom{10}{8}\frac{1}{27}\bar{f}ab^2\{1+\cos(\alpha-\upsilon)\} - \frac{1}{27}ab^2\cos 2(\upsilon+\alpha)]$
Backscattering	$\bar{\sigma}_c$	0
	$\bar{\sigma}_{02}$	$g_2'\ \bar{\sigma}_u^{-2}\ \ell^6[2ab^2 + a^2b]$
	$\bar{\sigma}_{11}^{\pm}$	$g_1'\ \bar{\sigma}_u^{-2}\ \ell^4[b^2 + \frac{1}{4}\bar{f}_1 a + \bar{K}_{1t}^{\pm}a^{\frac{1}{2}}b\sin(\upsilon+\phi_{1k}^{\pm}) \pm 2\bar{f}ab\cos(\alpha-\upsilon) \pm \frac{1}{4}\bar{f}a\bar{K}_{1t}^{\pm}(\bar{V}_1\sin\phi_{1k}^{\pm} + \bar{V}_2\cos\phi_{1k}^{\pm}) + \frac{1}{8}a\bar{K}_{1t}^{\pm 2}\{1-\cos 2(\upsilon+\alpha)\}]$
	$\bar{\sigma}_{22}^{\pm}$	$g_2'\ \bar{\sigma}_u^{-2}\ \ell^6[\frac{1}{2}a^{\frac{1}{2}}b\sin(\upsilon+\phi_{2k}^{\pm})\bar{K}_{2t}^{\pm} + \frac{1}{2}\bar{f}_1 ab \pm 2\bar{f}_1 a^{\frac{1}{2}}b\cos(\alpha-\upsilon) \pm \frac{1}{4}\bar{f}ab\bar{K}_{2t}^{\pm}(\bar{V}_1\sin\phi_{2k}^{\pm} + \bar{V}_2\cos\phi_{2k}^{\pm}) + \frac{1}{16}\bar{K}_{2t}^{\pm 2}ab\{1-\cos 2(\upsilon+\alpha)\}]$
	$\bar{\sigma}_{13}^{\pm}$	$g_3'\ \bar{\sigma}_u^{-2}\ \ell^8[\frac{1}{27}\bar{K}_{3t}^{\pm 2}\{1-\cos 2(\upsilon+\phi_{1k}^{\pm})\} \pm \frac{1}{27}\bar{f}\bar{K}_{1t}^{\pm}(\bar{V}_1\sin\phi_{1k}^{\pm} + \bar{V}_2\cos\phi_{1k}^{\pm}) - \frac{1}{6}\bar{f}_1]$
	$\bar{\sigma}_{33}^{\pm}$	$g_3'\ \bar{\sigma}_u^{-2}\ \ell^8[\frac{1}{3}\bar{f}_1 + \frac{1}{54}\bar{K}_{3t}^{\pm 2}\{1-\cos 2(\upsilon+\phi_{3k}^{\pm})\} \pm \frac{1}{27}\bar{f}\bar{K}_{3t}^{\pm}(\bar{V}_1\sin\phi_{3k}^{\pm} + \bar{V}_2\cos\phi_{3k}^{\pm})]$

Table 3-1. The Normalized Mean Scattering Cross Sections for Various Cases.

Quasi-Specular Scattering	$\bar{\sigma}_c$	$g_0 \cdot b$
	$\bar{\sigma}_{02}$	$g_2 \bar{\sigma}_u^{-2} \ell^6 \; ab[\frac{a}{2} + b\sin(v+\phi_k) - \frac{a}{2}\cos 2(v+\phi_k)]$
	$\bar{\sigma}_{11}^{\pm}$	$g_1 \bar{\sigma}_u^{-2} \ell^4 \cdot [b^2 + a^{\frac{1}{2}} b \bar{K}_{1t} \sin(v+\phi_k) + \frac{a}{8}\bar{K}_{1t}^2 - \frac{a}{8}\bar{K}_{1t}^2 \cos 2(v+\phi_{1k})]$
	$\bar{\sigma}_{22}^{\pm}$	$g_2 \bar{\sigma}_u^{-2} \ell^6 \cdot \frac{1}{2}\bar{K}_{2t} \cdot [\frac{ab}{8}\bar{K}_{2t} + a^{\frac{1}{2}} b^2 \sin(v+\phi_{1k}) - \frac{1}{8}\bar{K}_{2t}\, ab \cos 2(v+\phi_{2k})]$
	$\bar{\sigma}_{13}^{\pm}$	$g_3 \bar{\sigma}_u^{-2} \ell^8 \; \frac{1}{18} ab^2 \bar{K}_{\pm t} [1 - \cos 2(v+\phi_{1k})]$
	$\bar{\sigma}_{33}^{\pm}$	$g_3 \bar{\sigma}_u^{-2} \ell^8 \; \frac{1}{18} ab^2 \bar{K}_{3t} [1 - \cos 2(v+\phi_{1k})]$
Specular Scattering	$\bar{\sigma}_c$	$g_0 \cdot b$
	$\bar{\sigma}_{02}$	0
	$\bar{\sigma}_{11}^{\pm}$	$g_1 \bar{\sigma}_u^{-2} \ell^4 \; [b^2 + a^{\frac{1}{2}} b + \frac{1}{8} a]$
	$\bar{\sigma}_{22}^{\pm}$	$g_2 \bar{\sigma}_u^{-2} \ell^6 \; [\frac{1}{2} a^{\frac{1}{2}} b^2 + \frac{1}{16} ab]$
	$\bar{\sigma}_{13}^{\pm}$	$g_3 \bar{\sigma}_u^{-2} \ell^8 \; \frac{1}{9} ab^2$
	$\bar{\sigma}_{33}^{\pm}$	$g_3 \bar{\sigma}_u^{-2} \ell^8 \; \frac{1}{9} ab^2$
Backscattering	$\bar{\sigma}_c$	0
	$\bar{\sigma}_{02}$	$g_2' \bar{\sigma}_u^{-2} \ell^6 [2ab^2 + a^2 b]$
	$\bar{\sigma}_{11}^{\pm}$	$g_1' \bar{\sigma}_u^{-2} \ell^4 \; [b^2 + 2ab + a^2]$
	$\bar{\sigma}_{22}^{\pm}$	$g_2' \bar{\sigma}_u^{-2} \ell^6 \; [ab^2 + \frac{1}{2} a^2 b]$
	$\bar{\sigma}_{13}^{\pm}$	$g_3' \bar{\sigma}_u^{-2} \ell^8 \; \frac{4}{9} a^2 b^2$
	$\bar{\sigma}_{33}^{\pm}$	$g_3' \bar{\sigma}_u^{-2} \ell^8 \; \frac{4}{27} a^2 b^2$
notations for Tables		$a = \sin^2 u, \; b = \cos^2 u, \; \bar{V}_1 = \sin\alpha - \sin(\alpha+2v), \; \bar{V}_2 = \cos\alpha + \cos(\alpha+2v)$ $\bar{g}_m^h = g_0 \cdot g_m^h = \exp[-4b\ell^2] \cdot \exp[-a\ell^2\{1 - h\cos(v-v_s)\}/4m\bar{\sigma}_u^2]$ $\bar{K}_{mt}^{\pm 2} = [\sin u(\cos v - \cos v_s) \mp m\sin\alpha]^2 + [\sin u(\sin v - \sin v_s) \mp m\cos\alpha]^2$ $\phi_{mk}^{\pm} = \tan^{-1} \frac{\sin u(\cos v - \cos v_s) \mp m\sin\alpha}{\sin u(\sin v - \sin v_s) \mp m\cos\alpha}, \quad \bar{f}_1 = \bar{f} \cdot [1 + \cos 2(v-\alpha)]$

Table 3-2. The Normalized Mean Scattering Cross Sections
For Cross Wind.

A_0		$\frac{\sigma_u}{a} G_0$
$A_1 = \sum_{n=1}^{3} G_n \ell^m A^n_{1,m}$: [m=-1, 1, 3]	$A^1_{1,-1}$	$-\cos\Phi_1^2 \Lambda_1^2 \frac{\sin^2 u\, f^4 b^2}{2^6 A_t^2 \delta_1^4}$
	$A^1_{1,1}$	$\frac{\cos^2 u\, e^2}{2^3 \delta_1^2} + \frac{\sin^2 u\, f^2 b^2}{2^5 \delta_1^2}[(1-\cos\Phi_1^2\Lambda_1^2)+\sin\Phi_1^2\Lambda_1^2 \frac{f^2 a}{2^2\sigma_u^2\delta_1^2}]$
	$A^1_{1,3}$	$\frac{\cos^2 u\, e^2 f^2 A_t^2 a^2}{2^8 \sigma_u^4 \delta_1^4} - \sin\Phi_1^2\Lambda_1^2 \frac{f^2 a b \sin u}{2\delta_1^2}\frac{\sin^2 u\, A_t^2}{2^2}[(1-\cos\Phi_1^2\Lambda_1^2)(1-\frac{f^2a^2}{2^3\sigma_u^2\delta_1^2} - \frac{f^4 a^2 b}{2^8\sigma_u^4\delta_1^4})]$
	$A^2_{1,-1}$	$\cos\Phi_2^2\Lambda_2^2 \frac{3\sin^2 u\, f^4 b^3}{2^5\delta_2^4}(\frac{1}{A_t^2} - \frac{e^2}{2^5 A_t^2\delta_2^2} - \frac{3e^2f^2a^2}{2^{10}\sigma_u^2\delta_2^4}) + \sin\Phi_2^2\Lambda_2^2 \frac{\sin 2u\, f^4 b^3 e}{2^7\delta_2^6}(\frac{\cos u\, f b}{2^2 A_t^3} + \frac{3ea}{A_t^2} - \frac{3ef^2ab}{2^7\sigma_u^2\delta_2^2}) - \cos\Phi_2^1\Lambda_2^1 \frac{\sin 2u\, e f^3 a b^2}{2^9 A_t^2 \delta_2^4}$
	$A^2_{1,1}$	$(1-\cos\Phi_2^2\Lambda_2^2)\frac{3\sin^2 u}{2\delta_2^2}(\sigma_u^2 e^2 - \frac{f^2 b}{2^7} + \frac{e^2 f^2 b}{2^{10}\delta_2^2}) + \cos\Phi_2^2\Lambda_2^2 \frac{3\sin^2 u\, f^2 b^2}{2^8\delta_2^4}(\frac{\cos^2 u\, f a}{A_t} - \frac{e^2}{2^2} - \frac{e^2 f^2 a^2}{2^5\sigma_u^2\delta_2^2}) + \sin\Phi_2^2\Lambda_2^2 \frac{3\sin^2 u\, b f^2}{2^6\delta_2^2}(\frac{f^2 ab}{2^3\sigma_u^2\delta_2^2} - a - \frac{\cos^2 u\, e^2 b}{2^2 A_t^2 \delta_2^2} - \frac{3e^2 a}{2^3}) + \cos\Phi_2^1\Lambda_2^1 \frac{\sin 2u\, e f a}{2^6 A_t \delta_2^2} + \sin\Phi_2^1\Lambda_2^1 \frac{e f^3 a^2 b \sin 2u}{2^{10}\sigma_u^2\delta_2^4}$
	$A^2_{1,3}$	$(1-\cos\Phi_2^2\Lambda_2^2)\frac{3\sin^2 u\, A_t^2}{2^5\delta_2^2}(1+\frac{f^2e^2a^2}{2^3\delta_2^2} - \frac{f^2a^2}{2^4} + \frac{\cos u\, e a}{A_t} - \frac{3\cos u\, f^3 e a b}{2^6\sigma_u^2 A_t^2\delta_2^2} - \frac{e^2}{2^3} - \frac{3f^2e^2a^2}{2^7\delta_2^2} - \frac{3e^2f^2a^2b}{2^7\sigma_u^2\delta_2^4}) + \cos\Phi_2^2\Lambda_2^2\frac{3\sin^2 u\, f^4 a b^2}{2^8\sigma_u^2\delta_2^2} + \sin\Phi_2^2\Lambda_2^2 \frac{\cos u \sin^2 u\, f e b}{2^5\delta_2^2}(-A_t + \frac{A_t f^2 ab}{2^2\sigma_u^2\delta_2^2} - \frac{3fea}{\sigma_u^2\cos u}) + \cos\Phi_2^1\Lambda_2^1 \frac{\sin 2u\, A_t^2 e^4 a^3}{2^3\sigma_u^4\delta_2^4}$
	$A^3_{1,-1}$	$\cos\Phi_3^2\Lambda_3^2\frac{\cos^2 u \sin^2 u\, e^2 f^6 b^5}{2^6 3^3 A_t^3\delta_3^2} + \sin\Phi_3^2\Lambda_3^2\frac{\cos u\sin^2 u\, f^5 e b^4}{2^4 3^2 A_t}$
	$A^3_{1,1}$	$\cos\Phi_3^2\Lambda_3^2\frac{\cos^2 u\sin^2 u\, e f^3 b^2}{3^3\delta_3^2}(-\frac{a}{2^3 A_t} - \frac{efb}{3A_t^2\delta_3^2} - \frac{f^2 e a^2 b}{2^5\sigma_u^2\delta_3^4}) + \sin\Phi_3^2\Lambda_3^2\frac{\cos^2 u\sin^2 u\, e f^2 b^2}{3^2\delta_3^4}(-\frac{1}{2^4 A_t} + \frac{2ef^2ab}{3A_t^6\delta_3^2} - \frac{ef^4ab^2}{2^4 3^2\sigma_u^2\delta_3^4})$
	$A^3_{1,3}$	$(1-\cos\Phi_3^2\Lambda_3^2)\frac{\cos u\sin^2 u\, f e}{2^2 3^3\delta_3^2}(A_t^3 a - \frac{A_t ab}{2^5 3\sigma_u^2\delta_3^2} + \frac{\cos u\, f^2 e^2 b}{\delta_3^2} + \frac{A_t^2 f e a^2}{\sigma_u^2\delta_3}) + \cos\Phi_3^2\Lambda_3^2\frac{\cos^2 u\sin^2 u\, f e b^2}{3^3\delta_3^2}(\frac{f^3 a}{2^3 3\sigma_u^2} - \frac{fe}{\delta_3^2} + \frac{fea^2}{2^3 3\delta_3^4}) + \sin\Phi_3^2\Lambda_3^2\frac{\cos u\sin^2 u\, f e}{3^3\delta_3^2}(A_t b + \frac{A_t^2 f^2 a^2 b}{2^2 3\sigma_u^2\delta_3^2} - \frac{\cos u\, f e a b}{\delta_3^2} - \frac{A_t^2 f^3 a^3 b}{2^9 3\sigma_u^4\delta_3^4})$
$A_2 = \sum_m G_2 \cdot \ell^m \cdot A_{2,m}$ [m=-1, 1, 3]	$A_{2,-1}$	$(1-\cos\Phi_1^2\Lambda_1^2)\frac{\sin^2 u\, f^2}{2^5\delta_1^2} - \cos\Phi_1^2\Lambda_1^2\frac{\sin^2 u\, f^4 a^2}{2^7 A_t^2\delta_1^4} - \sin\Phi_1^2\Lambda_1^2\frac{\sin^2 u\, f a b}{2^2\delta_1^2}$
	$A_{2,1}$	$\sin\Phi_1^2\Lambda_1^2\frac{\cos^2 u \sin^2 u\, f a b}{2A_t\delta_1^2} - \frac{\sin^2 u\, f^4 ab}{2^{10}\sigma_u^2\delta_1^4}$
	$A_{2,3}$	$(1-\cos\Phi_1^2\Lambda_1^2)\frac{\sin^2 u\, A_t^2}{2^2}(1+\frac{f^2b^2}{2^3\sigma_u^2\delta_1^2} + \frac{f^4b^2}{2^8\sigma_u^4\delta_1^4}) + \sin u\cos^2 u(A_t + \cos\Phi_1^1\Lambda_1^1\frac{f A_t b^2}{2^3\sigma_u^2\delta_1^2})$
$A_3 = \sum_m G_3 \cdot \ell^m \cdot A_{3,m}$ [m=-1, 1, 3]	$A_{3,-1}$	$(1-\cos\Phi_3^2\Lambda_3^2)\frac{\sin^2 u\, f}{2^3 3^3\delta_3^2} + \cos\Phi_3^2\Lambda_3^2\frac{\sin^2 u\, f^4 a}{2^{10}3^3 A_t\delta_3^4} - \sin\Phi_3^2\Lambda_3^2\frac{\sin^2 u\, f^2 ab}{2^5 3^3\delta_3^2}$
	$A_{3,1}$	$-\frac{\sin^2 u\, f^4 ab}{2^{15}3^3\sigma_u^2\delta_3^4}$
	$A_{3,3}$	$(1-\cos\Phi_3^2\Lambda_3^2)\frac{\sin^2 u\, A_t^2}{2^5 3^3}(1+\frac{f^2b^2}{3^3 3\sigma_u^2\delta_3^2} + \frac{f^4b^2}{2^8 3\sigma_u^4\delta_3^4}) + \sin u\cos^2 u\, A_t(1+\cos\Phi_3^2\Lambda_3^2\frac{f b^2}{2^3\sigma_u^2\delta_3^2})$
		$f=\sin\alpha$, $e=\cos\alpha$, $a=\cos(\phi_A-\beta)$, $b=\sin(\phi_A-\beta)$, $A_t^2=4[(\tan\alpha\cos u)^2+\sin^2 u+\tan\alpha\sin 2u\cos(\beta-\upsilon)]$, $\phi_A=\tan^{-1}(\frac{\sin\alpha\sin\beta\cos u+\cos\alpha\sin\upsilon\sin u}{\sin\alpha\cos\beta\cos u+\cos\alpha\cos\upsilon\sin u})$,
		$G_0=\exp\{-\ell^2[\cos^2 u+\cot^2\alpha\sin^2 u+\cot\alpha\sin 2u\cos(\beta-\upsilon)]\}$, $G_m=\exp\{-\ell^2 A_t^2(\frac{2^4 m\sigma_u^2\delta_m^2-\sin^2\alpha\, a^2}{2^6 m^2\sigma_u^4\delta_m^2})\}$, $\delta_m^2=\frac{\sin^2\alpha}{4m\sigma_u^2}+\frac{\cos^2\alpha}{4}$, $\Phi_m^h=h(\phi_A-\upsilon-\frac{\sin^2\alpha\, ab}{m 2^4\sigma_u^2\delta_m^2})$, $\Lambda_m^h=\exp\{-\frac{h^2\sin^2\alpha\, b^2}{4^2\ell^2 A_t^2\delta_m^2}\}$.

Table 4-1. Spectral Magnitude Coefficients A_j.

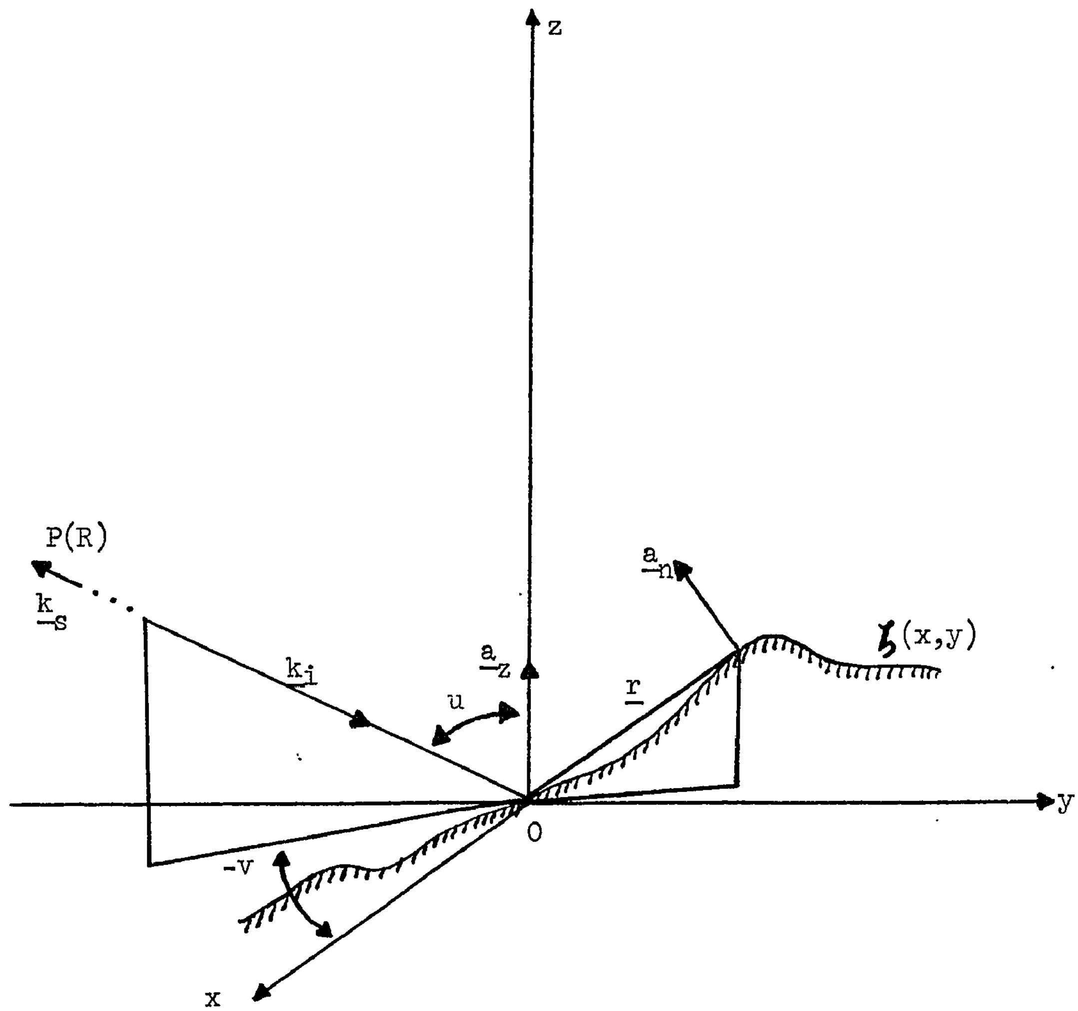

Figure 2-1.The coordinate of rough surface backscattering.

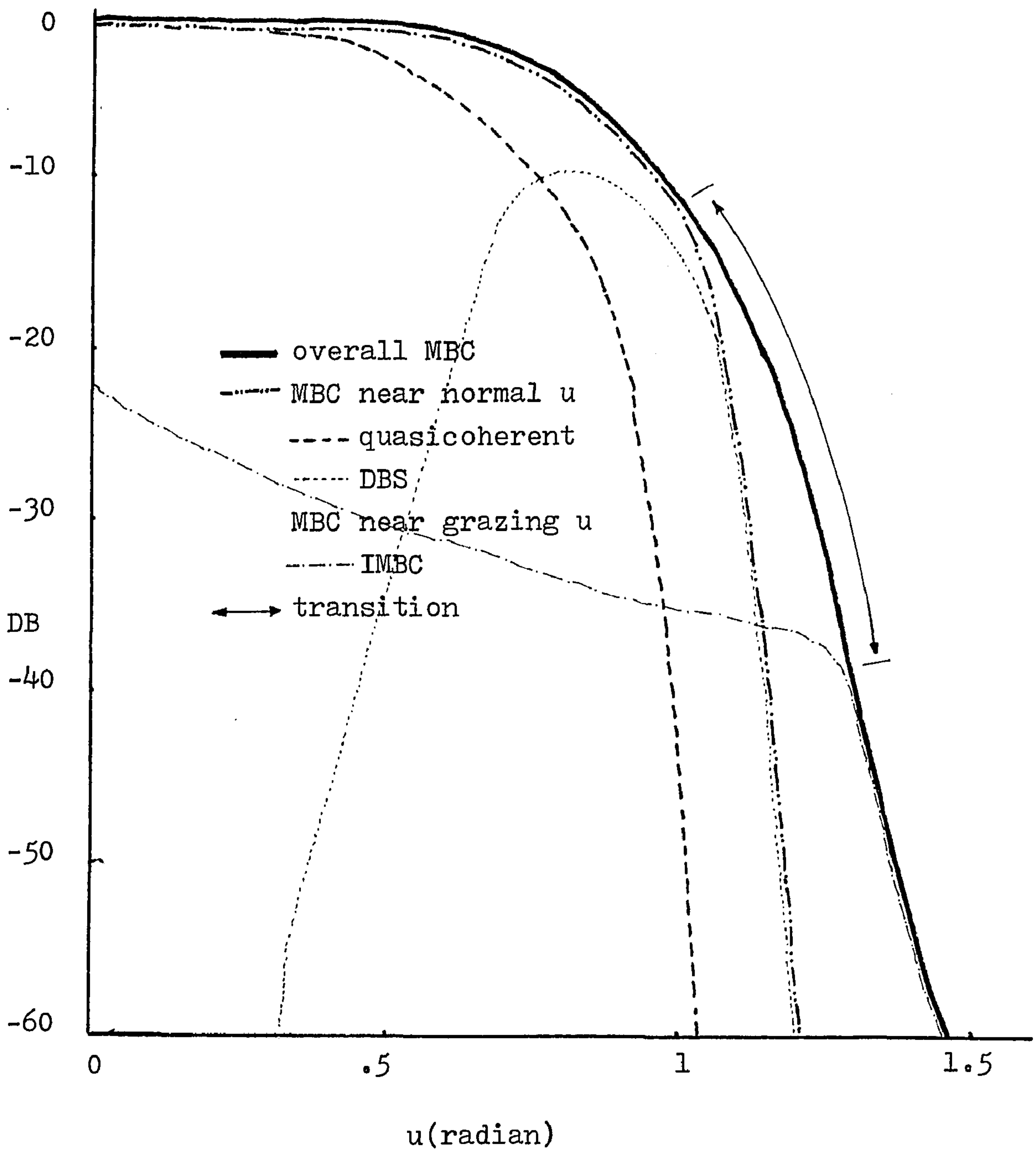

Figure 2-2. The total, quasicoherent MBC and DBS for near normal angle of incidence and IMBC for near grazing angle of incidence with transition region with values of k=1.5 and σ_n=.5.

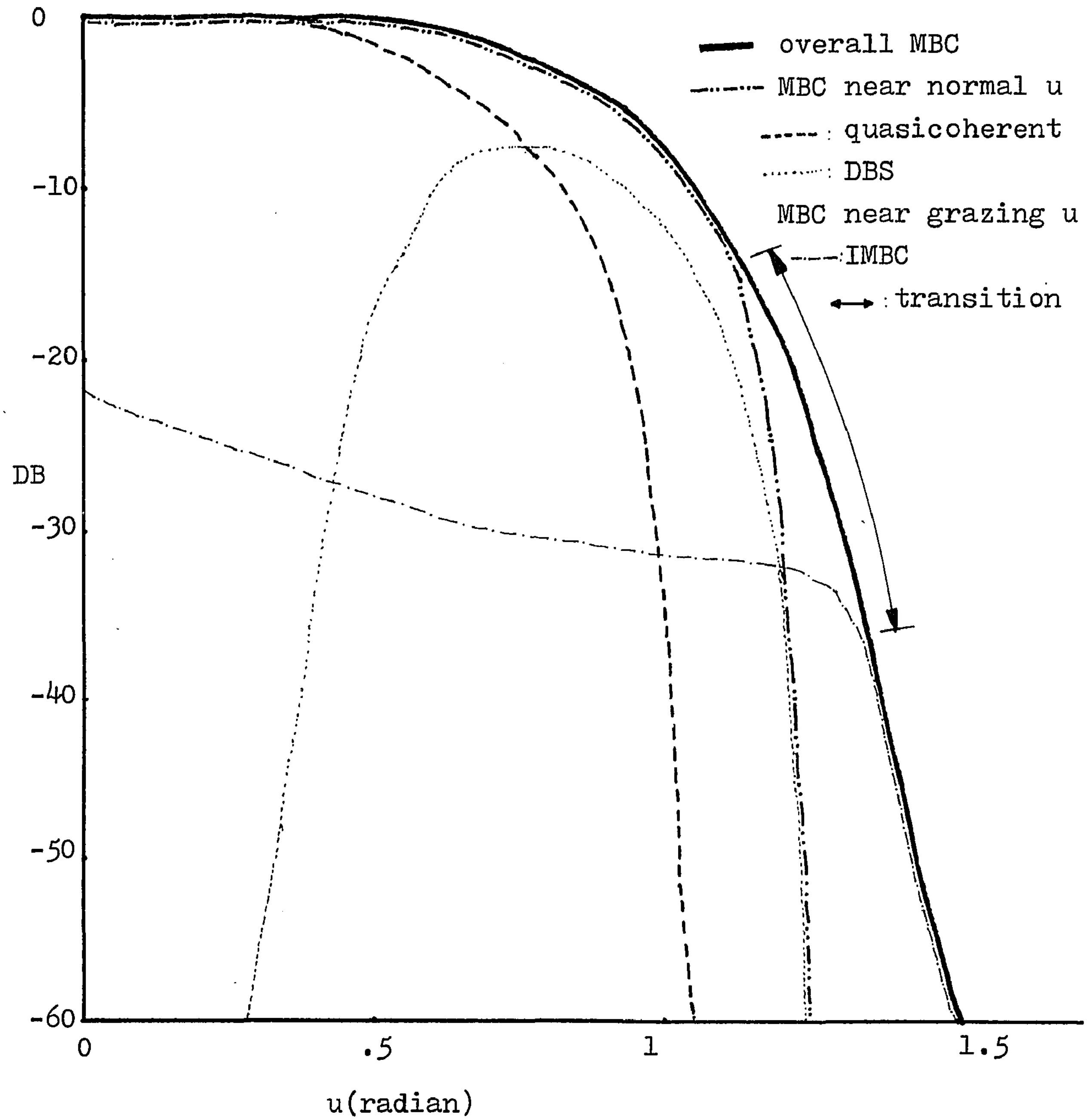

Figure 2-3. The total, quasicoherent MBC and DBS for near normal angle of incidence and IMBC for near grazing angle of incidence with transition region with values of ℓ=1.5 and σ_n=.8.

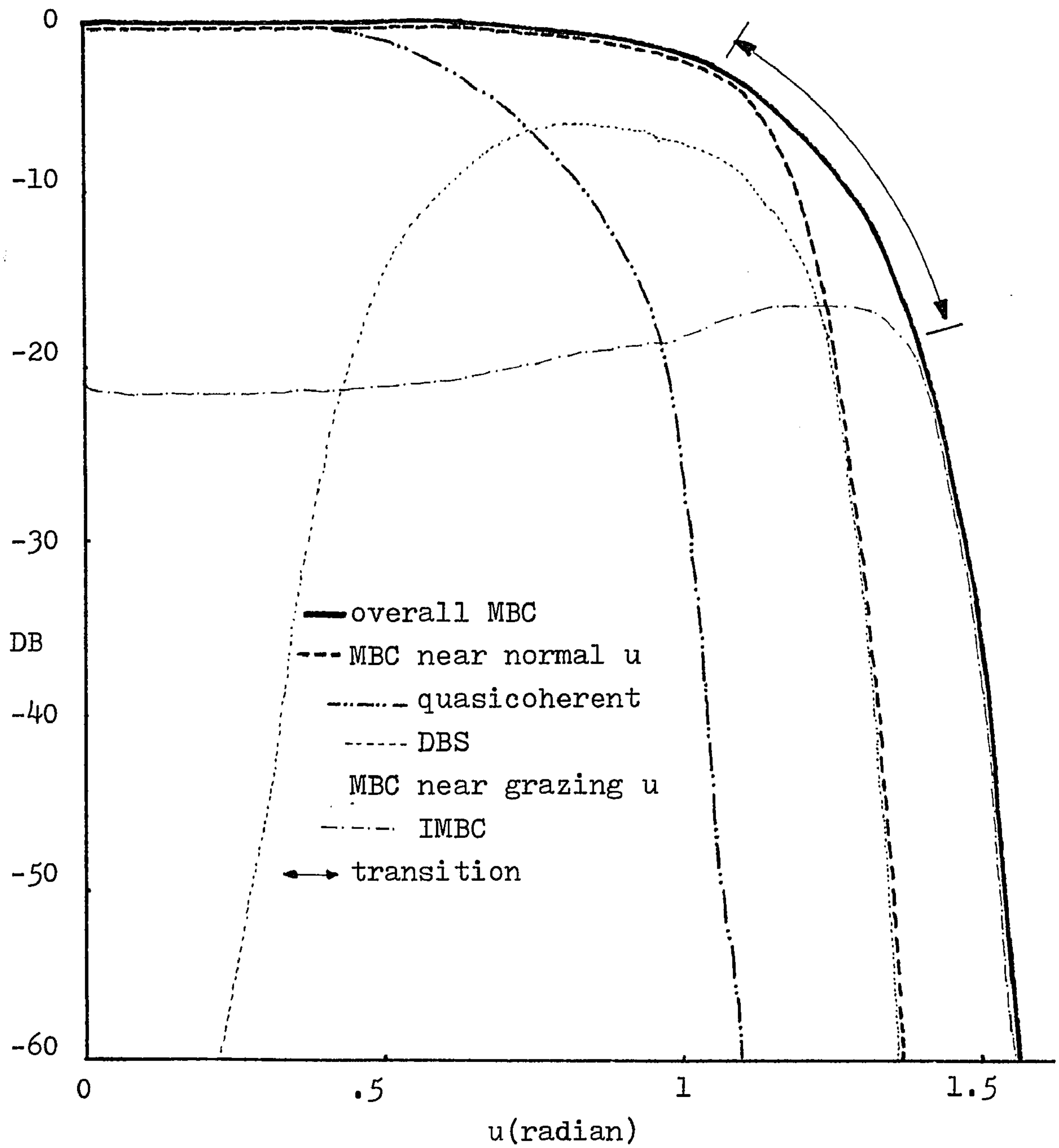

Figure 2-4. The total, quasicoherent MBC and DBS for near normal angle of incidence and IMBC for near grazing angle of incidence with transition region with values of $\ell=1.5$ and $\sigma_n=1$.

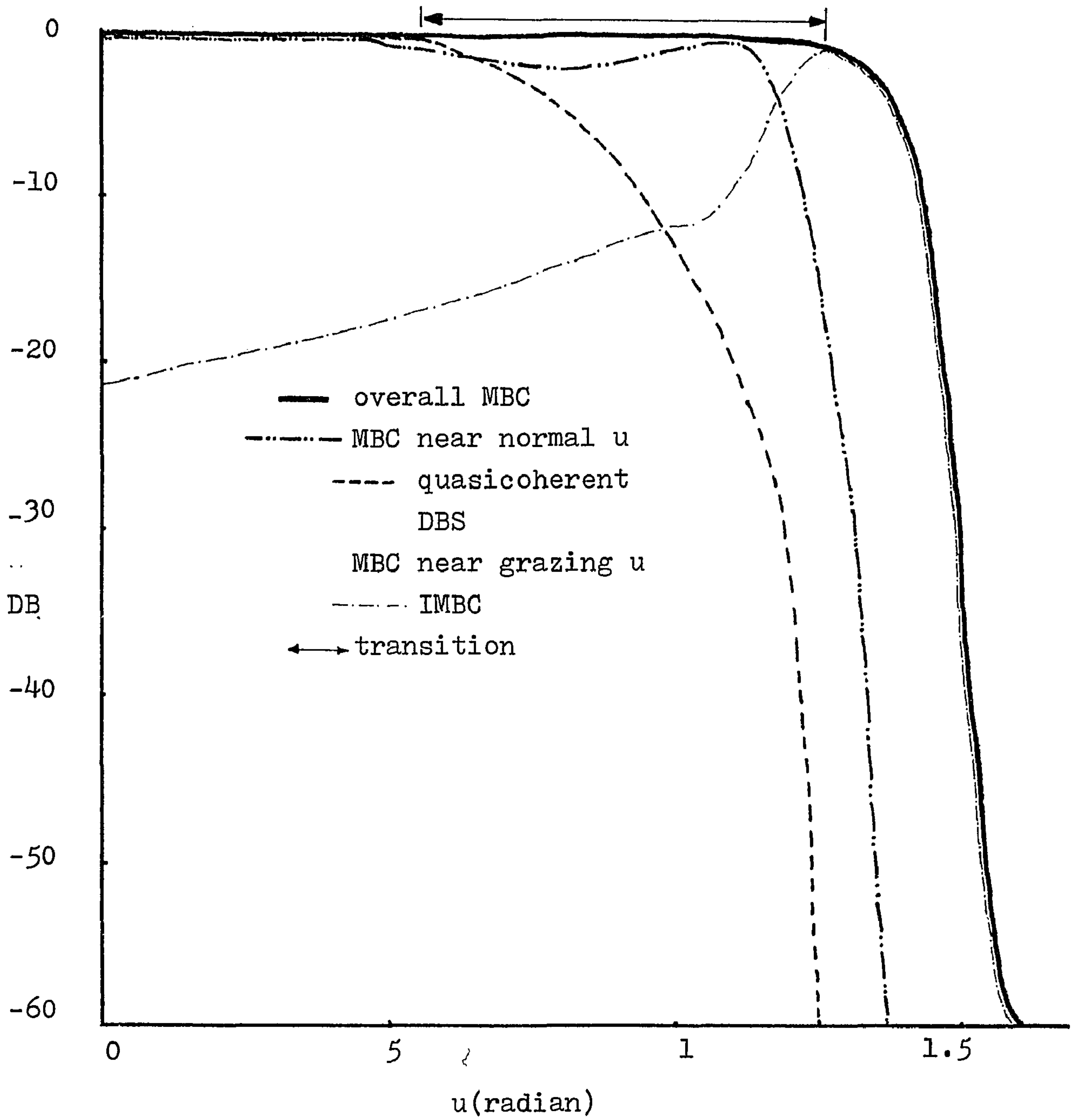

Figure 2-5. The total, quasicoherent MBC for near normal angle of incidence and IMBC for near grazing angle of incidence with transition region with values of ℓ=1.5 and $_n$=1.2.

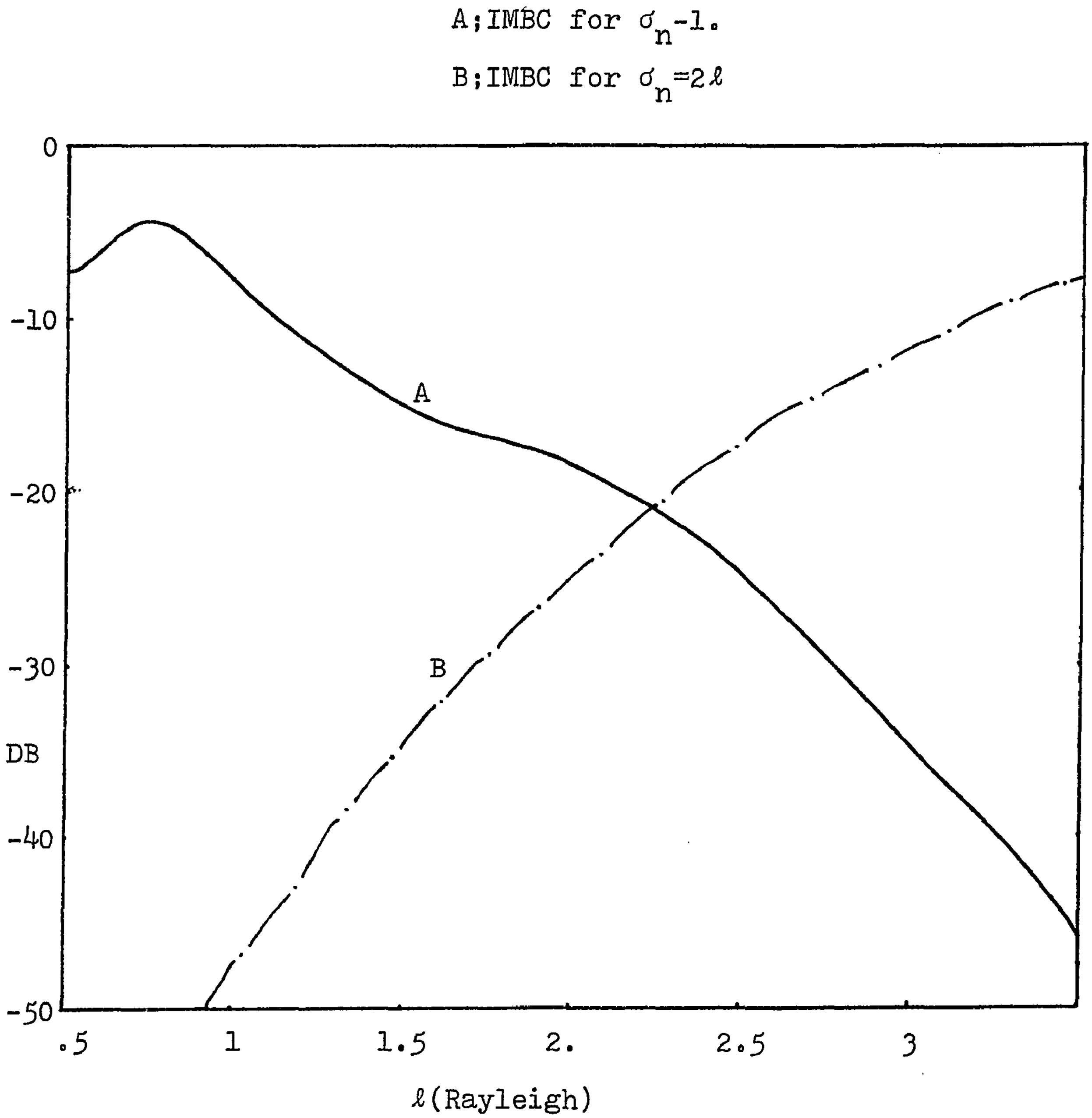

Figure 2-6. The MBC for fixed and varied σ_n for near grazing angle of incidence, with u=1.4.

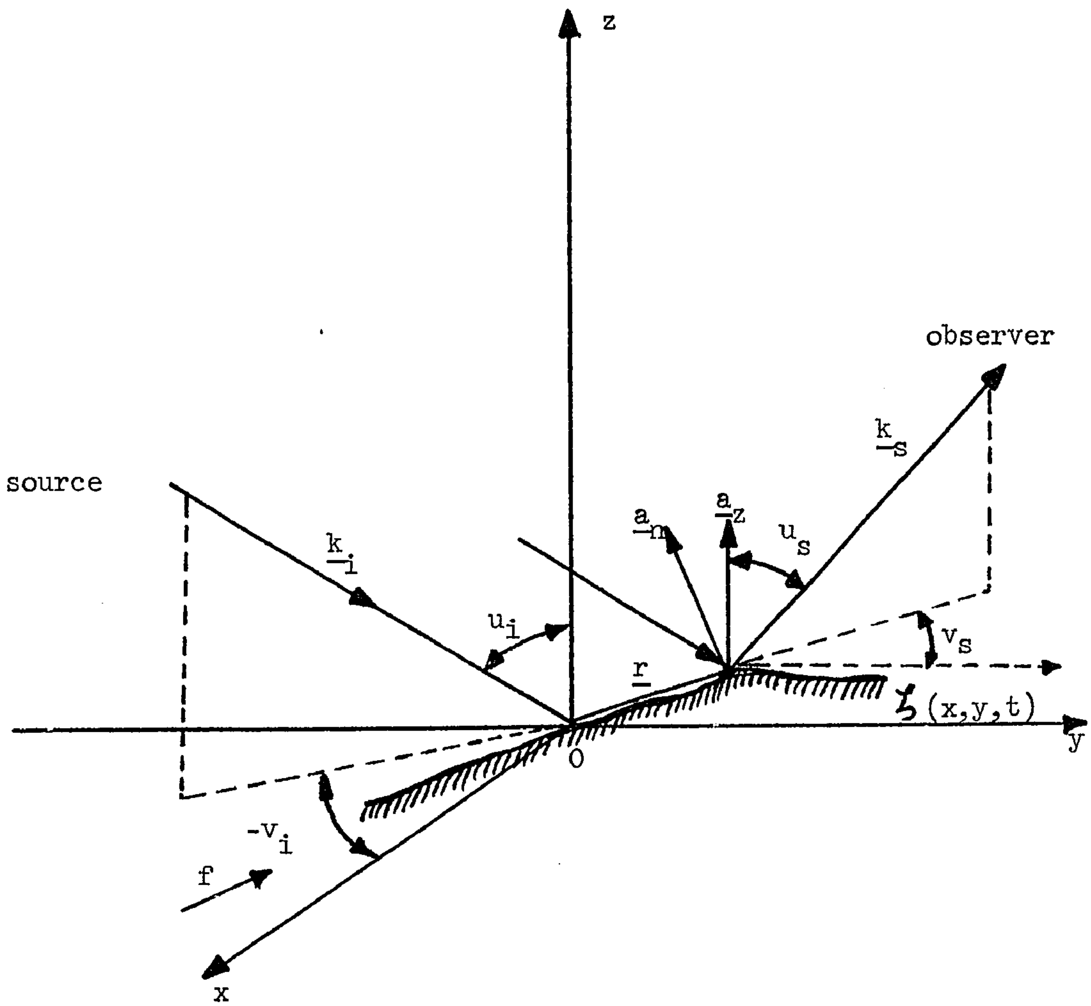

Figure 3-1. The coordinate of rough surface bidirectional scattering.

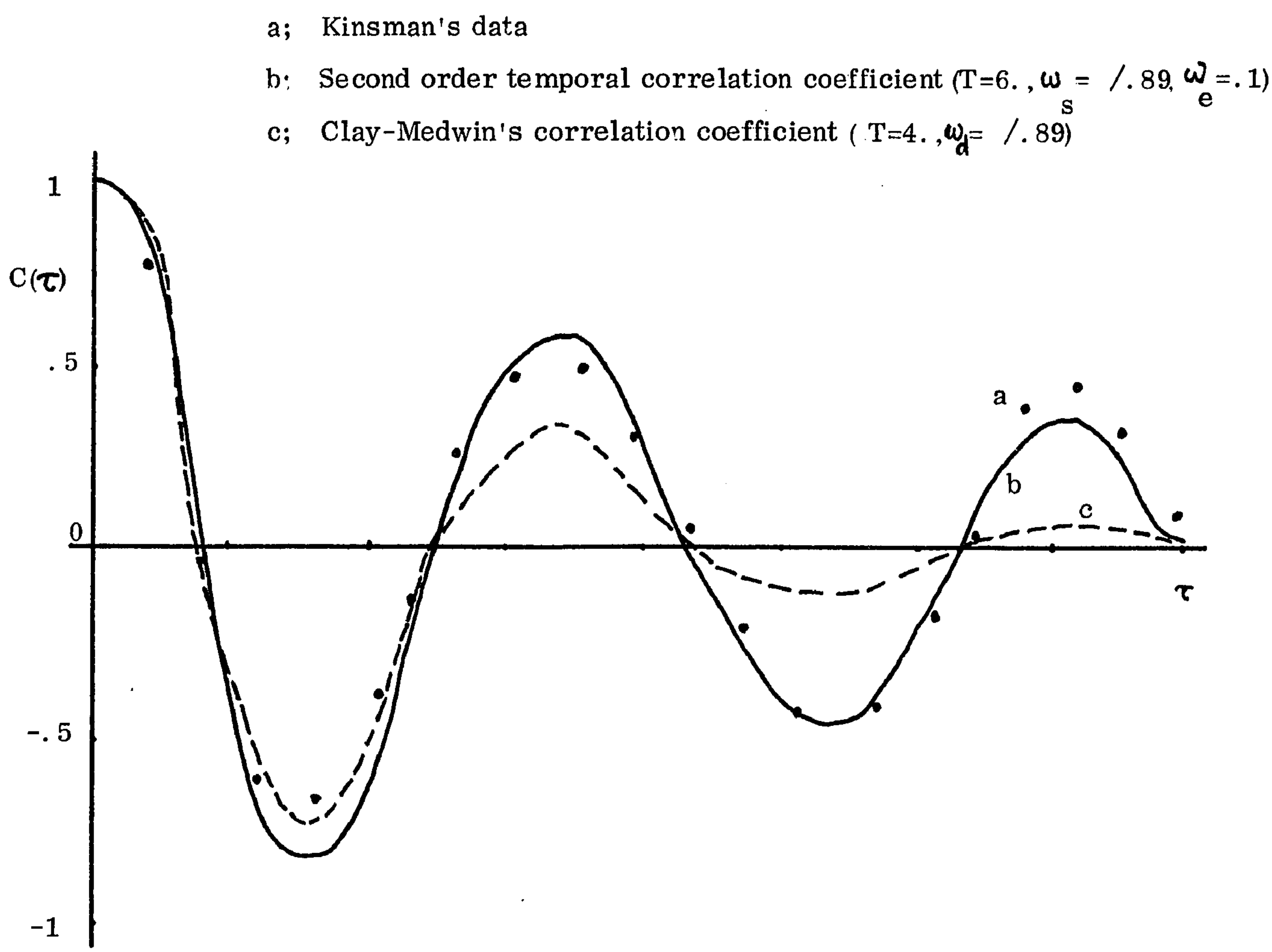

Figure 3 -2. The comparison between first and second order temporal correlation functions and Kinsman's data.

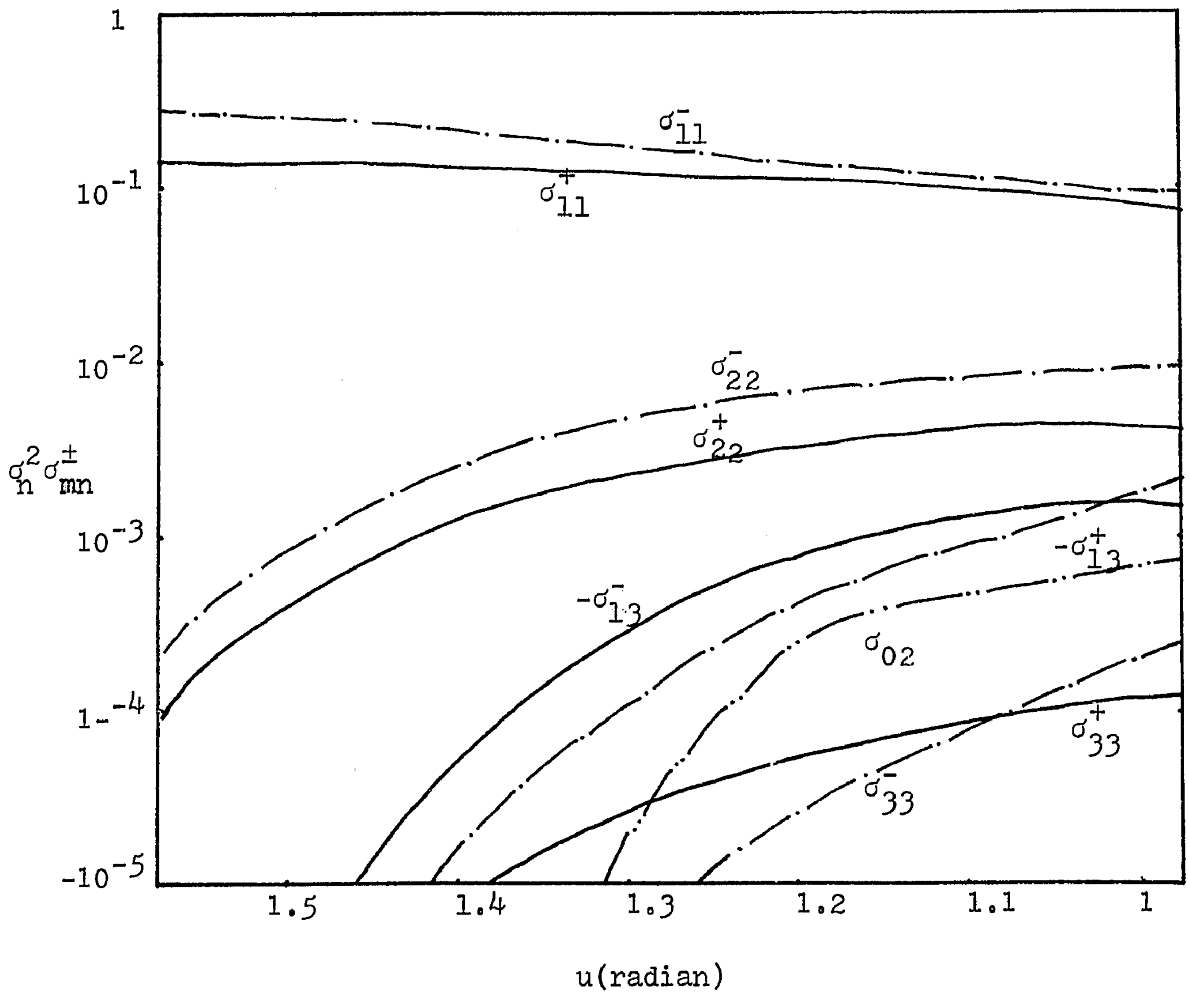

Figure 3-3. The partial MSCs for values α=.2, v=.2, v_s=.2, σ_n=.6, ℓ=1., u_s=1.5 and f=.01.

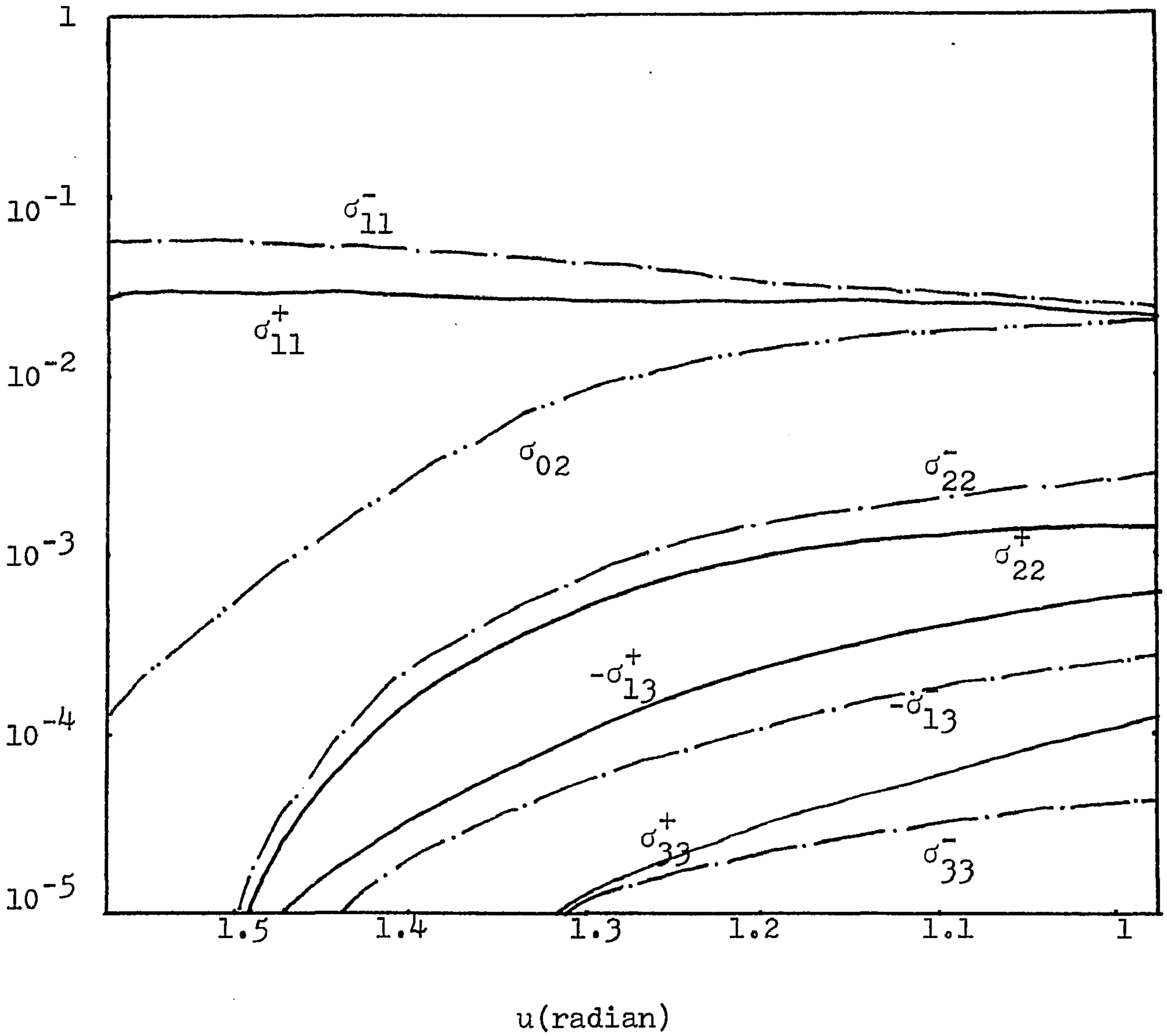

Figure 3-4. The partial MSCs for values α =.5, v=.2, v_s=1.76, σ_n=.6, ℓ=1., u_s=1.5 and f=.01.

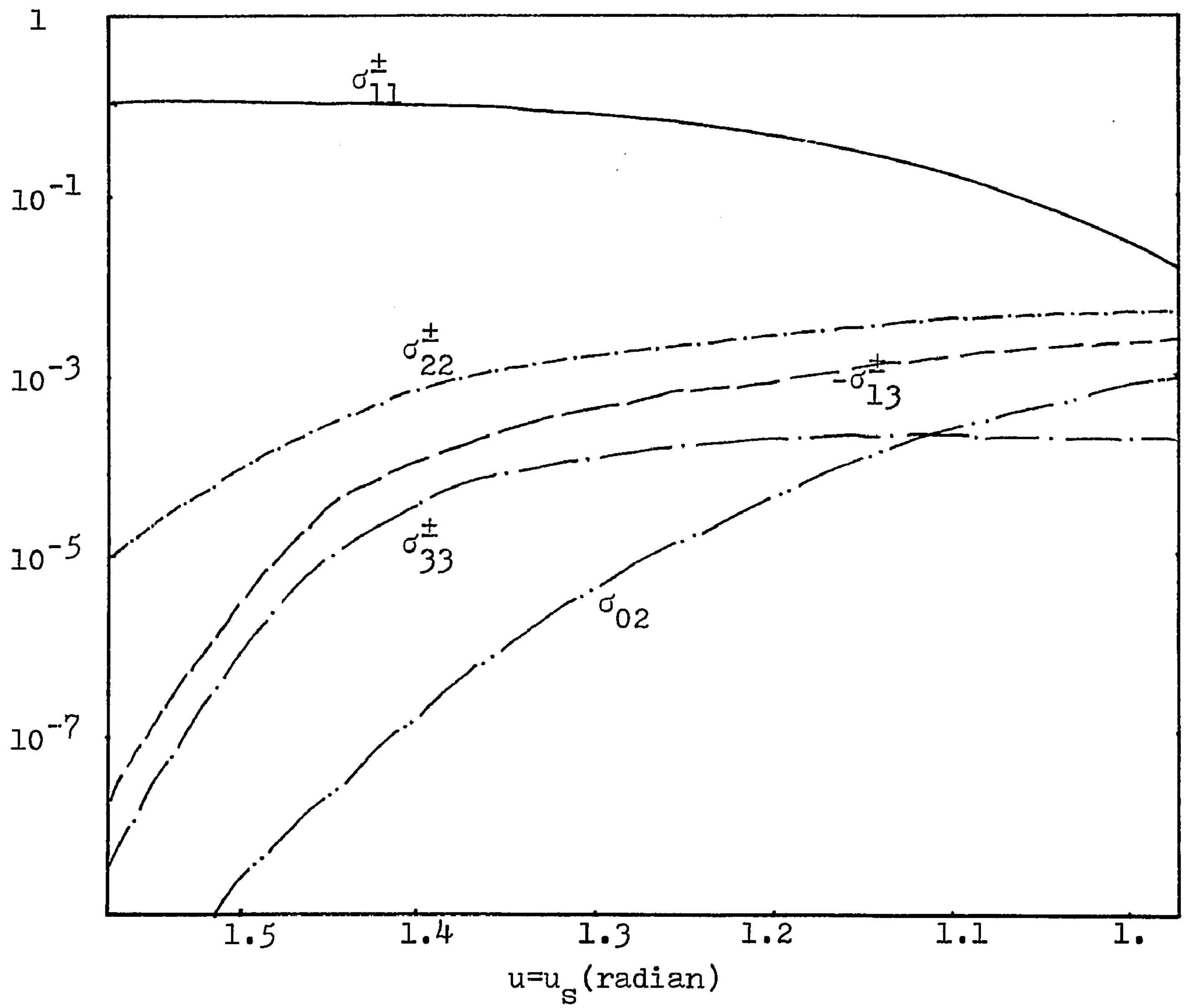

Figure 3-5. The partial MSCs for values $\alpha=1.57, v=v_s=o.$, $\sigma_n=1.$, $\ell=2.$ and $f=1.$

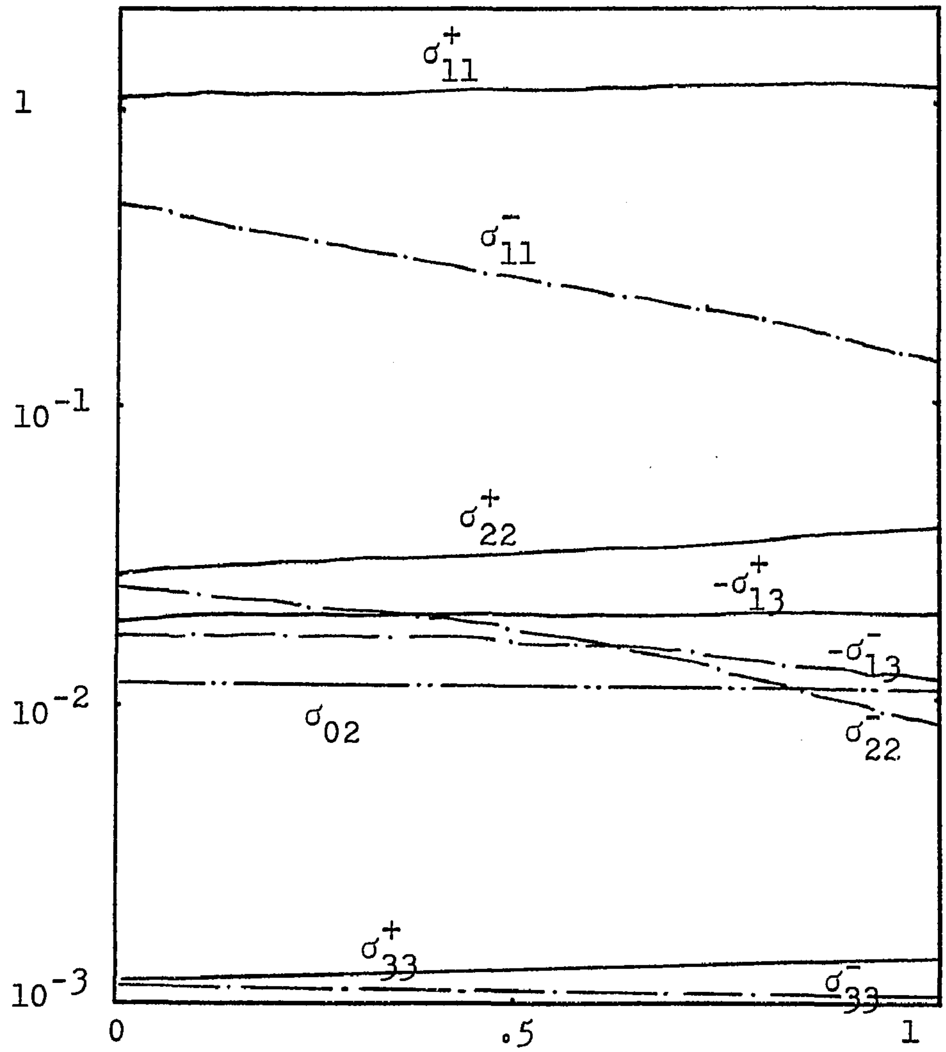

Figure 3-6. The partial MSCs for values α =.5, u_s=1.4,u=1.3, v_s=0., v=.2, σ_n=.8 and ℓ=2..

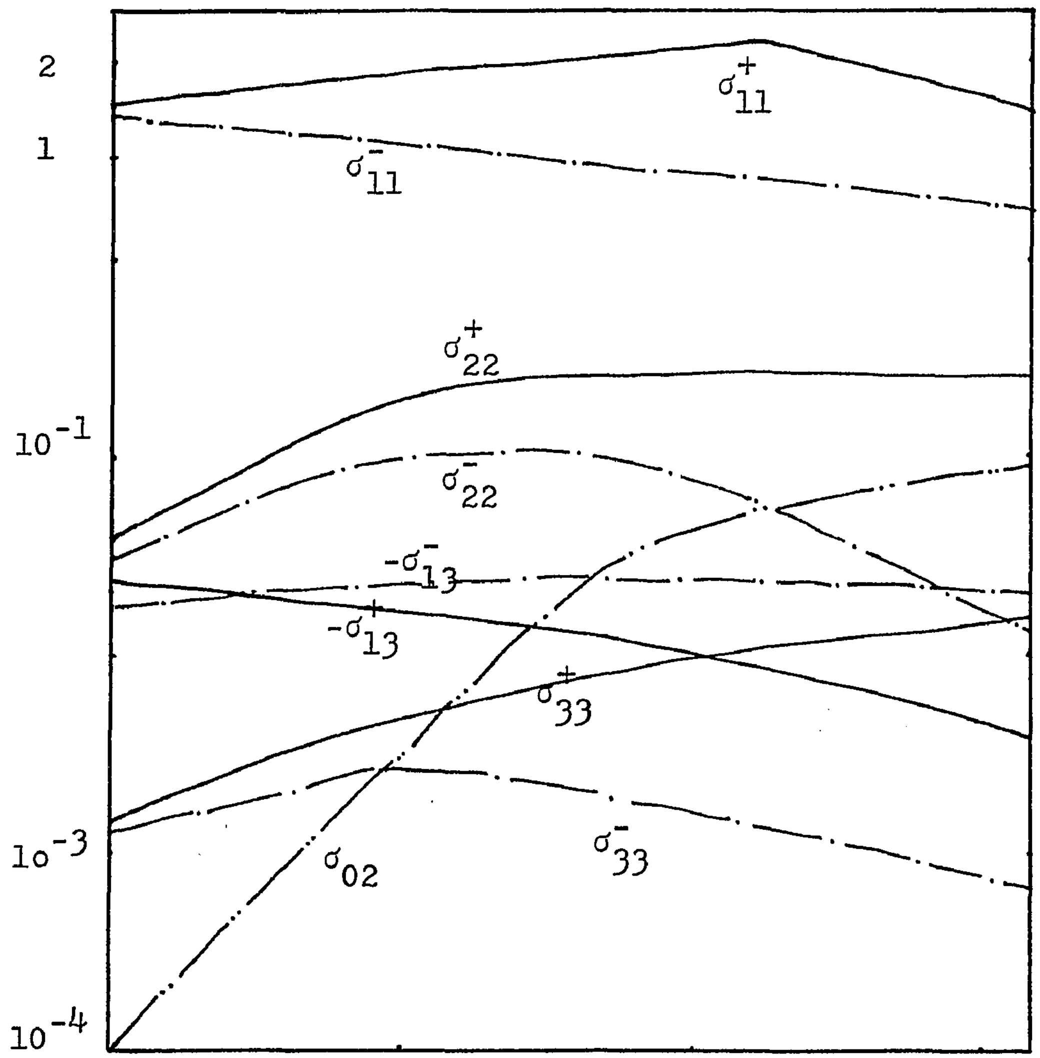

Figure 3-7. The partial MSCs for values α =.5, u=1.3, u_s=1.4, v_s=0., σ_n=2. ℓ=1. and f=.1.

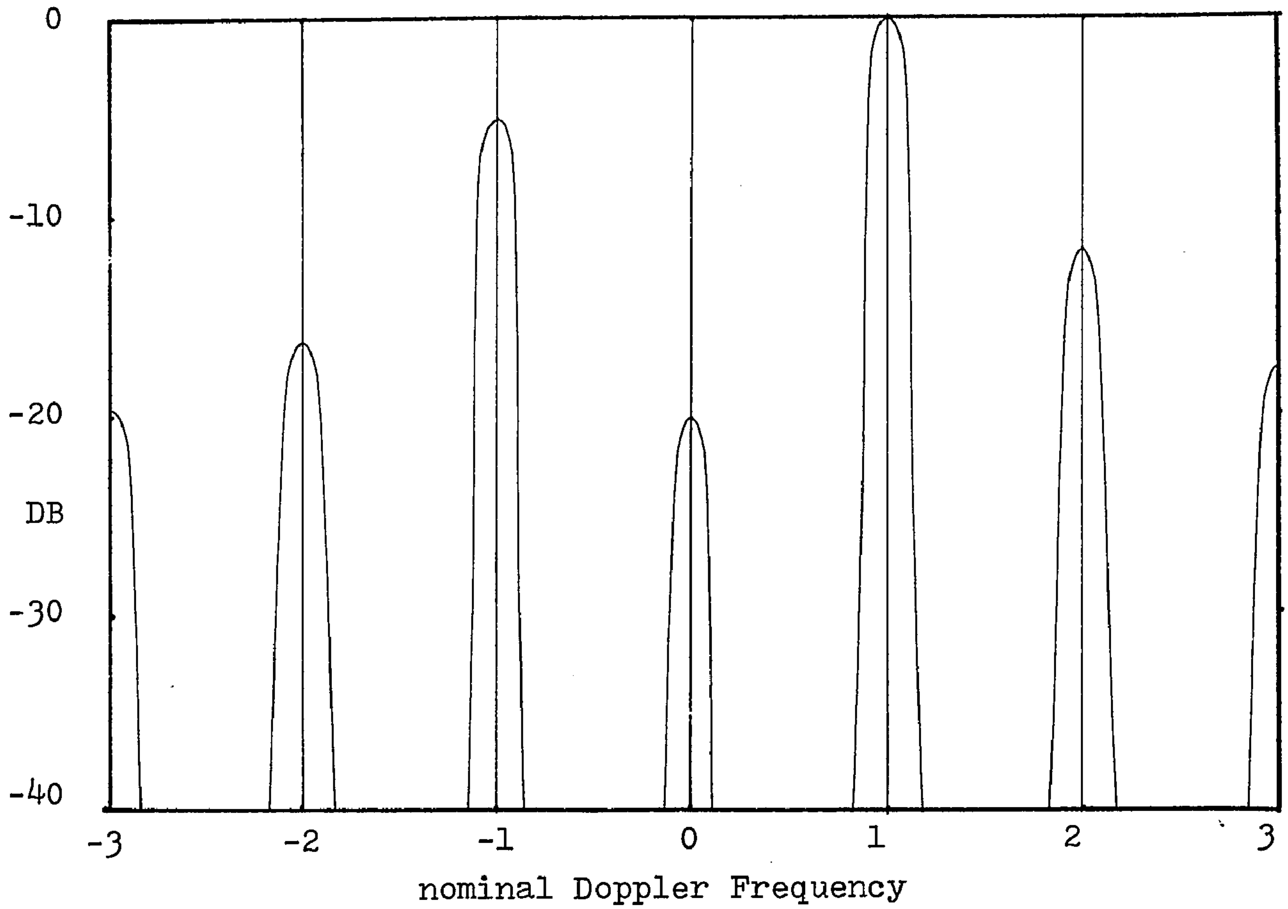

Figure 3-8. The partial TPS for values $\alpha = .5$, $u=1.2$, $u_s=1.4$, $v=.2$, $v_s=0.$, $\ell=.25$, $\sigma_n=.5$, $f=.1$ and $n=.01w_d$.

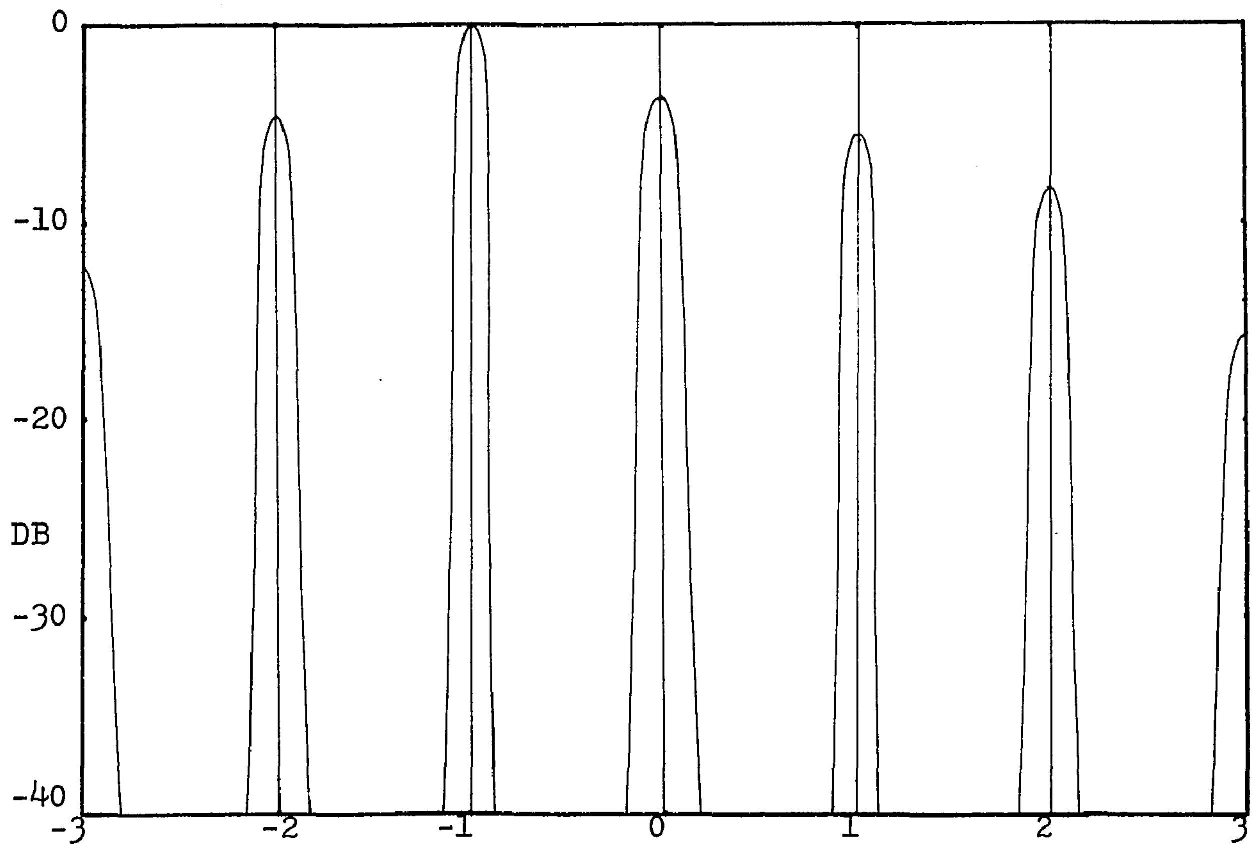

Figure 3-9. The partial TPS for values $\alpha=.2$, $u=1.2$, $u_s=1.5$, $\sigma_n=.6$, $v=.2$, $v_s=1.76$, $\ell=1$. $f=.01$ and $n=.01w_d$.

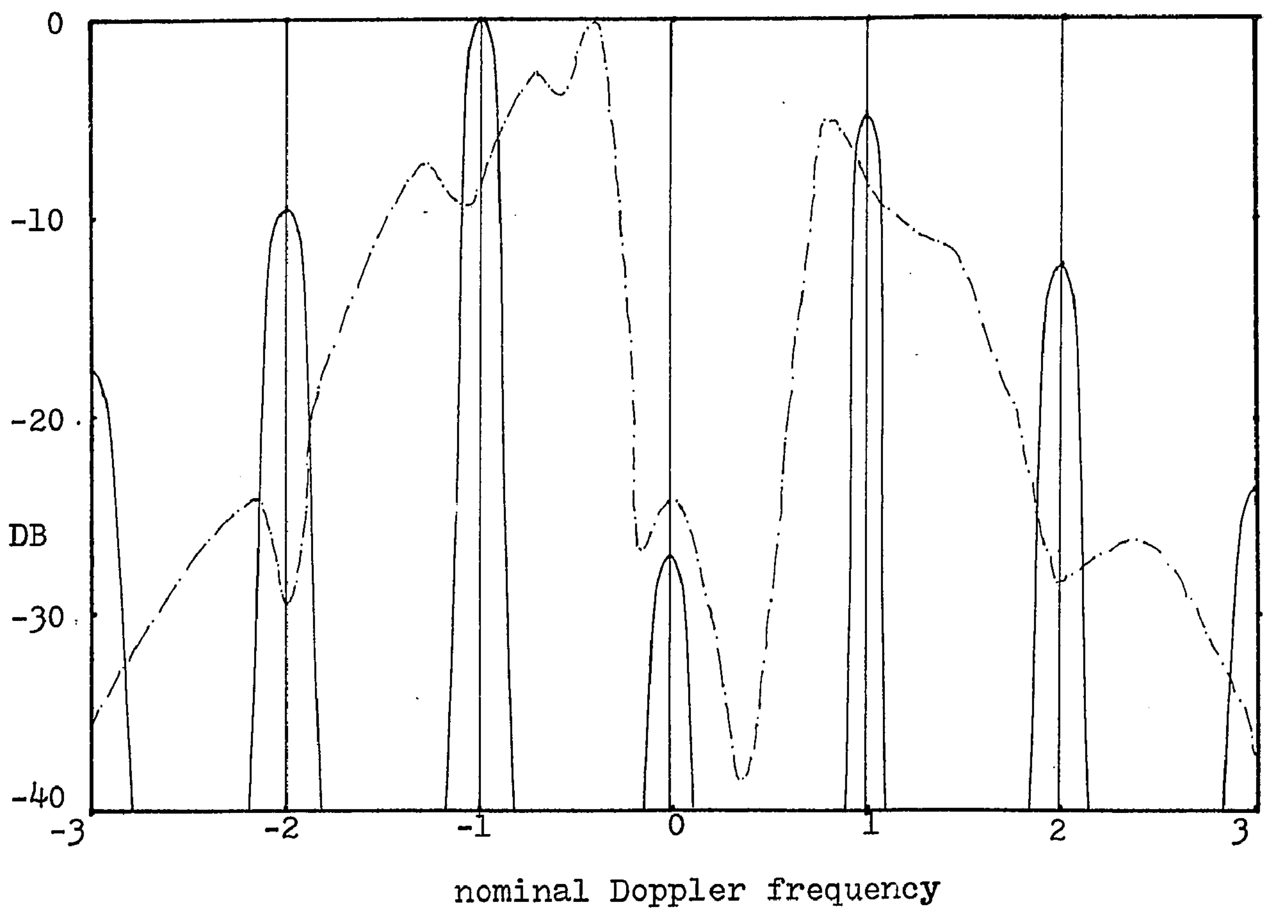

Figure 3-10. The comparison between first and second order partial TPS for values α=.5, u=1.4, u_s=1.5, v=.2, v_s=.2, ℓ=1., σ_n=.6, f=.01 and n=.01w_d.

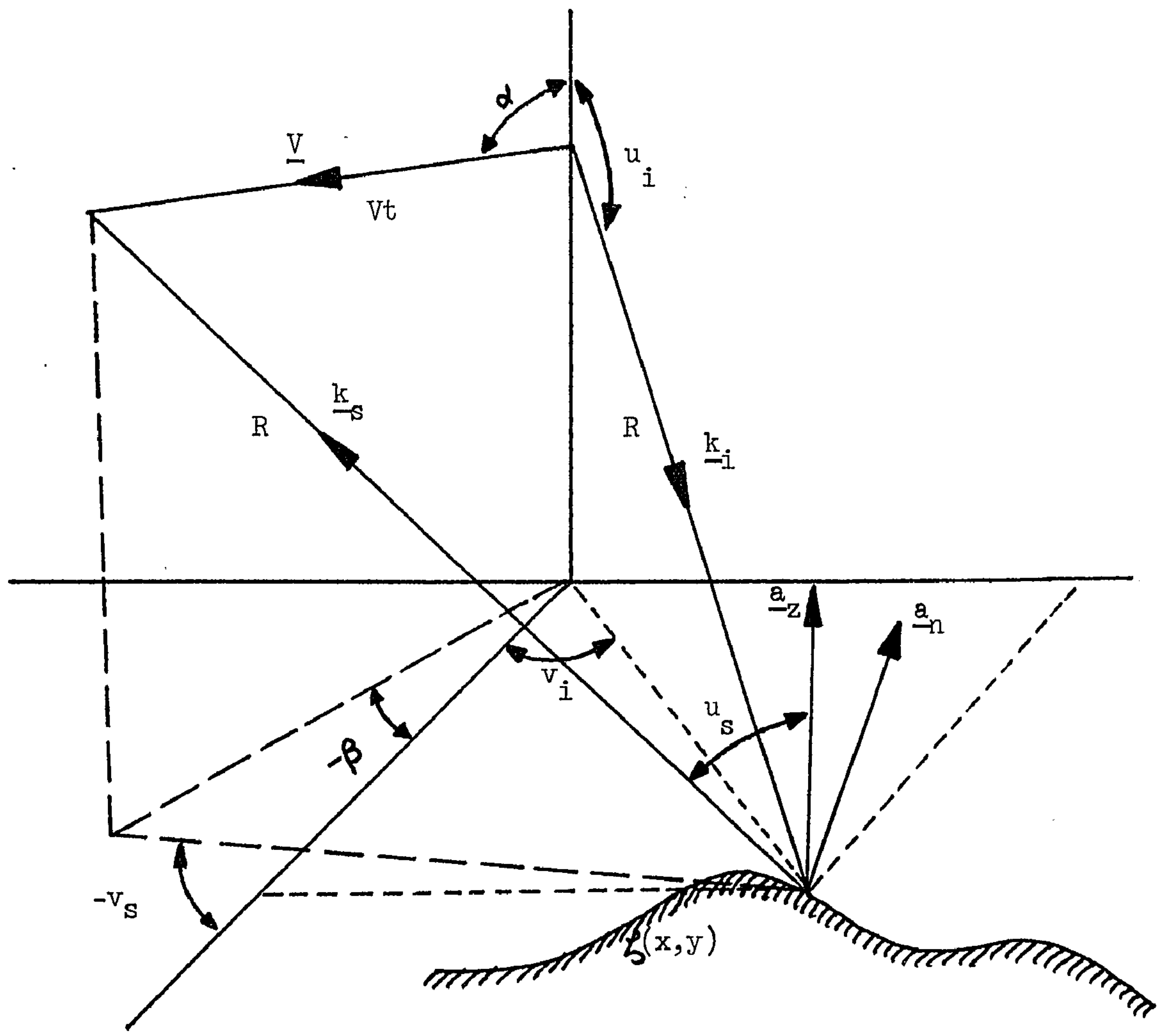

Figure 4-1 The coordinate of rough surface scattering by moving source.

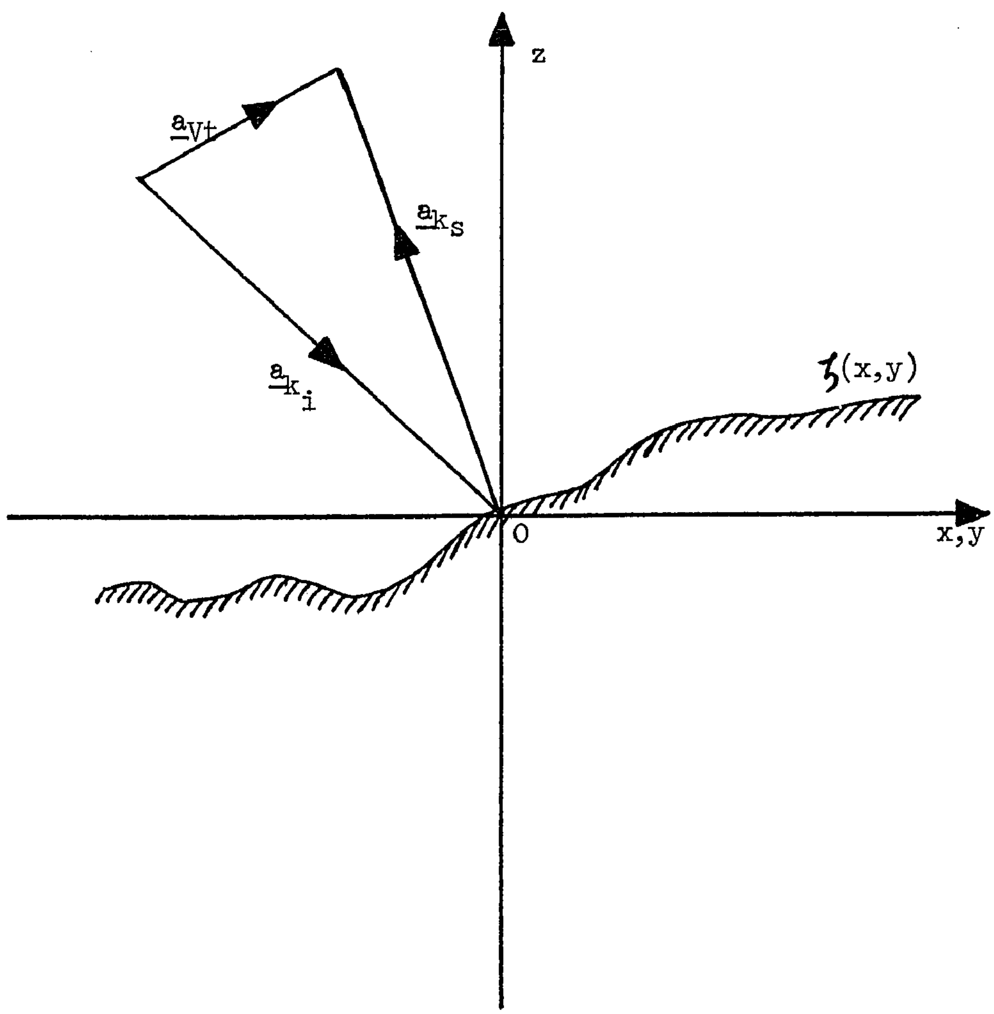

Figure 4-2 The coordinate of three units vectors.

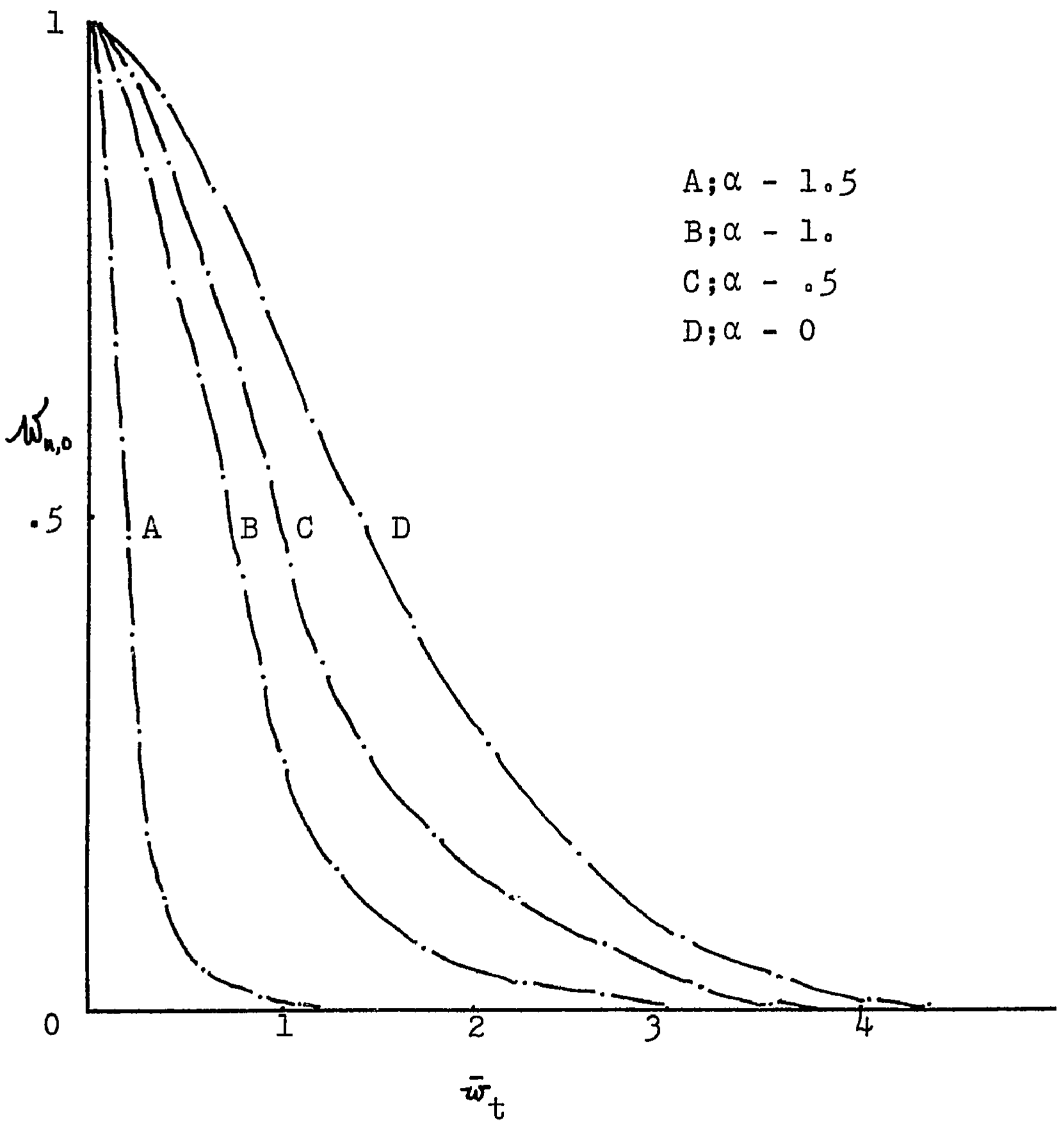

Figure 4-3. The dominant spectral shape with various values of the elevation angles of source velocity and $\ell=2$..

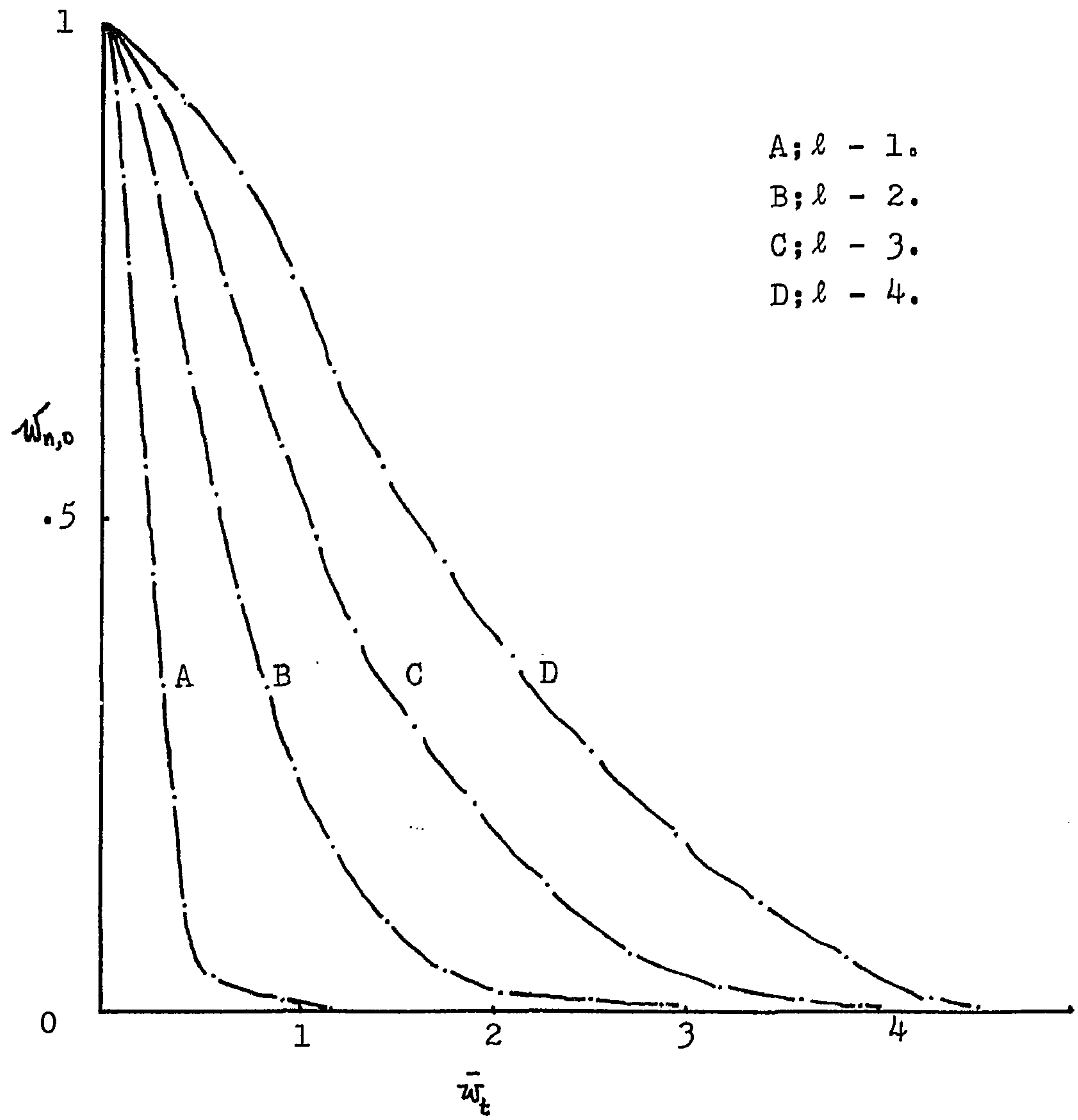

Figure 4-4. The dominant spectral shape with various values if the Rayleigh parameter and u=1..

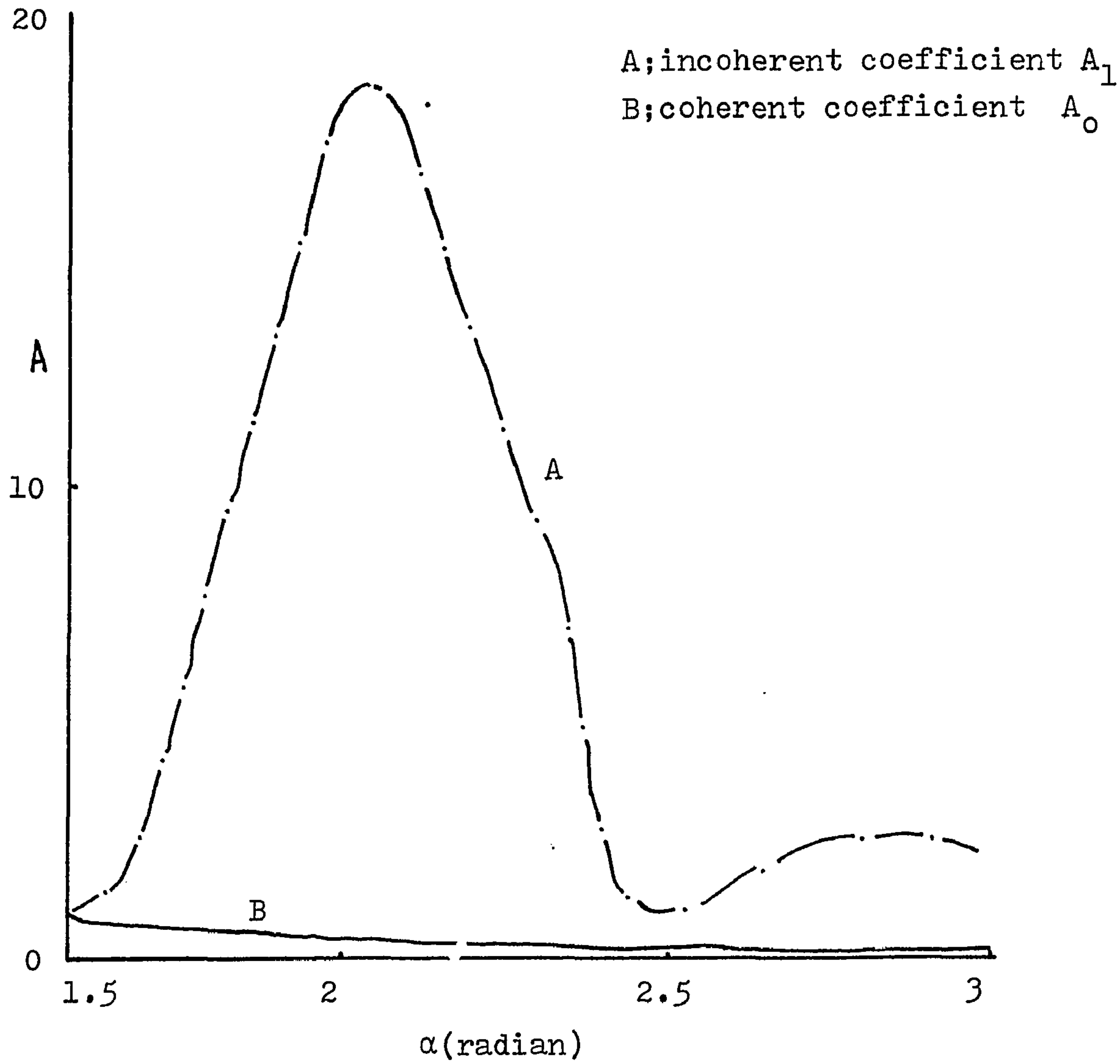

Figure 4-5. The magnitude coefficients of the Doppler spectrum with values of u_i=.6, v_i= .2, β=o., ℓ=.8 and σ_n=.6.

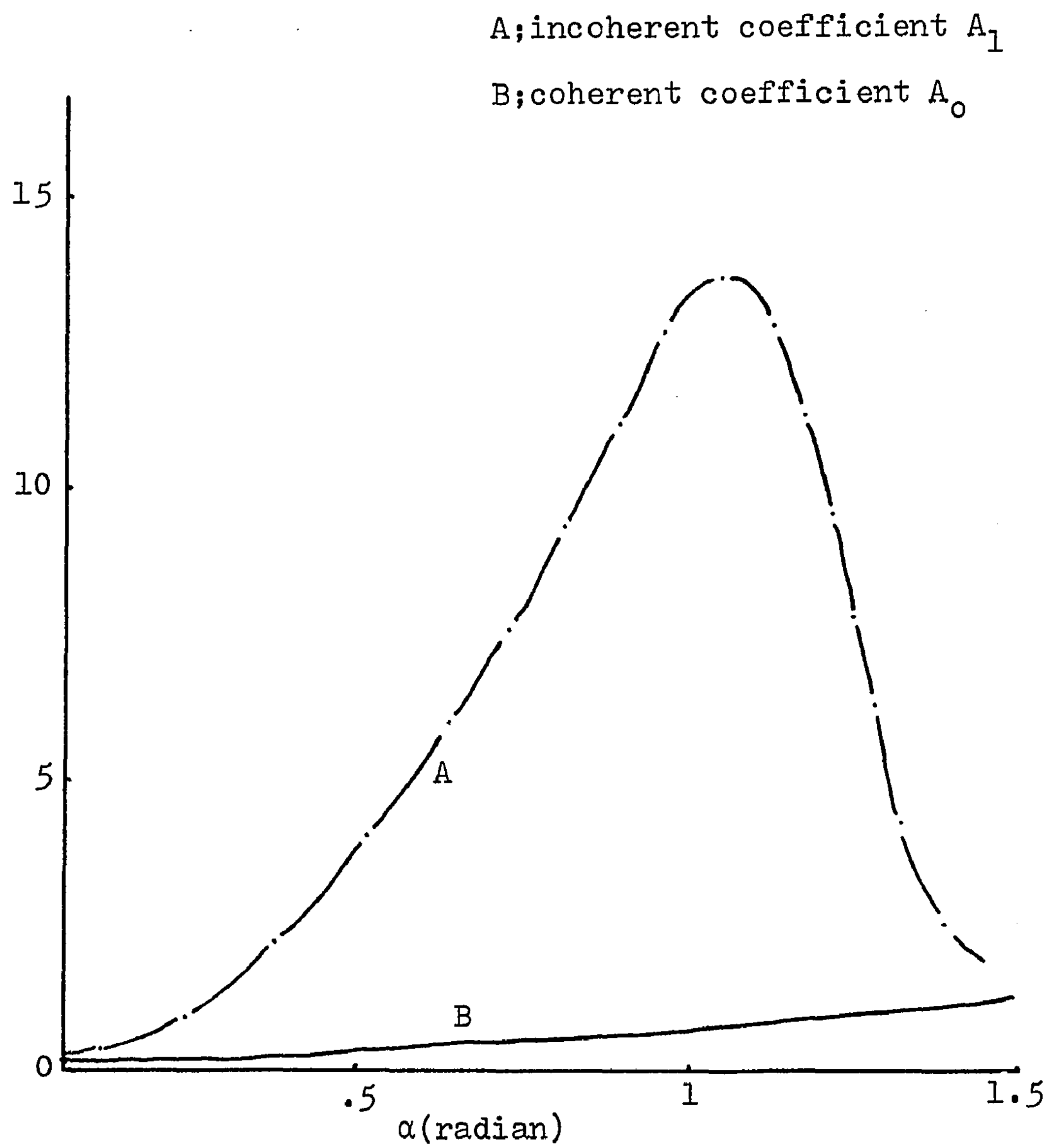

Figure 4-6. The magnitude cofficients of the Doppler spectrum with values of u_i=.6, v_i=.2, β=0., ℓ=.8 and σ_n=.6.

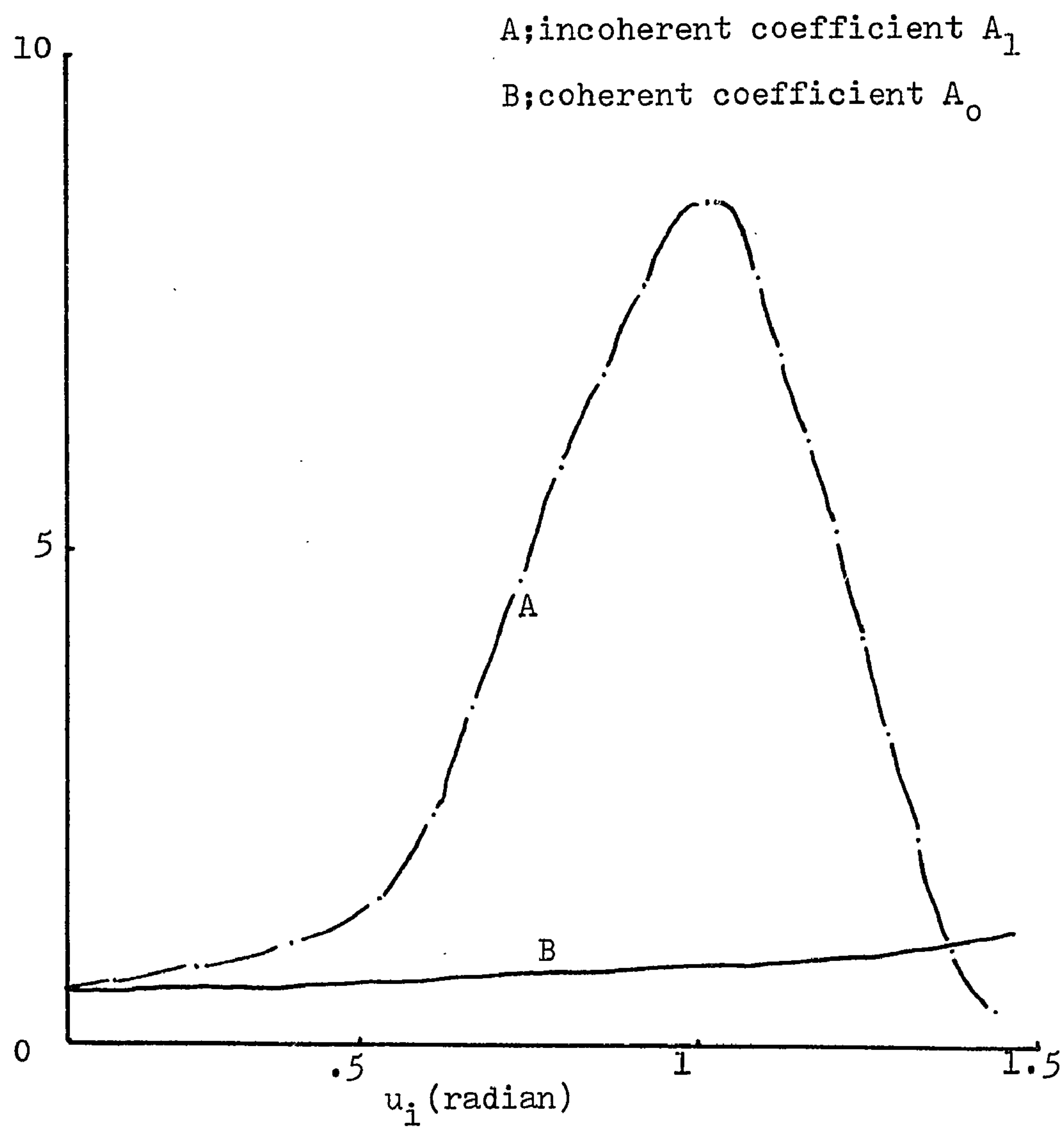

Figure 4-7. The magnitude coefficients of the Doppler spectrum with values of α =1.5, v_i=.2, β=0., ℓ=.8 and σ_n=.6.

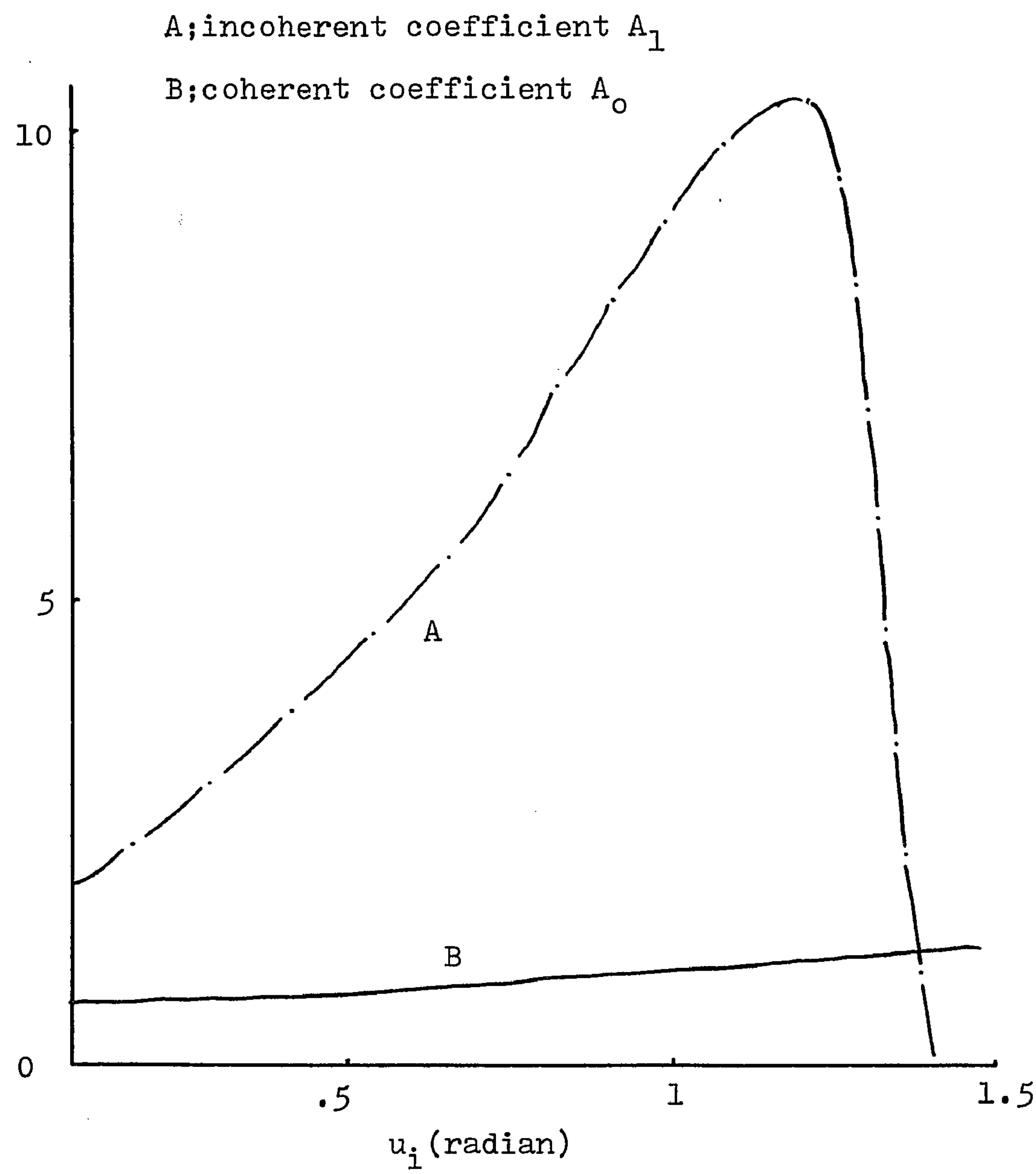

Figure 4-8. The magnitude coefficients of the Doppler spectrum with values of α=1.5, v_i=.2, β=0., ℓ=.8 and σ_n=.6.

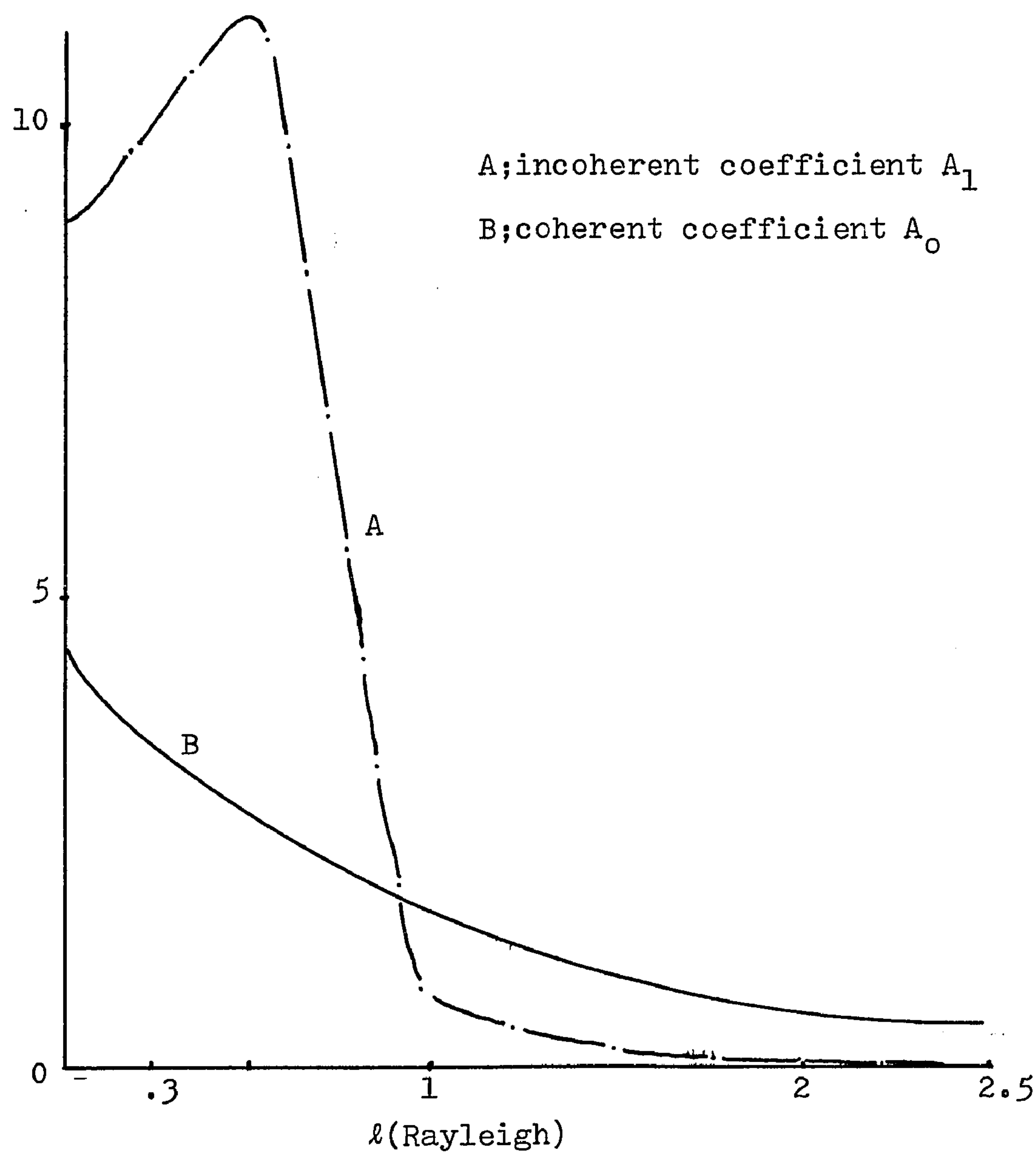

Figure 4-9. The magnitude coefficients of the Doppler spectrum with values of u_i=.3, α=1.4, v_i=.2, β=0. and σ_n=.6.

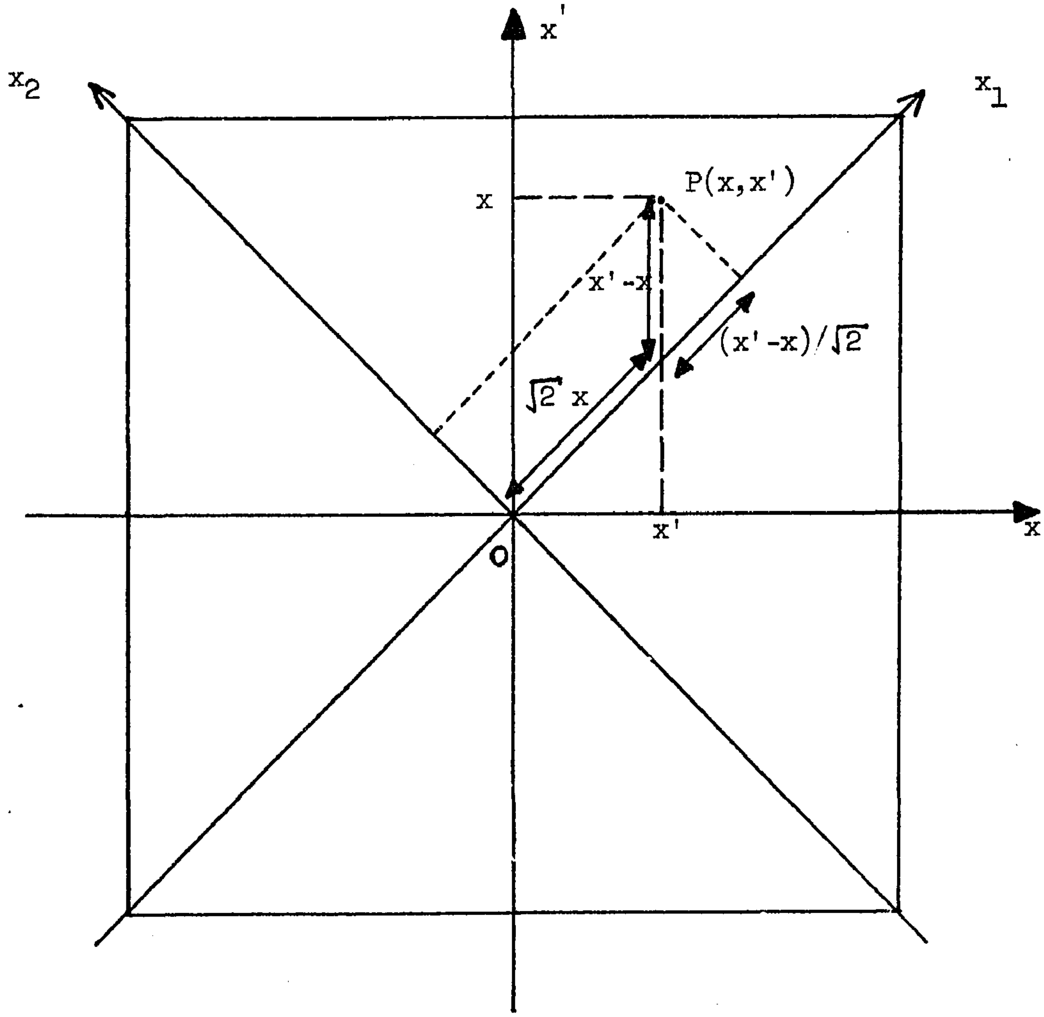

Figure 1-1. The coordinate transformation.

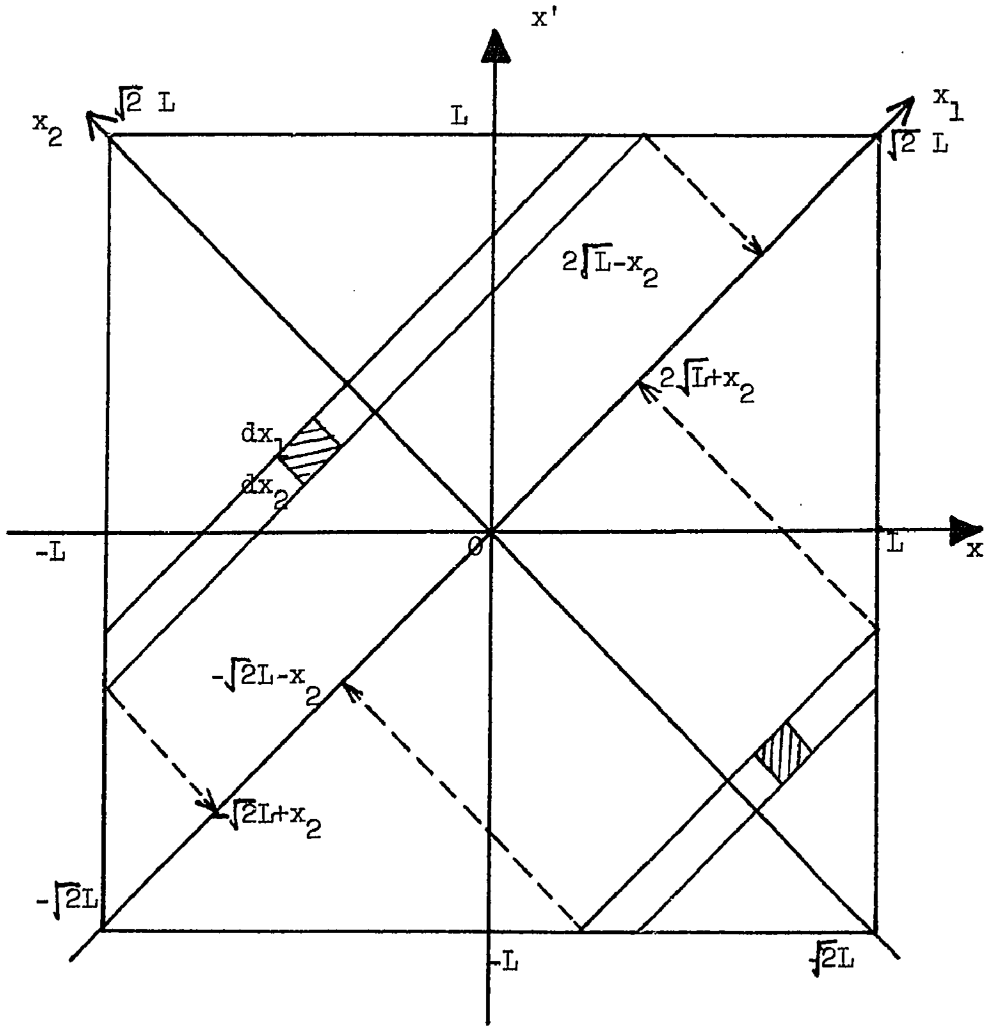

Figure 1-2. The intervals transformation.

Appendix A

The Transformation of I into Cylindrical Coordinates

The surface height correlation function for a stationary process is a function of displacement only

$$\sigma^2 C(x-x',y-y') = \langle \xi(x,y)\,\xi(x',y')\rangle \tag{A-1}$$

Introducing new variables, in cylindrical coordinates,

$$t = [(x-x')^2+(y-y')^2]^{\frac{1}{2}} \tag{A-2}$$

$$w = \tan^{-1}\left(\frac{x-x'}{y-y'}\right) \tag{A-3}$$

the surface height correlation function simplifies. Since the M functions involve partial differentiations with respect to x,x',y and y', we need the chain rule for the partial derivatives

$$\frac{\partial}{\partial x} = \frac{\partial t}{\partial x}\frac{\partial}{\partial t} + \frac{\partial w}{\partial x}\frac{\partial}{\partial w} \tag{A-4}$$

$$\frac{\partial}{\partial x'} = \frac{\partial t}{\partial x'}\frac{\partial}{\partial t} + \frac{\partial w}{\partial x'}\frac{\partial}{\partial w} \tag{A-5}$$

$$\frac{\partial}{\partial y} = \frac{\partial t}{\partial y}\frac{\partial}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial}{\partial w} \tag{A-6}$$

$$\frac{\partial}{\partial y'} = \frac{\partial t}{\partial y'}\frac{\partial}{\partial t} + \frac{\partial w}{\partial y'}\frac{\partial}{\partial w} \tag{A-7}$$

From Eqs.(A-2) and (A-3), we have

$$\frac{\partial}{\partial x} = \sin w\,\frac{\partial}{\partial t} + \frac{\cos w}{t}\frac{\partial}{\partial w} \tag{A-8}$$

$$\frac{\partial}{\partial x'} = -\sin w\,\frac{\partial}{\partial t} - \frac{\cos w}{t}\frac{\partial}{\partial w} \tag{A-9}$$

$$\frac{\partial}{\partial y} = \cos w\,\frac{\partial}{\partial t} - \frac{\sin w}{t}\frac{\partial}{\partial w} \tag{A-10}$$

$$\frac{\partial}{\partial y'} = -\cos w\,\frac{\partial}{\partial t} + \frac{\sin w}{t}\frac{\partial}{\partial w} \tag{A-11}$$

where we used

$$\frac{\partial t}{\partial x} = -\frac{\partial t}{\partial x'} = \sin w \tag{A-12}$$

$$\frac{\partial w}{\partial x} = -\frac{\partial w}{\partial x'} = \frac{\cos w}{t} \tag{A-13}$$

$$\frac{\partial t}{\partial y} = -\frac{\partial t}{\partial y'} = \cos w \tag{A-14}$$

$$\frac{\partial w}{\partial y}=\frac{\partial w}{\partial y'}=-\frac{\sin w}{t} \tag{A-15}$$

Similarly we have

$$\frac{\partial^2}{\partial x\partial x'}=-\sin^2 w\frac{\partial^2}{\partial t^2}-\frac{\cos^2 w}{t^2}\frac{\partial^2}{\partial w^2}-\frac{\sin 2w}{t}\frac{\partial^2}{\partial t\partial w}+\frac{\sin 2w}{t^2}\frac{\partial}{\partial w}-\frac{\cos^2 w}{t}\frac{\partial}{\partial t} \tag{A-16}$$

$$\frac{\partial^2}{\partial y\partial y'}=-\cos^2 w\frac{\partial^2}{\partial t^2}-\frac{\sin^2 w}{t^2}\frac{\partial^2}{\partial w^2}+\frac{\sin 2w}{t}\frac{\partial^2}{\partial t\partial w}-\frac{\sin 2w}{t^2}\frac{\partial}{\partial w}-\frac{\sin^2 w}{t}\frac{\partial}{\partial t} \tag{A-17}$$

$$\frac{\partial^2}{\partial x\partial y'}=-\frac{\sin 2w}{2}\frac{\partial^2}{\partial t^2}+\frac{\sin 2w}{2t^2}\frac{\partial^2}{\partial w^2}-\frac{\cos 2w}{t}\frac{\partial^2}{\partial t\partial w}+\frac{\cos 2w}{t^2}\frac{\partial}{\partial w}$$
$$+\frac{\sin 2w}{2t}\frac{\partial}{\partial t}=\frac{\partial^2}{\partial x'\partial y} \tag{A-18}$$

Substituting Eqs.(A-8)-(A-11) and Eqs.(A-15)-(A-18) into M functions, we have

$$M_1= e^{-p^2(1-C)} \tag{A-19}$$

$$M_2=-j\sigma p(2\sin w\frac{\partial C}{\partial t}+\frac{2\cos w}{t}\frac{\partial C}{\partial w})\ M_1 \tag{A-20}$$

$$M_3=-j\sigma p(2\cos w\frac{\partial C}{\partial t}-\frac{2\sin w}{t}\frac{\partial C}{\partial w})\ M_1 \tag{A-21}$$

$$M_4=\sigma^2[-\frac{\sin 2w}{2}\frac{\partial^2 C}{\partial t^2}+\frac{\sin 2w}{2t^2}\frac{\partial^2 C}{\partial w^2}-\frac{\cos 2w}{t}\frac{\partial^2 C}{\partial t\partial w}+\frac{\cos 2w}{t^2}\frac{\partial C}{\partial w}$$
$$+\frac{\sin 2w}{2t}\frac{\partial C}{\partial t}+p^2\{-\frac{\sin 2w}{2}(\frac{\partial C}{\partial t})^2+\frac{\sin 2w}{2t^2}(\frac{\partial C}{\partial w})^2-\frac{\cos 2w}{t}\frac{\partial C}{\partial t}\frac{\partial C}{\partial w}\}]\ M_1$$
$$= M_5 \tag{A-22}$$

$$\binom{M_6}{M_7}=\sigma^2[-\binom{\sin^2 w}{\cos^2 w}\frac{\partial^2 C}{\partial t^2}-\frac{\cos^2 w}{t^2}\frac{\partial^2 C}{\partial w^2}\mp\frac{\sin 2w}{t}\frac{\partial^2 C}{\partial t\partial w}\pm\frac{\sin 2w}{t^2}\frac{\partial C}{\partial w}$$
$$-\binom{\cos^2 w}{\sin^2 w}\frac{1}{t}\frac{\partial C}{\partial t}+p^2\{-\cos^2 w(\frac{\partial C}{\partial t})^2-\frac{\sin^2 w}{t^2}(\frac{\partial C}{\partial w})^2\mp\frac{\sin 2w}{t}\frac{\partial C}{\partial t}\frac{\partial C}{\partial w}\}]M_1 \tag{A-23}$$

Substituting Eqs.(A-19) through (A-25) into I, we have

$$I=e^{-p^2(1-C)}[\cos^2 u-\tfrac{1}{2}L_2\sigma^2\sin^2 u-j\sigma p\sin 2u\{\sin(w+v)\frac{\partial C}{\partial t}+\frac{\cos(w+v)}{t}\frac{\partial C}{\partial w}\}$$
$$+\tfrac{1}{2}\sigma^2\sin^2 u\{\sin 2v(-\sin 2wL_1-2\cos 2wL_3)+\cos 2v(\cos 2wL_1-2\sin 2wL_3)\}] \tag{A-24}$$

where

$$L_{1,2}=\frac{\partial^2 C}{\partial t^2} \mp \frac{1}{t^2}\frac{\partial^2 C}{\partial w^2} \mp \frac{1}{t}\frac{\partial C}{\partial t} + p^2[(\frac{\partial C}{\partial t})^2 \mp \frac{1}{t}(\frac{\partial C}{\partial w})^2] \qquad \text{(A-25)}$$

$$L_3=\frac{1}{t}\frac{\partial^2 C}{\partial t \partial w} - \frac{1}{t^2}\frac{\partial C}{\partial w} + p^2\frac{1}{t}\frac{\partial C}{\partial t}\frac{\partial C}{\partial w} \qquad \text{(A-26)}$$

For the isotropic surface the correlation function depends on the radial distance t only and is independent on the direction w. Therefore all partial differentiations with respect to w are zero and we have I for the isotropic surface

$$I=e^{-p^2(1-C)}[\cos^2 u-\tfrac{1}{2}\sigma^2 L_2 \sin^2 u - j\sigma p \sin 2u \sin(w+v)\frac{\partial C}{\partial t} + \tfrac{1}{2}\sigma^2 L_1 \sin^2 u \cos 2(v+w)] \qquad \text{(A-27)}$$

where L_3 becomes zero and $L_{1,2}$ are

$$L_{1,2}=\frac{\partial^2 C}{\partial t^2} \mp \frac{1}{t}\frac{\partial C}{\partial t} + p^2(\frac{\partial C}{\partial t})^2 \qquad \text{(A-28)}$$

Appendix B

The Evaluation of $\bar{J}$

The normalized surface correlation function is given (See Eq.(3.2-7))

$$C = C_o(\tau, z) . [\cos \; z(f_x \sin\theta + f_x \cos\theta) - w_d\tau] \tag{B-1}$$

where C_o is

$$C_o = \exp\{-\frac{\tau^2}{T^2} - \frac{z^2}{L^2}\} \tag{B-2}$$

The partial derivatives are

$$\frac{\partial C}{\partial z} = -\frac{2z}{L^2} . C - (f_x \sin\theta + f_y \cos\theta) . \bar{C} \tag{B-3}$$

where

$$\bar{C} = C_o \sin[\; z(f_x \sin\theta + f_y \cos\theta) - w_d\tau] \tag{B-4}$$

and

$$\frac{\partial C}{\partial \theta} = -z(f_x \cos\theta - f_y \sin\theta) \; . \; \bar{C} \tag{B-5}$$

$$\frac{\partial^2 C}{\partial z^2} = -[\; \frac{2}{L^2} + \frac{1}{2}(f_x^2 + f_y^2) + f_x f_y \sin 2\theta - \frac{1}{2}(f_x^2 - f_y^2)\cos 2\theta - \frac{4z^2}{L^2}]C + \frac{4z}{L^2}(f_x \sin\theta - f_y \cos\theta) . \bar{C} \tag{B-6}$$

$$\frac{\partial^2 C}{\partial \theta^2} = -z^2[\frac{1}{2}(f_x^2 + f_y^2) - f_x f_y \sin 2\theta + \frac{1}{2}(f_x^2 - f_y^2)\cos 2\theta] . C - z(f_x \cos\theta - f_y \sin\theta) . \bar{C} \tag{B-7}$$

and

$$\frac{\partial^2 C}{\partial \theta \partial z} = \frac{\partial^2 C}{\partial z \partial \theta} = -z[\frac{1}{2}(f_x^2 - f_y^2)\sin 2\theta = f_x f_y \cos 2\theta] . C - (1 - \frac{2z^2}{L^4}) . (f_x \cos\theta - f^y \sin\theta) . \; \bar{C} \tag{-8}$$

Substituting Eqs.(B-3) -(B-8) into Eqs.(A-27),(A-28), we have

$$L_1=-[2f_xf_y\sin 2\theta + \frac{1}{2}(f_x^2-f_y^2)\cos 2\theta - \frac{4z^2}{L^4}].C = \frac{4z}{L^2}(f_x\sin\theta + f_y\cos\theta)\bar{C}$$
$$+p^2[\frac{4z^2}{L^4}c^2 + 2f_xf_y\sin 2\theta\,\bar{C} + (f_y^2-f_x^2)\cos 2\theta\,\bar{C}^2 + \frac{4z}{L^2}(f_x\sin\theta + f_y\cos\theta).C\bar{C} \quad \text{(B-9)}$$

$$L_2=-[\frac{4}{L^4}=(f_x^2+f_y^2) - \frac{4z^2}{L^4}].C + \frac{4z}{L^2}(F_x\sin\theta + f_y\cos\theta).\bar{C}$$
$$+p^2[\frac{4z^2}{L^4}.C^2 + (f_x^2+f_y^2)\bar{C}^2 + \frac{4z}{L^2}(f_x\sin\theta + f_y\cos\theta)C\bar{C} \quad \text{(B-10)}$$

and

$$L_3=-[\frac{1}{2}(f_x^2-f_y^2)\sin 2\theta + f_xf_y\cos 2\theta].C + \frac{2z}{L^2}Cf_x\cos\theta - f_y\sin\theta)\bar{C}$$
$$+p^2[\frac{2z}{L^2}(f_x\cos\theta - f_y\sin\theta)C.\bar{C} + \{f_xf_y\cos 2\theta + \frac{1}{2}(f_x^2-f_y^2)\sin 2\theta\}\ \bar{C}^2] \quad \text{(B-11)}$$

Subsituting these L -indentities into the approximate integrand $\bar{J}$ defined by Eq.(3.3-33) and factoring the orders of travelling waves and the envelope of the surface correlation function, we have

$$\bar{J}=CN_{10}+\bar{C}N_{01}+C\bar{C}N_{11}+C^2N_{20}+\bar{C}^2N_{o2}+C^3N_{30}+C^2\bar{C}N_{21}+C\bar{C}^2N_{12}$$

$$=\sum_{mn}\bar{J}^{\pm}_{mn} \quad \text{(B-12)}$$

where N_{ij} are given by Eqs.(3.3-11) through (3.3-18) and $\bar{J}^{\pm}_{mn}$ are

$$\bar{J}_{02}=\frac{C_0^2}{2}(N_{20}+N_{02}) \quad \text{(B-13)}$$

$$\bar{J}^{\pm}_{11}=\frac{C_o}{2}C^{\pm}(N_{10}\mp jN_{01}) \quad \text{(B-14)}$$

$$\bar{J}^{\pm}_{22}=\frac{C_o^2}{4}C^{\pm 2}(N_{2o}-N_{02}\mp jN_{11}) \quad \text{(B-15)}$$

$$\bar{J}^{\pm}_{13}=\frac{C_o^3}{4}C^{\pm 1}(3N_{30}+N_{12}\mp jN_{21}) \quad \text{(B-16)}$$

$$\bar{J}^{\pm}_{33}=\frac{C_o^3}{4}C^{\pm 3}(N_{30}-N_{12}\mp jN_{21}) \quad \text{(B-17)}$$

Appendix C

The Evaluation of $Q^{\pm}_{mn}(\tau)$

The partial incoherent autocovariance functions $Q^{\pm}_{mn}(\tau)$ are from Eq.(3.3-35)

$$Q^{\pm}_{mn}(\tau)=ASe^{-p^2-j\omega_c\tau}\int_0^{\infty}\int_0^{2\pi}\mathfrak{J}^{\pm}_{mn}\cdot e^{-jzK_t\cos(\theta-\phi_k)}z\,d\theta\,dz \qquad (C-1)$$

where indices m and n are the orders of travelling wave and envelope of the surface correlation function respectively. We factored out the τ-dependencies from geometrical interations for the partial incoherent autocovariance functions as

$$Q^{\pm}_{mn}(\tau)=AS\exp\{-p^2-j\omega_c\tau-\frac{n\tau^2}{T^2}\mp jm\omega_d\tau\}.I^{\pm}_{mn} \qquad (C-2)$$

where

$$I^{\pm}_{mn}=\int_0^{\infty}\int_0^{2\pi}\exp\{-\frac{nz^2}{L^2}\pm jmz(f_x\sin\theta+f_y\cos\theta)-jzK_t\cos(\theta-\phi_k)\}f(N_{pq})z\,d\theta\,dz$$
$$=\int_0^{\infty}\int_0^{2\pi}\exp\{-\frac{nz^2}{L^2}-jzK^{\pm}_{mt}\cos(\theta-\phi^{\pm}_{mk})\}f(N_{pq})z\,dz\,d\theta \qquad (C-3)$$

with

$$K^{\pm 2}_{mt}=(K_x\mp mf_x)^2+(K_y\mp mf_y)^2 \qquad (C-4)$$

$$\phi^{\pm}_{mk}=\tan^{-1}\frac{K_x\mp mf_x}{K_x\mp mf_x} \qquad (C-5)$$

using $\bar{J}^{\pm}_{11}$, we have

$$I^{\pm}_{11}=\frac{1}{2}\int_0^{\infty}dz.z.\exp\{-\frac{z^2}{L^2}\}.\{[\cos^2up^2+\frac{\sigma^2}{2}\sin^2u(\frac{4}{L^2}+f_1-\frac{4z^2}{L^2})\pm\sigma p\cdot\sin2u(f_y\sin v+f_x\cos v)]\Phi^{\pm}_{01}+j\sigma p\sin2u\frac{2z}{L^2}(\cos v\Psi^{\pm}_{11}+\sin v\Phi^{\pm}_{11})$$
$$\pm j\frac{\sigma^2}{2}\sin^2u\frac{4z}{L^4}(V_1\Psi^{\pm}_{11}+V_2\Phi^{\pm}_{11})+\frac{\sigma^2}{2}\sin^2u\frac{4z^2}{L^4}(\cos2v\Phi^{\pm}_{21}-\sin2v\Psi^{\pm}_{21})\} \qquad (C-6)$$

where $\Phi^{\pm}_{\ell m}$ and $\Psi^{\pm}_{\ell m}$ are angular intergrals

$$\binom{\Phi^{\pm}_{\ell m}}{\Psi^{\pm}_{\ell m}} = \int_0^{2\pi} {\cos \atop \sin}(\ell\theta) e^{-jzK^{\pm}_{mt}\cos(\theta-\phi^{\pm}_{mk})} d\theta = 2\pi(-j)^{\ell} J_{\ell}(K^{\pm}_{mt}z) {\cos \atop \sin}(\ell\phi^{\pm}_{mk}) \tag{C-7}$$

The special cases are

$$\Phi^{\pm}_{om} = 2\pi J_o(K^{\pm}_{mt}z) \tag{C-8}$$

$$\binom{\Phi^{\pm}_{1m}}{\Psi^{\pm}_{1m}} = -j2\pi\ J_1(K^{\pm}_{mt}z) {\cos \atop \sin}(\phi^{\pm}_{mk}) \tag{C-9}$$

$$\binom{\Phi^{\pm}_{2m}}{\Psi^{\pm}_{2m}} = -2\pi\ J_2(K^{\pm}_{mt}z)\ {\cos \atop \sin}(2\phi^{\pm}_{mk}) \tag{C-10}$$

Substituting Eqs.(c-8)-(C-10) into I_{11}, we have

$$\begin{aligned} I^{\pm}_{11} = & \frac{1}{2} \int_0^{\infty} dz \cdot z \cdot \exp\{-\frac{z^2}{L^2}\} \cdot 2 \{ [\cos^2 u p^2 \pm \sigma p \sin 2u (f_x \sin v + f_y \cos v) \\ & + \frac{\sigma^2}{2} \sin^2 u (\frac{4}{L^2} + f_1 - \frac{4z^2}{L^4})\ J_o(K^{\pm}_{1t}z) + \frac{2\sigma p}{L^2} \sin 2u\ \sin(v+\phi^{\pm}_{1k}) \\ & \pm \frac{2\sigma^2}{L^2} \sin^2 u . (V_1 \sin\phi^{\pm}_{1k} + V_2 \cos\phi^{\pm}_{1k})\ ] . z\ J_1(K^{\pm}_{1t}z) - \frac{2\sigma^2}{L^4} \sin^2 u \cdot \\ & \cdot \cos 2(v+\phi^{\pm}_{1k})\ z^2 J_2(K^{\pm}_{1t}z)\} \end{aligned} \tag{C-11}$$

where

$$f_1 = (f_x^2 + f_y^2) + 2f_x f_y \sin 2v + (f_x^2 - f_y^2)\cos 2v \tag{C-12}$$

$$V_{1,2} = f_{x,y} - f_{y,x} \sin 2v \mp f_{x,y} \cos 2v \tag{C-13}$$

$$\begin{aligned} I^{\pm}_{11} = & \pi\{ [\cos^2 p^2 \pm \sigma p \sin 2u (f_x \sin v + f_y \cos v) + \frac{\sigma^2}{2} \sin\ 2u (\frac{4}{L^2} + f_1)\ ] . N^{\pm}_{o11} \\ & - \frac{\sigma^2}{2} \sin^2 u . \frac{4}{L^4} (N^{\pm}_{o11})^1 + [\frac{2\sigma p}{L^2}\ \sin 2u\ \sin(v+\phi^{\pm}_{1k}) \pm \frac{2\sigma^2}{L^2}\ \sin^2 u . \\ & . (V_1 \sin\phi^{\pm}_{1k} + V_2 \cos\phi^{\pm}_{1k}) - \frac{2\sigma^2}{L^4} \sin^2 u \cos 2(v+\phi^{\pm}_{1k})] . N^{\pm}_{211}\} \end{aligned} \tag{C-14}$$

where $I^{\pm}_{\ell mn}$ are the Weber integrals[24]

$$N^{\pm}_{\ell mn}=\int_0^{\infty} z^{\ell\pm1}\exp\{-\frac{nz^2}{L^2}\}J_{\ell}(K^{\pm}_{mt}z)\,dz=\frac{(K^{\pm}_{mt})^{\ell}}{(\frac{2n}{L^2})^{\ell\pm1}}.\exp\{-\frac{(LK^{\pm}_{mt})^2}{4n}\} \quad (C\text{-}15)$$

$(N_{o11})'$ is derivative with respect to (L^{-2})

$$(N^{\pm}_{o11})'=-\frac{d}{d(\frac{1}{L^2})}(N_{o11})=\frac{L}{2}[1-\frac{(K^{\pm}_{mt}L)^2}{4}]\ .\exp\{-\frac{(LK^{\pm}_{mt})^2}{4}\} \quad (C\text{-}16)$$

Substituting appropriate $I^{\pm}_{1mn}$ and Eq.(C-16) into $I^{\pm}_{11}$, we have

$$I^{\pm}_{11}=2\pi.\exp\{-(\frac{K^{\pm}_{1t}L}{2})^2\}\cdot\{\frac{p^2L^2}{4}\cos^2u\pm\frac{\sigma pL^2}{2}\sin2u(f_x\sin v\pm f_y\cos v)$$

$$+\frac{\sigma^2L^2f_1}{4}\sin^2u+\frac{\sigma^2L^2K^{\pm2}_{1t}}{8}\sin^2u.(1-\cos2(v+\phi^{\pm}_k))+\frac{\sigma pL^2K^{\pm}_{1t}}{4}\sin2u\sin(v\pm\phi^{\pm}_{1k})$$

$$\pm\frac{\sigma^2L^2K^{\pm}_{1t}}{4}\sin^2u(V_1\sin\phi^{\pm}_{1k}+V_2\cos\phi^{\pm}_{1k})\} \quad (C\text{-}17)$$

Similarly we evaluate the angular integrations of $I^{\pm}_{22}$ by using the Richard- Wolf indentities

$$I^{\pm}_{22}=\frac{\pi p^2}{4}\int_0^{\infty}dz.z.\exp\{-\frac{2z^2}{L^2}\}.\{[\frac{\sigma^2}{2}\sin^2u(\frac{4}{L^2}+(^{o}_{2f_1})-\frac{4z^2}{L^2})\pm\sigma p\sin2u(f_x\sin v$$

$$+f_y\cos v)]J_o(K^{\pm}_{2t}z)+[\sigma p\sin2u.\frac{2z}{L^2}\sin(v+\phi^{\pm}_{2k})\pm\frac{\sigma^2}{2}\sin^2u\frac{8z^2}{L^4}(V_1\sin\phi^{\pm}_{2k}$$

$$+V_2\cos\phi^{\pm}_{2k})]J_1(K^{\pm}_{2k}\ z)-\frac{\sigma^2}{2}\ \sin^2u\cdot\frac{8z^2}{L^4}\cos2(v+\phi^{\pm}_{2k})J_2(K^{\pm}_{2K}\ z)\}. \quad (C\text{-}18)$$

and the Weber integrals the radial integration of I_{22} using

$$I_{22}^{\pm}=2\pi\exp\{-\frac{(K_{2t}^{\pm}L)^2}{8}\}.\{\frac{\sigma^2p^2}{8}\sin^2u[\frac{L^2K_{2t}^{\pm 2}}{8}\pm(\frac{o}{2f_1})]\pm\frac{\sigma p^3L^2}{4}\sin2u(f_x\sin v$$

$$\mp f_y\cos v)\mp\frac{2\sigma p^3L^2K_{2t}^{\pm}}{4^3}\sin2u\ \sin(v\mp\phi_{2k}^{\pm})\pm\frac{\sigma^2p^2L^2K_{2t}^{\pm}}{4}\sin2u(f_x\sin v$$

$$\mp f_y\cos v)\ \frac{2\sigma p^3L^2K_{2t}^{\pm}}{4^3}\sin2u\ \sin(v\mp\phi_{2k}^{\pm})\pm\frac{\sigma^2p^2L^2K_{2t}^{\pm}}{4^2}\ \sin^2u(V_1\sin\phi_{2k}^{\pm}$$

$$\mp V_2\cos\phi_{2k}^{\pm})-\frac{\sigma^2p^2L^2K_{2t}^{\pm}}{4^2}\sin^2u\cos2(v\mp\phi_{2k}^{\pm})\} \quad \text{(C-19)}$$

Similarly

$$I_{13}=\frac{\pi p^4}{4}\int_0^{\infty}dz.z.\exp\{-\frac{3z^2}{L^2}\}.\{\frac{\sigma^2}{2}\sin^2u(-f_1-\frac{12z^2}{L^4})J_o(K_{1t}^{\pm}z)\pm\frac{2\sigma^2}{L^2}\sin^2u$$

$$(V_1\sin\phi_{1k}^{\pm}\ \mp V_2\cos\phi_{1k}^{\pm})\ z\ J_1(K_{1k}^{\pm}z)-\frac{6\sigma^2}{L^4}\sin^2u\cos2(v\mp\phi_{1k}^{\pm})z^2J_2(K_{1t}^{\pm}z)\}$$

and (C-20)

$$I_{13}^{\pm}=2\pi.\exp.\{-\frac{(K_{1t}^{\pm}L)^2}{12}\}.\{-\frac{\sigma^2p^4L^2f_1}{3.2^5}\sin^2u\mp\frac{\sigma^2p^4L^2K_{1t}^{\pm 2}}{3^2\ .2^5}\sin^2u$$

$$+\frac{\sigma^2p^4L^2K_{1t}^{\pm}}{3^2\ .2^5}\sin^2u(V_1\sin\phi_{1k}^{\pm}\ \mp V_2\cos\phi_{1k}^{\pm}\ )-\frac{\sigma^2p^4L^2K_{1t}^{\pm 2}}{3^2\ 2^5}\sin^2u\cos2(v\mp\phi_{1k}^{\pm})\}$$

Finally (C-21)

$$I_{33}^{\pm}=\frac{\pi p^4}{4}\int_0^{\infty}dz.z.\exp\{-\frac{3z^2}{L^2}\}.\{\frac{\sigma^2}{2}\sin^2u(f_1-\frac{4z^2}{L^4})J_o(K_{3t}^{\pm}z)$$

$$\pm\frac{2\sigma^2}{L^2}\sin^2u(V_1\sin\phi_{3k}^{\pm}\ \mp V_2\cos\phi_{3k}^{\pm})zJ_1(K_{3t}^{\pm}\ z)-\frac{2\sigma^2}{L^4}\sin^2u\cos2(v$$

$$+\phi_{3k}^{\pm})z^2J_2(K_{3t}^{\pm}\ z)\}$$ (C-22)

and

$$I_{33}^{\pm}=2\pi\exp\{\frac{(K_{3t}^{\pm}L)^2}{12}\}.\{\frac{\sigma^2p^4L^2f_1}{3.2^5}\sin^2u\pm\frac{\sigma^2p^4L^2K_{3t}^{\pm2}}{3^32^5}\sin^2u$$

$$+\frac{p^4L^2K_{3t}^{\pm}}{3^3\ 2^4}\sin^2u(V\sin\phi_{3k}^{\pm}\ +V_2\cos\phi_{3k}^{\pm})-\frac{\sigma^2p^4L^2K_{3t}^{\pm2}}{3^3 2^5}\sin^2u\cos(v\ +\phi_{3k}^{\pm})\}$$

(C-23)

Appendix D

The Evaluations of Second Order NAC and TPS for

The second order surface correlation function is

$$C_2 = C_{o2}(\cos a + \cos b) \tag{D-1}$$

where

$$C_{o2} = \frac{C_o}{2} = \tfrac{1}{2}\exp\left[-\frac{\tau^2}{T^2} - \frac{z^2}{L^2} \right] \tag{D-2}$$

$$\binom{a}{b} = z[(f_{sx} \pm f_{ex})\sin\theta = (f_{sy} \pm f_{ey})\cos\theta] - (\varpi_s \pm \varpi_e) \tag{D-3}$$

The partial derivatives of C_2 are

$$\frac{\partial C_2}{\partial z} = -\frac{2z}{L^2}C_2 - (f_{sx}\sin\theta + f_{sy}\cos\theta)\bar{C}_2^{+} - (f_{ex}\sin\theta + f_{ey}\cos\theta)\bar{C}_2^{-} \tag{D-4}$$

where

$$\bar{C}_2 = C_{o2}(\sin a \pm \sin b) \tag{D-5}$$

$$\frac{\partial C_2}{\partial \theta} = -z(f_{sx}\cos\theta - f_{sy}\sin\theta)\bar{C}_2^{+} - z(f_{ex}\cos\theta - f_{ey}\sin\theta)\bar{C}_2^{-} \tag{D-6}$$

$$\begin{aligned}\frac{\partial^2 C_2}{\partial z^2} = {} & \left[-\frac{2}{L^2} + \frac{4z^2}{L^4} - (f_{sx}\sin\theta + f_{sy}\cos\theta)^2 - (f_{ex}\sin\theta + f_{ey}\cos)^2 \right] C_2 \\ & + \frac{4z}{L^2}\left[(f_{sx}\sin\theta + f_{sy}\sin\theta)\bar{C}_2^{+} + (f_{ex}\sin\theta + f_{ey}\cos\theta)\bar{C}_2^{-} \right] \\ & + 2(f_{sx}\sin\theta + f_{sy}\cos\theta)(f_{ex}\sin\theta + f_{ey}\cos\theta)\bar{C}_2^{-}\end{aligned} \tag{D-7}$$

$$\begin{aligned}\frac{\partial^2 C_2}{\partial \theta^2} = {} & -z\left[(f_{sx}\cos\theta - f_{sy}\sin\theta)\bar{C}_2^{+} + (f_{ex}\cos\theta - f_{ey}\sin\theta)\bar{C}_2^{-} \right] \\ & - z^2\left[(f_{sx}\cos\theta - f_{sy}\sin\theta)^2 C_2 + (f_{ex}\cos\theta - f_{ey}\sin\theta)^2 C_2^{-} \right]\end{aligned} \tag{D-8}$$

where

$$C_2^{-} = C_{o2}(\cos a - \cos b) \tag{D-9}$$

Also

$$\frac{\partial^2 C_2}{\partial z \partial \theta} = \frac{\partial^2 C_2}{\partial \theta \partial z} = -(1-\frac{2z^2}{L^2})[(f_{sx}\cos\theta - f_{sy}\sin\theta)\bar{C}_2^+ + (f_{ex}\cos\theta - f_{ey}\sin\theta)\bar{C}_2^-]$$

$$-z[(f_{sx}\cos\theta - f_{sy}\sin\theta)(f_{sx}\sin\theta + f_{sy}\cos\theta)C_2$$

$$+(f_{ex}\cos\theta - f_{ey}\sin\theta)(f_{ex}\sin\theta + f_{ey}\cos\theta)C_2^-$$

$$+2(f_{sx}\cos\theta - f_{sy}\sin\theta)(f_{ex}\cos\theta - f_{ey}\sin\theta)C_2^-] \qquad \text{(D-10)}$$

using the inequality $f_s \gg f_e$, we may neglect f_e dependencies in Eqs.(D-4) through (D-10) in all but the phase of the second order correlation function C_2.

$$\frac{\partial C_2}{\partial z} = -\frac{2z}{L^2}C_2 - (f_{sx}\sin\theta + f_{sy}\cos\theta)\bar{C}_2^+ \qquad \text{(D-11)}$$

$$\frac{\partial C_2}{\partial \theta} = -z(f_{sx}\cos\theta - f_{sy}\sin\theta)C_2^+ \qquad \text{(D-12)}$$

$$\frac{\partial^2 C_2}{\partial z^2} = -[\frac{2}{L^2} + \tfrac{1}{2}(f_{sx}^2 + f_{sy}^2) + f_{sx}f_{sy}\sin 2\theta - \tfrac{1}{2}(f_{sx}^2 - f_{sy}^2)\cos 2\theta$$

$$-\frac{4z^2}{L^2}]C_2 + \frac{4z}{L^2}(f_{sx}\sin\theta + f_{sy}\cos\theta)\bar{C}_2^+ \qquad \text{(D-13)}$$

$$\frac{\partial^2 C_2}{\partial \theta^2} = -z^2[\tfrac{1}{2}(f_{sx}^2 + f_{sy}^2) - f_{sx}f_{sy}\sin 2\theta + \tfrac{1}{2}(f_{sx}^2 - f_{sy}^2)c\text{-}s2\theta]C_2$$

$$-z(f_{sx}\cos\theta - f_{sy}\sin\theta)\bar{C}_2^+ \qquad \text{(D-14)}$$

$$\frac{\partial^2 C_2}{\partial z \partial \theta} = -(1-\frac{2z}{L^2})(f_{sx}\cos\theta - f_{sy}\sin\theta)\bar{C}_2^+$$

$$-z[\tfrac{1}{2}(f_{sx}^2 - f_{sy}^2)\sin 2\theta + f_{sx}f_{sy}\cos 2\theta].C_2 \qquad \text{(D-15)}$$

Therefore, the approximated integrand $\bar{J}_2$ can be written in the same form as the first order model, see Eq.(B-12), Decomposing the second order surface correlation function into travelling waves, we have

$$C_2 = \frac{C_{o2}}{2}\left(C_a^{+} + C_a^{-} + C_b^{+} + C_b^{-} \right) \tag{D-16}$$

where

$$C_a^{\pm} = e^{-jn\left[\pm(w_s + w_e) \mp z\{(f_{sx} + f_{ex})\sin\theta + (f_{sy} + f_{ey})\cos\theta\}\right]} \tag{D-17}$$

$$C_b^{\pm} = e^{-jn\left[\pm(w_s - w_e) \mp z\{(f_{sx} - f_{ex})\sin\theta + (f_{sy} - f_{ey})\cos\theta\}\right]} \tag{D-18}$$

Similarly

$$\bar{C}^{+} = -j\frac{C_{o2}}{2}\left(C_a^{+} - C_a^{-} + C_a^{+} - C_a^{-} \right) \tag{D-19}$$

$$C_2^2 = C_{o2}^2 + \frac{C_{o2}^2}{2}\left(C_s^{+} + C_s^{-} + C_e^{+} + C_e^{-} \right) + \frac{C_{o2}^2}{4}\left(C_a^{+2} + C_a^{-2} + C_b^{+2} + C_b^{-2} \right) \tag{D-20}$$

$$\bar{C}_2^{+2} = C_{o2}^2 + \frac{C_{o2}}{2}\left(C_e^{+} + C_e^{-} - C_s^{+} - C_s^{-} \right) - \frac{\quad}{o2}\left(C_a^{+2} + C_a^{-2} + C_b^{+2} + C_b^{-2} \right) \tag{D-21}$$

$$C_2\bar{C}_2^{+} = -j\frac{C_{o2}}{2}\left(C_s^{+} - C_s^{-} \right) - j\frac{C_{o2}}{4}\left(C_a^{+2} - C_a^{-2} + C_b^{+2} - C_b^{-2} \right) \tag{D-22}$$

$$\begin{aligned} C_2^3 = \frac{C_{o2}^3}{8}\Big[& C_a^{+3} + C_a^{-3} + C_b^{+3} + C_b^{-3} + 3C_{2a,b}^{+} + 3C_{2a,b}^{-} + 3C_{a,2b}^{+} + 3C_{a,2b}^{-} \\ & + 3C_{2a,-b}^{+} + 3C_{2a,-b}^{-} + C_{a,-2b}^{+} + C_{a,-2b}^{-} + 9C_a^{+} + 9C_a^{-} + 9C_b^{+} + 9C_b^{-} \Big] \end{aligned} \tag{D-23}$$

$$\begin{aligned} C_2\bar{C}_2^{+2} = \frac{\quad}{8}\!\!\!\!o2\Big[& -C_a^{+3} - C_a^{-3} - C_b^{+3} - C_b^{-3}\ 3C_{2a,b}^{+} - 3C_{2a,b}^{-} - 3C_{a,2b}^{+} - 3C_{a,2b}^{-} \\ & + C_{2a,-b}^{+} + C_{2a,-b}^{-} + C_{a,-2b}^{+} + C_{a,-2b}^{-} + 3C_a^{+} + 3C_a^{-} + 3C_b^{+} + 3C_b^{-} \Big] \end{aligned} \tag{D-24}$$

$$\begin{aligned} C_2^2\bar{C}_2^{+} \doteq -j\frac{C_{o2}^3}{8}\Big[& C_a^{+3} - C_a^{-3} + C_b^{+3} - C_b^{-3} + 3C_{2a,b}^{+} - 3C_{2a,b}^{-} + 3C_{a,2b}^{+} - 3C_{a,2b}^{-} \\ & + C_{2a,-b}^{+} - C_{2a,-b}^{-} + C_{-a,2b}^{+} + C_{-a,2b}^{-} + 3C_a^{+} - 3C_a^{-} + 3C_b^{-} - 3C_b^{+} \Big] \end{aligned} \tag{D-25}$$

where

$$C^{\pm n}_{pa,qb}=e^{\mp jn[pa+qb]} \tag{D-26}$$

$$C^{\pm}_{2a,b}=e^{\mp j}\left[(3\omega_s+\omega_e)-z\{(3f_{sx}\sin\theta+3f_{sy}\cos\theta)+(f_{ex}\sin\theta+f_{ey}\cos)\}\right] \tag{D-27}$$

$$C^{\pm}_{a,2b}=e^{\mp j}\left[(3\omega_s-\omega_e)-z\{(3f_{sx}\sin\theta+3f_{sy}\cos\theta)-(f_{ex}\sin\theta+f_{ey}\cos)\}\right] \tag{D-28}$$

$$C^{\pm}_{2a,-b}=e^{\mp j}\left[(\omega_s+3\omega_e)-z\{(f_{sx}\sin\theta+f_{sy}\cos\theta)+(3f_{ex}\sin\theta+3f_{ey}\cos)\}\right] \tag{D-29}$$

$$C^{\pm}_{-a,2b}=e^{\mp j}\left[(\omega_s-3\omega_e)-z\{(f_{sx}\sin\theta+f_{sy}\cos\theta)-(3f_{ex}\sin\theta+3f_{ey}\cos)\}\right] \tag{D-30}$$

Substituting Eqs.(D-16)-(D-30) into $\bar{J}_2$, we have

$$\bar{J}_2=\frac{C_{o2}}{2}(C^{\pm}_a+C^{\pm}_b)N_{10}\mp j\frac{C_{o2}}{2}(C^{\pm}_a+C^{\pm}_b)+C^2_{o2}(N_{2o}+N_{o2})+\frac{C_{o2}}{2}C^{\pm}(N_{2o}-N_{o2})$$

$$+\frac{C_{o2}}{2}C^{\pm}(N_{o2}+N_{2o})+\frac{C^2_{o2}}{4}(C^{\pm2}_a+C^{\pm2}_b)(N_{2o}-N_{o2})\ \ j\frac{C^2_{o2}}{2}(C^{\pm}_s+\frac{C^{\pm2}_a}{2}+\frac{C^{\pm2}_b}{2})N_{11}$$

$$+\frac{C^3_{o2}}{8}(C^{\pm3}_a+C^{\pm3}_b+3C^{\pm}_{2a,b}+3C^{\pm}_{a,2b}+3C^{\pm}_{2a,-b}+3C^{\pm}_{-a,2b}+9C^{\pm}_a+9C^{\pm}_b)N_{3o}$$

$$\mp j\frac{C^3_{o2}}{8}(C^{\pm3}_a+C^{\pm3}_b+3C^{\pm}_{2a,b}+3C^{\pm}_{a,2b}+C^{\pm}_{2a,-b}+C^{\pm}_{-a,2b}3C^{\pm}_a\ +C^{\pm}_b\ )N_{21}$$

$$+\frac{C^3_{o2}}{8}(-C^{\pm3}_a-C^{\pm3}_b-3C^{\pm}_{2a,b}-3C^{\pm}_{a,2b}+C^{\pm}_{2a,-b}+C^{\pm}_{-a,2b}+\ C^{\pm}_a+3C^{\pm}_b)N_{12}$$

$$=\frac{C^2_o}{4}(N_{2o}+N_{o2})+\frac{C_o}{4}C^{\pm}(N_{10}\mp jN_{o1})+\frac{C_o}{4}C^{\pm}(N_{1o}\mp jN_{o1})$$

$$+\frac{C^2_o}{8}(C^{\pm}_s(N_{2o}-N_{o2}\mp jN_{11})-\frac{C^2_o}{8}C^{\pm}_e(N_{o2}+N_{2o})+\frac{C^2_o}{16}C^{\pm2}_a(N_{2o}-N_{o2}\mp jN_{11})$$

$$+\frac{C^2_o}{16}C^{\pm2}_b(N_{2o}-N_{o2}\mp jN_{11})+\frac{C^3_o}{64}C^{\pm}_a(3N_{3o}+N_{12}\mp jN_{21})+\frac{C^3_o}{64}C^{\pm}_b(3N_{3o}+N_{12}\ jN_{21})$$

$$+\frac{3C^3_o}{64}C^{\pm}_{2a,b}(N_{3o}-N_{12}\mp jN_{21})+\frac{3C^3_o}{64}C^{\pm}_{a,2b}(N_{3o}-N_{12}\mp jN_{21})+\frac{3C^3_o}{64}C^{\pm}_{2a,-b}(N_{3o}+N_{12}\mp jN_{21})$$

$$+\frac{3C^3_o}{64}{}^{\pm}_{-a,2b}(N_{3o}+N_{12}\mp jN_{21})+\frac{C^3_o}{64}C^{\pm3}(N_{3o}-N_{12}\mp jN_{21})+\frac{C^3_o}{64}C^{\pm3}(N_{3o}-N_{12}\mp jN_{21})$$

$$=\sum_{h\ell n}\bar{J}^{\pm}_{h\ell n}=\sum_{h\ell n}f(N_{pq})C_o^n e^{\mp j[(h\omega_s+\ell\omega_e)-z\{(hf_{sx}+\ell f_{ex})\sin\theta+(hf_{sy}+\ell f_{ey})\cos\theta\}]} \quad \text{(D-31)}$$

and appropriate $\bar{J}^{\pm}_{hln}$ are

$$\bar{J}_{oo2}=\frac{C_o^2}{4}(N_{2o}+N_{o2}) \quad \text{(D-32)}$$

$$\bar{J}^{\pm}_{111}=\frac{Co}{4}C_a^{\pm}(N_{1o}\mp JN_{o1}) \quad \text{(D-33)}$$

$$\bar{J}^{\pm}_{\bar{1}11}=\frac{C_o}{4}C_6^{\pm}(N_{1o}\mp jN_{o1}) \quad \text{(D-34)}$$

$$\bar{J}^{\pm}_{1o2}=\frac{C_o^2}{8}C_s^{\pm}(N_{2o}-N_{o2}\mp jN_{11}) \quad \text{(D-35)}$$

$$\bar{J}^{\pm}_{o12}=\frac{C_o^2}{8}C_e^{\pm}(N_{2o}-N_{o2}\mp jN_{11}) \quad \text{(D-36)}$$

$$\bar{J}^{\pm}_{222}=\frac{C_o^2}{16}C_a^{\pm 2}(N_{2o}-N_{o2}\mp jN_{11}) \quad \text{(D-37)}$$

$$\bar{J}^{\pm}_{\bar{2}22}=\frac{C_o^2}{16}C_b^{\pm 2}(N_{2o}-N_{o2}\mp jN_{11}) \quad \text{(D-38)}$$

$$\bar{J}^{\pm}_{113}=\frac{C_o^3}{64}C_a^{\pm}(3N_{3o}+N_{12}\mp jN_{21}) \quad \text{(D-39)}$$

$$\bar{J}^{\pm}_{\bar{1}13}=\frac{C_o^3}{64}C_b^{\pm}(3N_{3o}+N_{i2}\mp jN_{21}) \quad \text{(D-40)}$$

$$\bar{J}^{\pm}_{313}=\frac{3C_o^3}{64}C^{\pm}_{2a,b}(N_{3o}-N_{12}\mp jN_{21}) \quad \text{(D-41)}$$

$$\bar{J}^{\pm}_{\bar{3}13}=\frac{3C_o^3}{64}C^{\pm}_{a,2b}(N_{3o}-N_{12}\mp jN_{21}) \quad \text{(D-42)}$$

$$\bar{J}^{\pm}_{133}=\frac{3C_o^3}{64}C^{\pm}_{2a,-b}(N_{3o}+N_{12}\mp jN_{21}) \quad \text{(D-43)}$$

$$\bar{J}^{\pm}_{\bar{1}33}=\frac{3C_o^3}{64}C^{\pm}_{-a,2b}(N_{3o}+N_{12}\mp jN_{21}) \quad \text{(D-44)}$$

$$\bar{J}^{\pm}_{333}=\frac{C_o^3}{64}C_a^{\pm 3}(N_{3o}-N_{12}\mp jN_{21}) \quad \text{(D-45)}$$

$$\bar{J}_{\bar{3}33}=\frac{C_o^3}{64}C_b^{\pm 3}(N_{3o}-N_{12}\mp jN_{21}) \quad \text{(D-46)}$$

where

$$K^{\pm}_{hl}=\left[(K_x\mp hf_{sx}\mp lf_{ex})^2+(K_y\mp hf_{sy}\mp lf_{ey})^2\right]^{\frac{1}{2}} \qquad \text{(D-47)}$$

$$\phi^{\pm}_{hl}=\tan^{-1}\frac{K_x\mp hf_{sx}\mp lf_{ex}}{K_y\mp hf_{sy}\mp lf_{ey}} \qquad \text{(D-48)}$$

Since $\bar{J}_{hln}$ are the same as of the first order $\bar{J}_{mn}$, we can identify the results without integration. The second order NAC is

$$Q(\tau)=\exp\{-j\omega_c\tau\}\bar{\sigma}_c+\sum_{hln}\exp\{\mp j(h\omega_s+l\omega_e)-\frac{n^2}{T^2}\tau\}\bar{\sigma}_{h\pm ln} \qquad \text{(D-49)}$$

where partial MSC's are

$$\bar{\sigma}_{oo2}=\tfrac{1}{2}\bar{\sigma}_{o2}(f=f_s) \qquad \text{(D-50)}$$

$$\bar{\sigma}^{\pm}_{1\pm 11}=\tfrac{1}{2}\bar{\sigma}^{\pm}_{11}(f=f_s,\ K^{\pm}_{1t}=K^{\pm}_{1\ \pm 1},\ \phi^{\pm}_{1t}=\phi^{\pm}_{1\pm 1}) \qquad \text{(D-51)}$$

$$\bar{\sigma}^{\pm}_{102}=\tfrac{1}{2}\bar{\sigma}^{\pm}_{22}(f=f_s,\ K^{\pm}_{2t}=K^{\pm}_{1o},\ \phi^{\pm}_{2t}=\phi^{\pm}_{1o}) \qquad \text{(D-52)}$$

$$\bar{\sigma}^{\pm}_{2\pm 22}=\tfrac{1}{4}\bar{\sigma}^{\pm}_{22}(f=f_2,\ K^{\pm}_{2t}=K^{\pm}_{2\pm 2},\ \phi^{\pm}_{2t}=\phi^{\pm}_{2\pm 2}) \qquad \text{(D-53)}$$

$$\bar{\sigma}^{\pm}_{1\pm 13}=\tfrac{1}{16}\bar{\sigma}^{\pm}_{13}(f=f_s,\ K^{\pm}_{1t}=K^{\pm}_{1\pm 1},\ \phi^{\pm}_{1t}=\phi^{\pm}_{1\pm 1}) \qquad \text{(D-54)}$$

$$\bar{\sigma}^{\pm}_{3\pm 13}=\tfrac{1}{16}\bar{\sigma}^{\pm}_{33}(f=f_s,\ K^{\pm}_{3t}=K^{\pm}_{3\pm 1},\ \phi^{\pm}_{1t}=\phi^{\pm}_{3\pm 1}) \qquad \text{(D-55)}$$

$$\bar{\sigma}^{\pm}_{3\pm 33}=\tfrac{1}{16}\bar{\sigma}^{\pm}_{33}(f=f_s,\ K^{\pm}_{3t}=K^{\pm}_{3\pm 3},\ \phi^{\pm}_{3t}=\phi^{\pm}_{3\pm 3}) \qquad \text{(D-56)}$$

$$\bar{\sigma}^{\pm}_{o12}=\tfrac{1}{4}\bar{\sigma}^{\pm}_{o2}(f=f_s,\ K_t=K^{\pm}_{o1},\ \phi_t=\phi^{\pm}_{o1}) \qquad \text{(D-57)}$$

The term $\bar{\sigma}^{\pm}_{1\pm 33}$ can not be identified with the first order model because of the absence of the sum $N_{30}+N_{12}+jN_{21}$. Since the difference in the first order is only $-N_{12}$ we may correct the sign of corresponding terms and we have

$$O(\ell^8)=\frac{\ell^8}{\sigma_n^2}\left[\frac{\bar{p}^4 f_s^2}{24}\sin u(1+\sin 2v)\right] \qquad \text{(D-58)}$$

Therfore we have

$$\bar{\sigma}^{\pm}_{1\pm33}=\frac{3}{16}\bar{\sigma}^{\pm}_{13}\ (f=f_s, K^{\pm}_{3t}= {}^{\pm}_{1\pm3}, \phi^{\pm}_{3t}=\phi^{\pm}_{1\pm3}, -o(\ell^8)\ ) \qquad \text{(D-59)}$$

Appendix E

The Transfor of the spectrum $\Gamma_o(\omega_1,\omega_2)$

The partial bifrequency spectrum $\Gamma_o(\omega_1,\omega_2)$ is

$$\Gamma_o(\omega_1,\omega_2)=|A|^2\iint_S\!\!\iint dxdx'dydy'.H_o.\exp\{-p^2(1-C)-jq\cos v(x-x')$$
$$+\sin v(y-y')\}\ .I_t \qquad \text{(E-1)}$$

where I_t is integrations with respect to t_1 and t_2 as

$$I_t=\int_{-\infty}^{\infty}\!\!\int\exp\{-\frac{(pq_V)^2}{2}(t_1^2+t_2^2-2Ct_1t_2)+p^2q_V(1-C)(t_2+t_1)-j\omega_o(t_1-t_2)$$
$$-j\frac{k}{R}[t_1(V_xx+V_yy)-t_2(V_xx'+V_yy')-j(\omega_1t_1-\omega_2t_2)]\}dt_1dt_2 \qquad \text{(E-2)}$$

after integrating t_2 variable, we have

$$I_t=\int_{-\infty}^{\infty}\exp\{-\frac{(pq_V)^2}{2}t_1+p^2q_V(1-C)t_1-j(\omega_o+\omega_1)t_1-jf(x,y)t_1$$
$$+[p^2q^2Ct_1+p^2q_V(1-C)+j(\omega_o+\omega_2+f(x',y'))]^2\ ]/2p^2q_V^2\}dt_1\cdot\sqrt{\frac{2\pi}{p^2q_V^2}}$$

where (E-3)

$$f(x,y)=\frac{k}{R}(V_xx+V_yy) \qquad \text{(E-4)}$$

$$f(x',y')=\frac{k}{R}(V_xx'+V_yy') \qquad \text{(E-5)}$$

Integrating with respect to t_1, we habe

$$I_t=(2\pi)^2\exp\{-p^2(1-C)-\frac{j}{q_V}(z_1-z_2)\}.P(z_1,z_2) \qquad \text{(E-6)}$$

where

$$z_1=\omega_1+\omega_o+\frac{k}{R}(V_xx+V_yy) \qquad \text{(E-7)}$$

$$z_2=\omega_2+\omega_o+\frac{k}{R}(V_xx'+V_yy') \qquad \text{(E-8)}$$

and

$$P(z_1,z_2)=\frac{1}{2\pi p^2 q_V^2\sqrt{1-C^2}}\exp\{-\frac{z_1^2+z_2^2-2Cz_1z_2}{2p^2q_V^2(1-C^2)}\} \qquad \text{(E-9)}$$

Using the identity

$$P(z_1'z_2)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}\exp\{-\frac{(pq_V)^2}{2}(s_1+s_2)+j(s_1z_1+s_2z_2)\}ds_1ds_2 \qquad \text{(E-10)}$$

and substituting I_t into Eq.(E-1),

we have the partial bifrequency spectrum, $\Gamma_o(\omega_1,\omega_2)$,

$$\Gamma_o(\omega_1,\omega_2)=|A|^2\iint_{-\infty}^{\infty}ds_1ds_2\exp\{-\frac{(pq_V)^2}{2}(s_1+s_2)-j[(\omega_1+\omega_o)(s_1-\frac{1}{q_V}) +(\omega_2+\omega_o)(s_2-\frac{1}{q_V})]\}.P_o \qquad \text{(E-11)}$$

where

$$P_o=\iiiint dxdx'dydy'.H_o\exp\{-Cp^2q_V^2s_1s_2-j(x-x')[\frac{kV_x}{R}(\frac{1}{q_V}+\frac{s_1-s_2}{2})+q\cos v] -j(y-y')[\frac{kV_y}{R}(\frac{1}{q_V}+\frac{s_1-s_2}{2})+q\sin v]-j[\frac{kV_x}{2R}(s_1+s_2)(x+x') +\frac{kV_y}{2R}(s_1+s_2)(y+y')]\} \qquad \text{(E-12)}$$

where we used the relationship

$$s_1x+s_2x'=\frac{1}{2}[(s_1-s_2)(x-x')+(s_1+s_2)(x+x')] \qquad \text{(E-13)}$$

Similarly, we have higher order partial bifrequency spectra

$$\Gamma_{12}(\omega_1,\omega_2)=|A|^2\iint_{-\infty}^{\infty}[P_1(-j\frac{\partial}{\partial\omega_1})+P_2(j\frac{2}{\partial\omega_2})]\exp\{-\frac{(pq_V)^2}{2}(s_1+s_2) -j[(\omega_1+\omega_o)(s_1-\frac{1}{q_V})+(\omega_2+\omega_o)(s_2-\frac{1}{q_V})\}ds_1ds_2 \qquad \text{(E-14)}$$

and

$$\Gamma_3(\omega_1,\omega_2)=|A|^2\iint_{-\infty}^{\infty}P_3(\frac{\partial^2}{\partial\omega_1\partial\omega_2})\exp\{-\frac{(pq_V)^2}{2}(s_1+s_2) -j[(\omega_1+\omega_o)(s_1-\frac{1}{q_V})+(\omega_2+\omega_o)(s_2-\frac{1}{q_V})]\}ds_1ds_2 \qquad \text{(E-15)}$$

where P_1, P_2 and P_3 are the geometric integrals as in Eq.(E-12) except H_1, H_2 and H_3 instead of H_o as

$$P_{12} = B.(-T_{12}) = B.2\pi(-T_{12}^1 + p^2 q_V^2 s_1 s_2 T_{12}^2) \quad \text{(E-16)}$$

$$P_3 = B.T_3 = B.2\pi(T_3^1 - p^2 q_V^2 s_1 s_2 T_3^2) \quad \text{(E-17)}$$

and

$$-T_{12} = \frac{4\sigma^2}{L^4}\sin^2 u p^2 q_V^2 \left(-I_{3(\frac{2}{3})o} - \cos\gamma I_{3(\frac{2}{3})2}\right) + \frac{\sigma p q_V}{L^2}\sin 2u\cos\gamma I_{2(\frac{1}{2})1} \quad \text{(E-18)}$$

$$T_3 \quad \frac{2\sigma^2 p^2 q_V^2}{L^4}\sin^2 u\left(-I_{3(\frac{2}{3})o} - \cos 2\gamma I_{3(\frac{2}{3})2}\right) \quad \text{(E-19)}$$

By the use of the Weber integrals, we have

$$-T_{12}^1 = S_1 \cdot \frac{1}{2^4}\sigma p q_V L^2 \sin 2u \cdot d\cos\gamma + S_2 \cdot \frac{1}{2^4}\sigma p^2 q_V L^2 \sin^2 u\, d^2(1-\cos\gamma) + \ldots \quad \text{(E-20)}$$

$$-T_{12}^2 = S_2 \cdot \frac{1}{2^4}\sigma p q_V L^2 \sin 2u \cdot d\cos^{\gamma} + S_3 \cdot \frac{1}{2.3^3}\sigma^2 p^2 q_V L^2 \sin^2 u \cdot d^2(1-\cos 2\gamma) + \ldots \quad \text{(E-21)}$$

and

$$T_3^2 = S_3 \cdot \frac{1}{2.3^3}\sigma^2 p^2 q_V L^2 \sin u^2 d^2(1-\cos 2\gamma) \quad \text{(E-22)}$$

The partial differentiations with respect to the two radian frequencies ω_1 and ω_2 to the integrands lead

$$\left(-j\frac{\partial}{\partial\omega_1} + j\frac{\partial}{\partial\omega_2}\right) \Rightarrow -\frac{2}{q_V} - \tau \quad \text{(E-24)}$$

$$\frac{\partial^2}{\partial\omega_1 \partial\omega_2} \Rightarrow \left(\frac{1}{q_V} + \frac{\tau}{2}\right)^2 - \left(\frac{t}{2}\right)^2 \quad \text{(E-25)}$$

Therefore the higher order partial mean bifrequency spectral components are

$$W_{12} = 2\pi 1 A 1^2 S \int\int_{-\infty}^{\infty} \frac{J_1(\omega_u t)}{\omega_u t} \exp\{-(\frac{p q_V}{2})^2 (t^2 + \tau^2) - j\omega_t t\} (\frac{2}{q_V} + \tau) .\ T_{12}^1$$

$$+\frac{(pq_V)^2}{2}(\tau^2-t^2)T_{12}^2]\ dt\ d\tau \qquad \text{(E-26)}$$

$$W_3=2\pi|A|^2S\int\int_{-\infty}^{\infty}\frac{J_1(\omega_u t)}{\omega_u t}\ \exp\{-(\frac{pq_V}{2})^2(t^2+\tau^2)-j\omega_t t\}[(\frac{1}{q_V}+\frac{\tau}{2})^2$$

$$-(\frac{t}{2})^2].[T_3^1+\frac{(pq_V)^2}{2}(\tau^2-t^2)T_3\ dtd\tau \qquad \text{(E-27)}$$

Using the expression for j_n, the t-integration, we have

$$W_{12}=2\pi|A|^2S\int_{-\infty}^{\infty}d\tau\ \exp\{-(\frac{pq_V}{2})^2\tau^2\}\ \{(\frac{2}{q_V}+\tau)\ [T_{12}^1+\frac{(pq_V)^2}{2}\ \tau^2T_{12}^2]\dot{j}_o$$

$$+(\frac{2}{q_V}+\tau)\frac{(pq_V)^2}{2}\cdot T_{12}^2.\dot{j}_2\} \qquad \text{(E-28)}$$

$$W_3=2\pi|A|^2S\int_{-\infty}^{\infty}d\tau.\exp\{-(\frac{pq_V}{2})^2\tau^2\}.\{(\frac{1}{q_V}+\frac{\tau}{2})^2.\ [T_3^1+\frac{(pq_V)^2}{2}\tau^2T_3^2]\dot{j}_o$$

$$+[\tfrac{1}{4}T_3^1+\frac{(pq_V)^2}{2}(\frac{1}{q_V}+\frac{\tau}{2})^2\tau^2T_3^2].\dot{j}_2+[\tfrac{1}{4}T_3^2\ \frac{(pq_V)^2}{2}\ ]\dot{j}_4\} \qquad \text{(E-29)}$$

where

$$j_4=\frac{\dot{j}_c(\bar{\omega}_t)}{2(pq_V)^4}\cdot\sum_{m=o}^{\infty}\frac{(\frac{\bar{\omega}_t}{2pq_V})^{2m}}{m!(m+1)!}\ H_{2m+4}(\bar{\omega}_t) \qquad \text{(E-30)}$$

By using the τ-integrations ,$K(n)_i^{1,2}$, and factoring the order of Hermite polynomials, we have

$$W_{12}=2\pi|A|^2S\ j_c(\bar{\omega}_t)\cdot\sum_m\frac{(\frac{\omega_u}{2pq_V})^{2m}}{m!(m+1)!}[B_1H_{2m}(\bar{\omega}_t)+B_2H_{2m+2}(\bar{\omega}_t)] \qquad \text{(E-31)}$$

$$W_3=2\pi|A|^2S\ j_c(\bar{\omega}_t)\cdot\sum_m\frac{(\frac{\omega_u}{2pq_V})^{2m}}{m!(m+1)!}[C_1H_{2m}(\bar{\omega}t)$$

$$+C_{12}H_{2m+2}(\bar{\omega}_t)+C_3H_{2m+4}(\bar{\omega}_t)] \qquad \text{(E-32)}$$

where

$$B_1 = \frac{2}{q_V} \overset{1}{\underset{12}{K}}(0) + \overset{1}{\underset{12}{K}}(1) + p^2 q_V \overset{2}{\underset{12}{K}}(2) + \frac{p^2 q_V^2}{2} \overset{2}{\underset{12}{K}}(3) \qquad \text{(E-33)}$$

$$B_2 = \frac{1}{q_V} \overset{2}{\underset{12}{K}}(0) + \tfrac{1}{2} \overset{2}{\underset{12}{K}}(1) \qquad \text{(E-34)}$$

$$C_1 = \frac{1}{q_V^2} \overset{1}{\underset{3}{K}}(0) + \frac{1}{q_V} \overset{1}{\underset{3}{K}}(1) + \tfrac{1}{4} \overset{1}{\underset{3}{K}}(2) + \overset{2}{\underset{3}{K}}(2) + p^2 q_V \overset{2}{\underset{3}{K}}(3) + \frac{p^2 q_V^2}{8} \overset{2}{\underset{3}{K}}(4) \qquad \text{(E-35)}$$

$$C_2 = \frac{1}{4p^2 q_V^2} \overset{1}{\underset{3}{K}}(0) + \tfrac{1}{2} \overset{1}{\underset{3}{K}}(2) + 2q_V \overset{1}{\underset{3}{K}}(3) + \frac{1}{8} \overset{1}{\underset{3}{K}}(4) \qquad \text{(E-36)}$$

and

$$C_3 = \frac{1}{8p^2 q_V^2} \overset{2}{\underset{3}{K}}(0) \qquad \text{(E-37)}$$

Appendix F

Reduction of a Quadruple to a Double Integral.

Considering an integral with finite integration interval with 2L as

$$I = \int\int_{-L}^{L} f(x=x')\,dx\,dx' \qquad \text{(F-1)}$$

where the integrand is a function of the skew coordinate variables only. Introducing the new variables

$$x_1 = \frac{x + x'}{\sqrt{2}} \qquad \text{(F-2)}$$

$$x_2 = \frac{x' - x}{\sqrt{2}} \qquad \text{(F-3)}$$

where Eqs.(F-2) and (F-3) are abtained by finding the point p(x,x') in the new coordinate system (See Fig. 1. F-1).

Considering the integration intervals in the new coordinate system, which is a rhombus, we have (See Fig. 1 . F-2)

$$I = \left[\int_{-\sqrt{2L}}^{0} dx_2 \int_{-\sqrt{2L}-x_2}^{\sqrt{2L}+x_2} dx_1 + \int_{0}^{\sqrt{2L}} dx_2 \int_{-\sqrt{2L}+x_2}^{\sqrt{2L}-x_2} dx_1 \right] f(\sqrt{2X_1})\, g(-\sqrt{2X_2}) \qquad \text{(F-4)}$$

Letting

$$\bar{X}_2 = -\sqrt{2X_2} = X - X' \qquad \text{(F-5)}$$

$$\bar{X}_2 = \sqrt{2X_1} = X + X' \qquad \text{(F-6)}$$

we may rewrite the integral I as

$$I = \int_{-2L}^{2L} g(X_2) \int_{0}^{2L-|\bar{x}_2|} f(\bar{X}_1)\, d\bar{X}_1\, d\bar{X}_2 \qquad \text{(F-7)}$$

Unless one knows the explicit expressions of the function $f(\bar{X}_1)$, and $\delta(\bar{X}_2)$, Eq.(F-7) is a final transformed form in the new skewed coordinate system. For example, if $f(\bar{X}_1)$ is unity, the integral I of Eq.(F-7) can be more simplified as integrating I with respect to $\bar{x}_1$

$$I = \int_{-2L}^{2L} (2L-|\bar{x}_2|)\ g(\bar{X}_2)\ d\bar{x}_2 \tag{F-8}$$

$$\doteq \int_{-\infty}^{\infty} G_T(\bar{X}_2)\ (2L-|\bar{X}_2|)g(\bar{X}_2)\ d\bar{X}_2 \tag{F-9}$$

where $G_T(\bar{X}_2)$ is a gate function as

$$G_T(X_2)=\begin{cases}1 & : \bar{X}_2 \leq 2L \\ 0 & : \bar{X}_2 > 2L\end{cases} \tag{F-10}$$

This result can be extended for a mul dimensional integrals i.e

$$J= \iiiint_{-L}^{L} g(x-x')(y-y')f(x+x',y+y')\ dxdx'dydy' \tag{F-11}$$

$$= \iint_{-2L}^{2L} g(\bar{x}_2,\ \bar{y}_2) \int_0^{2L-|x_2|} \int_0^{2L-|y_2|} f(\bar{x}_1,\bar{y}_1)d\bar{x}_1 f\bar{y}_1 . d\bar{x}_2 d\bar{y}_2$$

where

$$\bar{y}_1 = y+y' \tag{F-12}$$

$$\bar{y}_2 = y-y' \tag{F-13}$$

For the case of $f(\bar{x}_1,\bar{y}_1)$ unity, J becomes

$$J= \iint_{-\infty}^{\infty} G_T(\bar{x}_2)G_T(\bar{y}_2)(2L-|\bar{x}_2|)(2L-|\bar{y}_2|)g(\bar{x}_2,\bar{y}_2)d\bar{x}_2 d\bar{y}_2 \tag{F-14}$$

When the following condition is satisfied

$$g(\bar{x}_2,\bar{y}_2) \cong 0 \qquad ,\text{for } |x_2|,|y_2| >> T \tag{F-15}$$

where

$$T << L \tag{F-16}$$

The integral J is further simplified as

$$J= \iint_{-\infty}^{\infty} 4L^2 \; g\;(\bar{x}_2,\bar{y}_2)d\bar{x}_2 \; d\bar{y}_2 \tag{F-17}$$

If

$$f(\bar{x}_1,\bar{y}_1) = e^{-j(\bar{x}_1+\bar{y}_1)} \tag{F-18}$$

with the condition of Eq.(F-15), we approximate

$$J=\iint_{-\infty}^{\infty} g(\bar{x}_2,\bar{y}_2)d\bar{x}_2 d\bar{y}_2 \iint_{-2L}^{2L} e^{-j(x_1+y_1)} \; d\bar{x}_1 d\bar{y}_1 \tag{F-19}$$

Using cykindrical coordinate variable, we have

$$J= \int_0^{\infty}\int_0^{2\pi} g(t_2,w_2)t_2 dw_2 dt_2 \int_0^{A}\int_0^{2\pi} e^{-jt_1(\sin w_1+\cos w_1)} t_1 dx_1 dt_1$$

where

$$t_2=\sqrt{\bar{x}_2^2 + \bar{y}_2^2} \tag{F-20}$$

$$w_2= \tan^{-1} \frac{x_2}{\bar{y}_2} \tag{F-21}$$

$$t_1=\sqrt{\bar{y}_1^2 + \bar{x}_1^2} \tag{F-22}$$

$$w_1= \tan^{-1} \frac{\bar{x}_1}{\bar{y}_1} \tag{F-23}$$

with A is approximate circular interval

Appendix G

The Evalution of the Coefficient A_j

The coefficient A_j are determined by the τ-integrations of $K_i^{1,2}(n)$

$$K_i^{1,2}(n) = \int_{-\infty}^{\infty} \tau^n T_i^{1,2}(\tau)\ \exp\{-(\frac{pq_V}{2}\tau)^2\}\ d \qquad \text{(G-1)}$$

where $T_i^{1,2}$ are given by Eqs.(4.3-61), (4.3-62), (E-20)-(E-23)

The coefficient A_j are given as

$$A_o = \sqrt{\pi} k^2 \sqrt{\pi} \int_{-\infty}^{\infty} L d\tau \exp\{-(\frac{pq_V}{2}\tau)^2\} \delta(d) \qquad \text{(G-2)}$$

$$A_1 = \sqrt{\pi} k^2 \int_{-\infty}^{\infty} d\tau \exp\{-(\frac{pq_V}{2}\tau)^2\} (a_{10} + a_{11}\tau + a_{12}\tau^2 + a_{13}\tau^3) \qquad \text{(G-3)}$$

$$A_2 = \sqrt{\pi} k^2 \int_{-\infty}^{\infty} d\tau \exp\{-(\frac{pq_V}{2}\tau)^2\} (a_{20}\tau + a_{21}\tau^2) \qquad \text{(G-4)}$$

$$A_3 = \sqrt{\pi} K^2 \int_{-\infty}^{\infty} d\tau \exp\{-(\frac{pq_V}{2}\tau)^2\} a_{30}\tau^2 \qquad \text{(G-5)}$$

where

$$a_{10} = T^1 + \frac{2}{q_V} T^1_{12} + \frac{1}{q_V^2} T_3^{\ 1}$$
$$= S_1 \cdot \{\frac{\sigma^2 L^2}{2^2} \sin^2 u\ d^2(1-\cos\gamma) + \frac{\sigma p L^2}{2} \sin 2u d \cos\gamma\}$$
$$+ S_2 \cdot \{\frac{3L^2\sigma^2}{2^5} \sin^2 u\ d^2(1-\cos\gamma)\} \qquad \text{(G-6)}$$

$$a_{11} = T_{12} + \frac{1}{q_V} T_3 = S_2 \cdot q_V \{\frac{p\sigma L^2}{2^4} \sin 2u\ d\cos\gamma + \frac{p^2\sigma^2 L^2}{2^5} \sin^2 u\ d^2(1-\cos 2\gamma)\}$$
$$+ S_3 \cdot q_V \{\frac{p^2\sigma^2 L^2}{2,3^3} \sin^2 u\ d^2(1-\cos 2\gamma)\} \qquad \text{(G-7)}$$

$$a_{12} = \frac{p^2\sigma_V^{\ 2}}{2} T_o^{\ 2} + \frac{1}{4} T_3^{\ 1} + \frac{p^2}{2} T_3^{\ 3} = S_1 q_V^{\ 2} \frac{p^2 L^2}{4} \cos u$$
$$+ S_2 \cdot q_V^2 \{\frac{1}{2^6} p^2 \sigma^2 L^2 \sin^2 u\ d^2(1-\text{cis}2\gamma) + \frac{\sigma p^2 L^2}{2^5} \sin 2u\ d\cos\gamma\ \}$$
$$+ S_3 \cdot q_V^2 \{\frac{p^4\sigma^2 L^2}{2^2 \cdot 3^3} \sin^2 u\ d^2 (1-\cos 2\gamma)\} \qquad \text{(G-8)}$$

$$a_{13}=\frac{p^2 q_V{}^2}{2}T_{12}^2=S_2\, q_V{}^2\frac{p^3\sigma L^2}{2^5}\sin 2u\ d\cos\gamma$$

$$+S_3 q_V\,\frac{p^4\sigma^2 L^2}{2^2\,3^3}\sin^2 u\ \ d^2(1-\cos 2\gamma) \qquad \text{(G-9)}$$

$$a_{20}=\tfrac{1}{2}T_o^2+\frac{1}{q_V}T_{12}^2+\frac{1}{4p^2 q_V{}^2}T_3^1=S_2\{\frac{\sigma^2 L^2}{2^3}\ \sin^2 u\ d^2(1-\cos 2\gamma)$$

$$+\frac{\sigma p L^2}{2}\sin 2u p\cos\gamma\}+S_3\{\frac{p^2\sigma^2 L^2}{2^6}\ \sin^2 u\ \ d^2(1-\cos 2\gamma)$$

$$+\frac{\sigma p L^2}{2^4}\sin 2u\ d\cos\gamma\} \qquad \text{(G-10)}$$

$$a_{21}=\tfrac{1}{2}T_{12}^2=\ S_2 q_V\ \frac{\sigma p L^2}{2^5}\sin 2u\ d\cos\gamma+S_3 q_V\frac{p^2\sigma^2 L^2}{2^2\ 3^3}\sin u(1-\cos 2\gamma) \qquad \text{(G-11)}$$

and

$$a_{30}=\frac{1}{8p^2 q_V^2}\ T_3^2=S_3\frac{\sigma^2 L^2}{2^5 3^3}\ \sin^2 u\ d^2(1-\cos\gamma) \qquad \text{(G-12)}$$

where d and ν are function of τ .

$$d^2=\ A_t^2+B_1\tau\ +B_2\tau^2 \qquad \text{(G-13)}$$

$$\nu\ =B_3\tau \qquad \text{(G-14)}$$

where

$$B_1=\frac{kA_t V_t}{R}\ \sin(\phi_A+\phi_V) \qquad \text{(G-15)}$$

$$B_2=(\frac{kV_t}{2R})^2 \qquad \text{(G-16)}$$

$$B_3=\ \frac{kV_t}{2RA_t}\ \cos(\phi_A+\phi_V) \qquad \text{(G-17)}$$

We integrate A_o of Eq.(G-2) by using the identity

$$\delta(ax+b)=\ \frac{1}{|a|}\delta(x+\frac{b}{a}) \qquad \text{(G-18)}$$

and the approximation

$$d\doteq\ A_t(1-\frac{B_1}{2A_t^2}\ \tau) \qquad \text{(G-19)}$$

Result is

$$A_o=\sqrt{\pi}\ k^2\sqrt{\pi}L\ \left|\frac{2A_t}{B_1}\right| \exp\{-(\frac{pq_v}{2})^2 \cdot (\frac{2A_t}{B_1})^2\} \qquad (G-20)$$

Letting

$$S_m=\exp\{-\frac{L^2d^2}{4m}\}=\exp\{-\frac{L^2A_t^2}{4m} -\frac{L^2B_1}{4m}\tau-\delta_m^2\ \tau^2\} \qquad (G-21)$$

where

$$\delta_m^2=(\frac{L^2B_2}{4m}+\ \frac{p^2q_v^2}{4m}) \qquad (G-22)$$

and decomposing

$$2\cos[h(\gamma_A-v-\nu]=e^{jh(\gamma_A-v-\nu)}+e^{-jh(\gamma_A-v-\nu)} \qquad (G-23)$$

we have A_{10} which is the integration due to a_{10} of Eq.(G-3)

$$A_{10}=\frac{\sqrt{\pi}}{pq_v}k^2\{\frac{L^2\sigma^2}{2}\sin^2u[A_t^2(N_{010}+\frac{3}{8}N_{020})-\frac{A_t^2}{2}(N_{012}e^{j2(\gamma_A-v)}+*$$
$$+\frac{3}{8}N_{022}e^{j2(\gamma_A-v)}+*)+B_1(N_{110}+\frac{3}{8}N_{120})-\frac{B_1}{2}(N_{112}e^{j2(\gamma_A-v)}+*$$
$$+\frac{3}{8}N_{122}e^{j2(\gamma_A-v)}+*)+B_2(N_{210}+\frac{3}{8}N_{220})-\frac{B_2}{2}(N_{212}e^{j2(\gamma_A-v)}+*$$
$$+\frac{3}{8}N_{222}e^{j2(\gamma_A-v)}+*)]+\frac{\sigma pL}{2}\sin2u[A_t(N_{o11}e^{j(\gamma_A-v)}+*)$$
$$+\frac{B_1}{2A_t}(N_{111}e^{j(\gamma_A-v)}+*)]\} \qquad (G-24)$$

with

$$N_{nmh}=\int_{-\infty}^{\infty}\tau^n\exp\{-\delta_m^2\tau^2-(\frac{L^2B_1}{4m}-jhB_3)\tau\}d\tau \qquad (G-25)$$

and * stands for the complex conjugate.

The integral N_{omh}

$$N_{omh}=\ D_{mh}\cdot\frac{\sqrt{\pi}}{\delta_m} \qquad (G-26)$$

with

$$D_{mh}=\exp\{\frac{(\frac{L^2B_1}{4m}-jhB_3)^2}{4\delta_m^2}\} \quad ;\quad m=1,2,3 \quad h=1,2 \qquad (G-27)$$

and N_{nmh} can be evaluated by sucessive differentiations with respect $-\delta_m^2$ and $-(\frac{L^2B_1}{4m}-jhB_3)$.

The results are

$$N_{1mh}=\frac{\sqrt{\pi}}{2\,\delta_m^3}(\frac{L^2B_1}{4m}-jhB_3)D_{mh} \tag{G-28}$$

$$N_{2mh}=\frac{\sqrt{\pi}}{4\,\delta_m^3}\left[1+\frac{(\frac{L^2B_1}{4m}-jhB_3)^2}{2\,\delta_m^2}\right]D_{mh} \tag{G-29}$$

$$N_{3mh}=\frac{\sqrt{\pi}}{4\,\delta_m^5}(\frac{L^2B_1}{4m}-jhB_3)\left[3+\frac{(\frac{L^2B_1}{4m}-jhB_3)^2}{2\delta_m^2}\right]D_{mh} \tag{G-30}$$

$$N_{4mh}=\frac{\sqrt{\pi}}{4\,\delta_m^5}\left[3+\frac{6(\frac{L^2B_1}{4m}-jhB_3)^2}{\delta_m^2}+\frac{(\frac{L^2B_1}{4m}-jhB_3)^4}{\delta_m^2}\right]D_{mh} \tag{G-31}$$

However, by adding the corresponding complex conjugate terms, all expressions are real.

$$N_{omh}e^{jh(\gamma_A-v)}+N_{omh}^{*}\,e^{-jh(\gamma_A-v)}=2.\frac{\sqrt{\pi}}{m}\cos\Phi_m^h\bar{D}_{mh} \tag{G-32}$$

where

$$\Phi_m^h\triangleq h(\gamma_A-v-\frac{L^2B_1B_3}{8m\delta_m^2}) \tag{G-33}$$

$$\bar{D}_{mh}\triangleq\bar{D}_{mo}.\lambda_m^h \tag{G-34}$$

with

$$\bar{D}_{mo}=\exp\{\frac{L^4B_1^2}{4^3m^2\delta_m^2}\} \tag{G-35}$$

$$\lambda_m^h=\exp\{-\frac{h^2B_3^2}{4\,\delta_m^2}\} \tag{G-36}$$

Similary

$$N_{1mh}\, e^{jh(\gamma_A - v)} + * = -2\frac{\sqrt{\pi}}{2\delta_m^2}\left[\frac{L^2B_1}{4m}\cos\Phi_m^h + hB_3 \sin\Phi_m^h\right]\cdot\bar{D}_{mh} \qquad \text{(G-37)}$$

$$N_{2mh}e^{jh(\gamma_A - v)} + * = 2\frac{\sqrt{\pi}}{2\delta_m^3}\left[\left(1+\frac{\frac{B_1^2 L^4}{(4m)^2} - h^2B_3^2}{\delta_m^2\, 2}\right)\cos\Phi_m^h + \frac{hB_3\frac{L^2B_1}{4m}}{\delta_m^2}\sin\Phi_m^h\right]\cdot\bar{D}_{mh} \qquad \text{(G-38)}$$

$$N_{3mh}e^{jh(\gamma_A - v)} + * = -2\frac{\sqrt{\pi}}{4\delta_m^5}\left[\left\{3\frac{L^2B_1}{4m} + \frac{1}{2\delta_m^2}\left(\frac{L^6B_1^3}{(4m)^3} - \frac{L^2B_1}{4m}\, h^2B_3^2 - \frac{2h^2B_3^2L^2B_1}{4m}\right)\right\}\cos\Phi_m^h + \left\{3hB_3 + \frac{3hB_3L^4\gamma^2}{2\cdot(4m)^2\delta_m^2} - \frac{(hB_3)^3}{2\delta_m^2}\right\}\sin\Phi_m^h\right]\cdot\bar{D}_{mh} \qquad \text{(G-39)}$$

$$N_{4mh}e^{jh(\gamma_A - v)} + * = 2\frac{\sqrt{\pi}}{4\delta_m^5}\left[\left\{3+\frac{6\frac{L^4B_1^2}{(4m)^2} - 6h^2B_3^2}{\delta_m^2} + \frac{\left(\frac{L^2B_1}{4m}\right)^4 + (hB_3)^4 - 6\left(\frac{L^2B_1}{4m}\right)^2 h^2B_3^2}{4\delta_m^4}\right\}\cdot\cos\Phi_m^h + \left\{\frac{12\frac{L^2B_1}{4m}hB_3}{\delta_m^2} + \frac{4\left(\frac{L^2B_1}{4m}\right)^3 hB_3 - 4\frac{L^3B_1}{4m}(hB_3)^3}{4\delta_m^4}\right\}\sin\Phi_m^h\right]\cdot\bar{D}_{mh}$$

(G-40)

Subsituting Eqs.(G-26) through Eq.(G-40) into Eq.(G-24),

we have

$$A_{10} = \frac{\pi k^2}{\delta_1} d_{10}\cdot\left[\frac{L^2\sigma^2}{4}\sin^2 u\;(1-\cos\Phi_1^2\lambda_1^2)\left(A_t^2 - \frac{L^2B_1^2}{8\delta_1^2} + \frac{B_2}{2\delta_1^2} + \frac{L^2B^2B_2}{2^6\delta_1^4\, 4}\right)\right.$$

$$-\cos\Phi_1^2\Lambda_1^2\frac{B_2B_3}{2\delta_1^4}+\sin\Phi_1^2\Lambda_1^2\left(\frac{L^2B_1B_2B_3}{2^2\delta_1^4}-\frac{B_1B_3}{\delta_1^2}\right)\}+\frac{\sigma pL^2}{2}[\sin 2u$$

$$\cdot\{A_t\cos\Phi_1^1\Lambda_1^1-\frac{L^2B_1^2}{2^4A_t\delta_1^2}\cos\Phi_1^1\Lambda_1^1-\frac{B_1B_3}{2A_t\delta_1^2}\sin\Phi_1^1\Lambda_1^1\}]$$

$$+\frac{\pi k}{\delta_2}d_{20}\cdot\frac{3}{\lfloor 2^5}L^2\sigma^2\sin^2u\{(1-\cos\Phi_2^2\Lambda_2^2)\left(A_t^2-\frac{L^2B_1}{2^4\delta_2^2}+\frac{B_2}{2\delta_2^2}+\frac{L^4B^2B_2}{2^8\delta_2^2}\right)$$

$$-\frac{B_2B_3}{\delta_2^4}\cos\Phi_2^2\Lambda_2^2+\left(\frac{L^2B_1B_2B_3}{2^3\delta_2^4}-\frac{B_1B_3}{\delta_2^2}\right)\sin\Phi_2^2\Lambda_2^2\}] \qquad \text{(G-41)}$$

where

$$d_{mo}=\bar{D}_{mo}\ \cdot\exp\{-\frac{L^2A_t^2}{4m}\} \qquad \text{(G-42)}$$

similarly we have A_{1n}

$$A_{11}=\frac{\pi k^2}{\delta_2}d_{20}\Big[\frac{q_V}{2\delta_2^2}\cdot\frac{p^2\sigma^2L^2}{2^5}\sin^2u\{(1-\cos\Phi_2^2\Lambda_2^2)\left(\frac{A_t^2L^2B_1}{2^3}+B_1+\frac{L^4B^3_1}{2^7\delta_2^2}-\frac{L^2B\ B_2}{2^4\delta_2^2}-\frac{L^6B^3B}{2^{11}\delta_2^4}\right)$$

$$+\cos\Phi_2^2\Lambda_2^2\left(-\frac{2B_1B_3}{\delta_2^2}+\frac{3L^2B_1B_2B_3}{2\delta_2^2}\right)+\sin\Phi_2^2\Lambda_2^2\left(-2B_3A_t^2+\frac{L^2B_1^2B_3}{2^33\delta_2^2}+\frac{2B_2B_3^3}{\delta_2^4}-\frac{3B_2B_3}{\delta_2^2}-\frac{L^4B_1^2B_2B_3}{2^7\delta_2^4}\right)\}$$

$$+\frac{q_V}{2\delta_2^2}\cdot\frac{p^2\sigma L^2}{2^4}\sin 2u\{\cos\Phi_2^1\Lambda_2^1\left(\frac{B_1}{2A_t}+\frac{L^2B_1}{2^8\delta_2^2A}-\frac{B_1B_3}{2^2\delta_2^2A}-\frac{A_tL^2_1}{2^3}\right)$$

$$+\sin\Phi_2^1\Lambda_2^1\left(\frac{L^2B_1^2B_3}{2^4\delta_2^2A_t}-A_tB_3\right)\}\Big]+\frac{\pi k^2}{\delta_3}d_{30}\Big[\frac{q_Vp\sigma L^2}{2\delta_3^2 2^3}\sin^2u\{(1-\cos\Phi_3^2\Lambda_3^2)\left(-\frac{A_t^2L^2B_1}{2^2 3}\right)$$

$$+B_1+\frac{L^4B_1^3}{2^53^2}-\frac{L^2B_1B_2}{2^3\delta_3^2}-\frac{L^6B_1^3B_2}{2^83^2\delta_3^2}\Big)+\cos\Phi_3^2\Lambda_3^2\left(\frac{L^6B_1^3B_2B_2}{2\delta_3^2}-\frac{2B_1B_3^2}{\delta_3^2}\right)$$

$$+\sin\Phi_3^2\Lambda_3^2\cdot\left(-2B_3A_t^2+\frac{L^2B_1^2B_3}{2.3\delta_3^2}+\frac{2B_2B_3^2}{\delta_3^4}-\frac{2B_2B_3}{\delta_3^2}-\frac{L^4B_1^2B_2B_3}{2^53\delta_3^4}\right)\}\Big] \qquad \text{(G-43)}$$

$$A_{12}=\frac{\pi k^2}{\delta_1}d_{10}\Big[\frac{q_V^2}{2\delta_1^2}\cdot\frac{p^2\sigma^2}{2}\cos^2u\left(1+\frac{L^4B_1}{2^5\delta_1^2}\right)\Big]+\frac{\pi k^2}{pq_V\delta_2}d_{20}\Big[\frac{q_V^2}{2\delta_2^2}\cdot\frac{3p^2\sigma^2}{2^4}\sin^2u\left(1+\frac{L^4B_1^2}{2^7\delta_2^2}\right)$$

$$+\frac{q_V^2}{2\delta_2^2}\cdot\frac{3p^2\sigma^2L^2}{2^7}\sin^2u\{(1-\cos\Phi_2^2\Lambda_2^2)(A_t^2+\frac{A_t^2L^4B_1^2}{2\delta_2^2}-\frac{3L^2B^2}{2^4\delta_2^2}-\frac{L^6B_1^4}{2^{1}\delta_2^4}$$

$$+\frac{3B_2}{2\delta_2^2}+\frac{3L^4B_1^2B_2}{2^6\delta_2^4}+\frac{L^8B_1^4B_2}{2^{15}\delta_2^6})+\cos\Phi_2^2\Lambda_2^2\cdot(-\frac{2A_t^2B_3^2}{\delta_2^2}+\frac{L^2B_1^2B_3^2}{2^3\delta_2^4}+\frac{B_1^2L^2B_3^2}{2^2\delta_2^2}-\frac{12B_1B_3^2}{\delta_2^4}$$

$$+\frac{2B_2B_3^4}{\delta_2^6}-\frac{3L^2B_1^2B_2B_3^2}{2^3\delta_2^6})+\sin\Phi_2^2\Lambda_2^2\cdot(\frac{L^2A_t^2B_1B_3}{2^2\delta_2^2}-\frac{3B_2B_3}{\delta_2^2}-\frac{L^4B_1^3B_3}{2^7\delta_2^4}+\frac{2B_1B_3^3}{\delta_2^4}+\frac{3L^2B_1B_2B_3}{2\delta_2^4}$$

$$+\frac{L^6B_1^3B_2B_3}{2^9\delta_2^6}-\frac{L^2B_1B_2B_3^3}{2\delta_2^6})\}+\frac{q_V^2}{2\delta_2^2}\cdot\frac{p^3\sigma L^2}{2^5}\sin 2u\{\cos\Phi_2^1\Lambda_1^1\cdot(A_t^2+\frac{L^4B_1^2A_t^2}{2^7\delta_2^2}-\frac{2A_t^2B_3^2}{\delta_2^2}$$

$$-\frac{3L^2B_1^2}{2^5A_t\delta_2^5}-\frac{L^6B_1^4}{2^9A_t\delta_2^4}+\frac{L^2B_1^2B_3^2}{2^4A_t\delta_2^4}+\frac{L^2B_1^2B_3^2}{2^3A_t\delta_2^4})+\sin\Phi_2^1\Lambda_2^1(\frac{A_t^2L^2B_1B_3}{2^3\delta_2^2}-\frac{3B_1B_2}{2A_t\delta_2^2}$$

$$-\frac{L^4B_1^3B_3}{2^8A_t\delta_2^4}+\frac{B_1B_3^3}{A_t\delta_2^4})\Big]$$

$$+\frac{\pi k^2}{\delta_3}d_{30}\cdot\frac{q_V^2}{2\delta_3^2}\cdot\frac{p^4\sigma^2L^2}{3^3}\sin^2u\ (1-\cos\Phi_3^2\Lambda_3^2)(A^2+\frac{L^2A_t^2B_1^2}{2^5 3^2\delta_3^2}-\frac{L^2B_1^2}{2^3\delta_3^2}-\frac{L^2B_1^4}{2^3 3^3\delta_3^4}$$

$$+\frac{3B_2}{2\delta_3^2}+\frac{L^4B_1^2B_2}{2^6\cdot 3\delta_3^4}+\frac{L^8B_1^4B_2}{2^{11}3^4\delta_3^6})+\cos\Phi_3^2\Lambda_3^2\cdot(-\frac{2A_t^2B_3^2}{\delta_3^2}+\frac{L^2B_1^2B_3^2}{2^2\cdot 3\delta_3^4}+\frac{L^2B_1^2B_3^2}{2\cdot 3\delta_3^2}-\frac{12B_2B_3^2}{\delta_3^4}$$

$$+\frac{2B_2B_3^4}{\delta_3^6})+\sin\Phi_3^2\Lambda_3^2\cdot(\frac{L^2A_t^2B_1B_3}{2\cdot 3\delta_3^2}-\frac{3B_1B_3}{\delta_3^2}-\frac{L^4B_1^3B_3}{2^6 3\delta_3^4}+\frac{2B_1B_3^3}{\delta_3^4}+\frac{L^2B_1B_2B_3}{\delta_3^4}$$

$$+\frac{L^6B_1^3B_2B_3}{2^6 3^3\delta_3^6}-\frac{L^2B_1B_2B_3^3}{3\delta_3^6})\} \qquad \text{(G-44)}$$

Therefore $A=A_{10}+A_{11}+A_{12}$ and using normalized parameter, we have

$$A_1 = \sum_{n=1}^{3} G_n (\ell^{-1} A_{1,-1}^n + \ell A_{1,1}^n + \ell^3 A_{1,3}^n) \tag{G-45}$$

Similarly

$$A_2 = G_2 (\ell^{-1} A_{2,-1} + \ell A_{2,1} + \ell^3 A_{2,3}) \tag{G-46}$$

$$A_3 = G_3 (\ell^{-1} A_{3,1} + \ell A_{3,1} + \ell^3 A_{3,3}) \tag{G-47}$$

with

$$A_{1,-1}^1 = -\cos\bar{\Phi}_1^2 \Lambda_1^2 \frac{\sin^2 u \sin^4\alpha\, b_1^2}{2^6 \bar{A}_t^2 \bar{\delta}_1^4} \tag{G-48}$$

$$A_{1,1}^1 = \frac{\cos^2 u \cos^2\alpha}{2^3 \bar{\delta}_1^2} + \frac{\sin^2 u}{2^2}\left[(1-\cos\bar{\Phi}_1^2\Lambda_1^2)\frac{\sin^2\alpha\, b_1^2}{2^3 \bar{\delta}_1^2} + \sin\bar{\Phi}_1^2\Lambda_1^2 \frac{\sin^4\alpha\, a_1 b_1^2}{2^5 \sigma_u^2 \bar{\delta}_1^4}\right] \tag{G-49}$$

$$A_{1,3}^1 = \frac{\cos^2 u \cos^2\alpha \sin^2\alpha\, \bar{A}_t^2 a_1^2}{2^8 \sigma_u^4 \bar{\delta}_1^4} + \frac{\sin^2 u}{2^2}\left[(1-\cos\bar{\Phi}_1^2\Lambda_1^2)\cdot\left(\bar{A}_t^2 - \frac{\bar{A}_t^2 \sin^2\alpha\, a_1^2}{2^3 \sigma_u^2 \bar{\delta}_1^2} + \frac{\bar{A}_t^2 \sin^4\alpha\, a_1^2 b_1}{2^8 \sigma_u^4 \bar{\delta}_1^4}\right) - \sin\bar{\Phi}_1^2\Lambda_1^2 \frac{\sin^2\alpha\, a_1 b_1}{2\bar{\delta}_1^2}\right] \tag{G-50}$$

$$A_{1,-1}^2 = \cos\bar{\Phi}_2^2\Lambda_2^2 \frac{3\sin^2 u \sin^4\alpha\, b_1^3}{2^5 \bar{\delta}_2^4}\left(\frac{1}{\bar{A}_t^2} - \frac{\cos^2\alpha}{2^5 \bar{A}_t^2 \bar{\delta}_2^2} - \frac{3\cos^2\alpha \sin^2\alpha\, a_1^2}{2^{10}\sigma_u^2 \bar{\delta}_2^4}\right) + \cos\bar{\Phi}_2^1 \Lambda_2^1 \frac{\sin 2u \cos\alpha \sin^3\alpha\, a_1 b_1^2}{2^9 \bar{A}_t^2 \bar{\delta}_2^4}$$
$$+ \sin\bar{\Phi}_2^2\Lambda_2^2 \frac{\sin^2 u \sin^4\alpha\, b_1^3}{2^7 \bar{\delta}_2^6}\left(\frac{\cos u \cos\alpha \sin\alpha}{2^2 \bar{A}_t^2} + \frac{3\cos^2\alpha\, a_1}{\bar{A}_t^2} + \frac{3\cos^2\alpha \sin^2\alpha\, a_1 b_1}{2^7 \sigma_u^2 \bar{\delta}_2^2}\right) \tag{G-51}$$

$$A_{1,1}^2 = (1-\cos\bar{\Phi}_2^2\Lambda_2^2)\frac{\sin^2 u}{2\bar{\delta}_2^2}\left(\frac{3\sigma_u^2\cos^2\alpha}{1} + \frac{3\sin^2\alpha\, b_1\cos^2\alpha}{2^3\, \bar{\delta}_2^2} - \frac{3\sin^2\alpha\, b_1}{2^7}\right) + \cos\bar{\Phi}_2^1\Lambda_2^1 \frac{\sin 2u \sin\alpha \cos\alpha\, a_1}{2^6 \bar{A}_t \bar{\delta}_2^2}$$
$$+ \cos\bar{\Phi}_2^2\Lambda_2^2 \frac{3\sin^2 u\, b_1^2}{2^8 \bar{\delta}_2^4}\left(-\frac{\cos^2 u \sin^2\alpha}{\bar{A}_t} - \frac{3\cos^2\alpha \sin^2\alpha}{2} - \frac{3\cos^2\alpha \sin^4\alpha\, a_1^2}{2^5 \sigma_u^2 \bar{\delta}_2^2}\right) + \sin\bar{\Phi}_2^2\Lambda_2^2 \frac{3\sin^2 u \sin^4\alpha\, a_1 b_1^2}{2^9 \sigma_u^2 \bar{\delta}_2^4} \tag{G-52}$$

$$A_{1,3}^2 = (1-\cos\bar{\Phi}_2^2\Lambda_2^2)\frac{\sin^2 u \cdot 3}{2^5 \bar{\delta}_2^2}\left(\frac{\bar{A}_t^2 \sin^2\alpha \cos^2\alpha\, a_1^2}{2^3 \bar{\delta}_2^2} + \bar{A}_t^2 + \cos u\, \bar{A}_t \cos\alpha\, a_1\right) + \cos\bar{\Phi}_2^2\Lambda_2^2 \frac{3\sin^2 u \sin^4\alpha\, a_1 b_1^2}{2^8 \sigma_u^2 \bar{\delta}_2^2} \tag{G-53}$$
$$+ \sin\bar{\Phi}_2^2\Lambda_2^2 \frac{\cos u \sin^2 u}{2^5 \bar{\delta}_2^2}\left(-\frac{\bar{A}_t \cos\alpha \sin^5\alpha\, b_1^2 a_1^2}{2^{10}\sigma_u^4 \bar{\delta}_2^4} - \frac{\bar{A}_t \cos\alpha \sin\alpha\, b_1}{1} + \frac{\bar{A}_t \cos\alpha \sin^3\alpha\, a_1^2 b_1}{2^2 \sigma_u^2 \bar{\delta}_2^2}\right)$$

$$A_{1,-1}^3 = \cos\bar{\Phi}_3^2\Lambda_3^2 \frac{\cos^2 u \sin^2 u \cos^2\alpha \sin^6\alpha\, b_1^5}{2^6\, 3^3\, \bar{A}_t^4 \bar{\delta}_3^2} + \sin\bar{\Phi}_3^2\Lambda_3^2 \frac{\cos u \sin^2 u \cos\alpha \sin^5\alpha\, b_1^4}{2^4\, 3^2\, \bar{A}_t} \tag{G-54}$$

$$A_{1,1}^3 = \cos\bar{\Phi}_3^2\Lambda_3^2 \frac{\cos^2 u \sin^2 u \sin^2\alpha\, b_1^2}{3^2 \bar{\delta}_3^4}\left(\frac{\cos\alpha}{2^4 \bar{A}_t} + \frac{\cos^2\alpha \sin^2\alpha\, a_1 b_1}{\bar{A}_t^2 \bar{\delta}_3^2} - \frac{\cos^2\alpha \sin^4\alpha\, a_1 b_1^2}{2^3\, 3^4\, \bar{\delta}_3^4}\right)$$
$$+ \sin\bar{\Phi}_3^2\Lambda_3^2 \frac{\cos^2 u \sin^2 u\, b_1^2}{3^2 \bar{\delta}_3^4}\left(-\frac{\cos\alpha \sin^2\alpha}{2^4 \bar{A}_t} + \frac{2\cos^2\alpha \sin^4\alpha\, a_1 b_1}{\bar{A}_t^6 \bar{\delta}_3^2} - \frac{\cos^2\alpha \sin^6\alpha\, a_1 b_1^2}{2^4\, 3^2 \sigma_u^2 \bar{\delta}_3^4}\right) \tag{G-55}$$

$$A_{1,3}^3 = (1-\cos\bar{\Phi}_3^2\Lambda_3^2)\frac{\cos u \sin^2 u}{3^3\, 2\, \bar{\delta}_3^2}\left(\sin\alpha \cos\alpha\, \bar{A}_t^3 a_1 - \frac{\bar{A}_t \cos\alpha \sin\alpha\, a_1 b_1}{2^5 \cdot 3\sigma_u^2 \bar{\delta}_3^2}\right) + \cos\bar{\Phi}_3^2\Lambda_3^2 \frac{\cos^2 u \sin^2 u \cos\alpha \sin\alpha\, a_1 b_1^2}{2^6\, 3^3 \sigma_u^2 \bar{\delta}_3^2} \tag{G-56}$$

$$A_{2,-1} = \frac{\sin^2 u}{2^3 \bar{\delta}_1^2}\left\{\frac{\sin^2\alpha}{2^2}(1-\cos\bar{\Phi}_1^2\Lambda_1^2) - \sin\bar{\Phi}_1^2\Lambda_1^2\, 2\sin^2\alpha\, a_1 b_1 - \cos\bar{\Phi}_1^3\Lambda_1^2 \frac{\sin^4\alpha\, a_1^2}{2^4 \bar{\delta}_1^2 \bar{A}_t^2}\right\} = A_{3,-1}(\delta_1 = \delta_3) \tag{G-57}$$

$$A_{2,1} = \frac{\sin^2\alpha\, a_1 b_1}{\bar{\delta}_1^2}\left\{ \sin\Phi_1^1 \Lambda_1^1 \frac{\cos^2 u \sin u}{2\bar{A}_t} - \frac{\sin^2 u \sin^2\alpha}{2^{10}\sigma_u^2 \bar{\delta}_1^2} \right\} \tag{G-58}$$

$$A_{2,3} = (1-\cos\Phi_1^2\Lambda_1^2)\frac{\sin^2 u\, \bar{A}_t^2}{2^2}\left(1+\frac{\sin^2\alpha\, b_1^2}{2^3\sigma_u^2\bar{\delta}_1^2}\right) + \cos^2 u \sin u\left(\bar{A}_t + \cos\Phi_1^1\Lambda_1^1 \frac{\bar{A}_t \sin^2\alpha\, b_1^2}{2^3\sigma_u^2\bar{\delta}_1^2}\right) \tag{G-59}$$

$$A_{3,1} = -\frac{\sin^2 u \sin^4\alpha\, a_1 b_1}{2^{15}\, 3^3 \sigma_u^2 \bar{\delta}_3^4} \tag{G-60}$$

$$A_{3,3} = (1-\cos\Phi_3^2\Lambda_3^2)\frac{\sin^2 u\, \bar{A}_t^2}{2^5\, 3^3}\left(1+\frac{\sin^2\alpha\, b_1^2}{2^3\, 3\,\sigma_u^2\bar{\delta}_3^2}\right) + \cos^2 u \sin u\, A_t (1+\cos\Phi_3^1\Lambda_3^1)\frac{\sin^2\alpha\, b_1^2}{2^3\sigma_u^2\bar{\delta}_3^3} \tag{G-61}$$

and

$$G_0 = \exp\left\{-\ell^2\left[\cos^2 u + \cot^2\alpha \sin^2 u + \cot\alpha \sin 2u \cos(\beta - \upsilon)\right]\right\} \tag{G-62}$$

$$G_m = \exp\left\{-\ell^2 \bar{A}_t^2\left(\frac{2^4 m \sigma_u^2 \bar{\delta}_m^2 - \sin^2\alpha\, a_1^2}{2^6 m^2 \sigma_u^4 \bar{\delta}_m^2}\right)\right\} \tag{G-63}$$

$$a_1 = \cos(\varphi_A - \beta) \tag{G-64}$$

$$b_1 = \sin(\varphi_A - \beta) \tag{G-65}$$

$$\bar{A}_t^2 = 4\left[(\tan\alpha \cos u)^2 + \sin^2 u + \tan\alpha \sin 2u \cos(\beta - \upsilon)\right] \tag{G-66}$$

$$\bar{\delta}_m^2 = \left(\frac{\sin^2\alpha}{4m\sigma_u^2} + \frac{\cos^2\alpha}{4}\right) \tag{G-67}$$

$$\Phi_m^h = h\left(\varphi_A - \upsilon - \frac{\sin^2\alpha\, a_1 b_1}{2^4 m \sigma_u^2 \bar{\delta}_m^2}\right) \tag{G-68}$$

$$\Lambda_m^h = \exp\left\{-\frac{h^2 \sin^2\alpha\, b_1^2}{4^2 \ell^2 \bar{A}_t^2 \bar{\delta}_m^2}\right\} \tag{G-69}$$

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